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SWITCHES AND CROSSINGS.

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FORMULÆ

FOR ASCERTAINING

THE ANGLES OF CROSSINGS, THE LENGTHS OF SWITCHES, AND THE DISTANCES OF THE POINTS OF THE CROSSINGS AND THE HEELS OF THE SWITCHES FROM THE SPRINGING OF THE CURVE.

BY

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ERRATA.

Pages 222, 223, and 224. The denominators in the formulæ which give the positions of the heels of the switches and the points of the fast points ought not to be $\sqrt{4g - d}$ or $\sqrt{4s - d}$, but $\sqrt{d}$ in each case.

Page 256. The multipliers are in each case $1\frac{1}{8}$, $1\frac{1}{10}$ and $\frac{1}{4}$, and not $\frac{3}{8}$, $\frac{1}{7}$ and $\frac{1}{3}$.

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INTRODUCTION.

In the following investigations, a symbol is used to denote every element, so that the formulæ will be applicable to every gauge, and to every section of rail.

The formulæ are based on the assumption that a sufficient degree of approximation to exactness is obtained by making the sine of the angle of the crossing equal to its circular measure. They have been still further simplified by leaving out all squares, products, and terms of higher dimensions, of the spaces between the centre lines of contiguous rails, when these are not multiplied by factors greater than unity.

All the ordinary cases of junctions, three throws, cross-over and through roads, have been discussed, so that no one who has mastered the method adopted will have any difficulty in making out similar formulæ for any special cases which may arise.

SWITCHES AND CROSSINGS.

CHAPTER I.

EXPLANATION OF THE SYMBOLS.

In order to obtain a system of symbols, which show at once the particular crossing to which the formula refers, the rails of the branch and main lines are numbered in order. Those rails which touch each other have the same number. Thus the 1st, 2nd, 3rd, &c., rails of the main line touch the 1st, 2nd, 3rd, &c., rails of the branch. The rails of the branch line give the order of numbering; that rail which is most remote from the centre of the branch curve being number one.

Therefore, in a single-line junction of a single-gauge road, the 1st rail of the branch line will cross the 2nd rail of the main line; of a double-gauge road, in which one rail is common to both gauges, the 1st rail of the branch line will cross the 2nd and 3rd rails of the main line, and the 2nd rail of the branch line the 3rd rail of the main line.

In a double-line junction of a single-gauge road, the 1st rail of the branch line will cross the 2nd, 3rd, and 4th rails of the main line; the 2nd rail of the branch, the 3rd and 4th of the main line, and the 3rd rail of the branch the 4th rail of the main line; of a double-gauge road, in which one rail is common to both gauges, the 1st rail of the branch will cross the 2nd, 3rd, 4th, 5th, and 6th rails of the main line; the 2nd rail of the branch, the 3rd, 4th, 5th, and 6th rails of the main line; the 3rd rail of the branch the 4th, 5th, and 6th rails of the main line; the 4th rail of the branch, the 5th and 6th rails of the main line; and the 5th rail of the branch, the 6th rail of the main line.

The symbol c_n^m is used to designate the crossing of the m^{th}

rail of the main line by the n^{th} rail of the branch, the suffix n referring to the number of the branch rail, and the power m to the number of the main-line rail. Thus, the symbols denoting the crossings of the main-line rails by the branch rails, will be:

Single-gauge Roads.

Case I.—Single-line junction, c_1^3.

Case II.—Double-line junction, $c_1^3\ c_1^3\ c_1^4$, $c_2^3\ c_2^4$, c_3^4.

Double-gauge Roads, one Rail common to both Roads.

Case I.—Single-line junctions, $c_1^3\ c_1^3$, c_2^3.

Case II.—Double-line junctions, $c_1^3\ c_1^3\ c_1^4\ c_1^5\ c_1^6$, $c_2^3\ c_2^4\ c_2^5\ c_2^6$, $c_3^4\ c_3^5\ c_3^6$, $c_4^5\ c_4^6$, c_5^6.

Although in a single-line junction there cannot be more than two or three, and in a double-line junction more than four or six, branch-road rails, since the branch-road rails after leaving the main-road rails which they touch, may cross other roads parallel to the first, there may be any number of main-road rails. These are numbered in sequence after those in contact with the branch-road rails.

The symbol c_n^m is likewise used to designate the circular measure of the angle of the crossing of the m^{th} main-line rail by the n^{th} branch-line rail, and also its position on any plan of the roads.

The points, where the centre lines of the 1st, 2nd, 3rd, &c., rails of the branch road touch the centre lines of the 1st, 2nd, 3rd, &c., rails of the main line at the springing of the curve, are designated by the symbols S_1, S_2, S_3, &c., and the positions of the heels of the switches of the 1st, 2nd, 3rd, &c., main and branch rails by the symbols H_1, H_2, H_3, &c.

The points, where normals through the springing of the branch curve and through the heels of the switches cut any main-line rails parallel to those, which the branch rails meet at a common tangent, are likewise designated by the symbols S_m, &c., H_m, &c., respectively, the suffix m referring to the number of the main-line rail.

Symbols which have the same Meaning for both Single and Double-gauge Roads.

R = radius of main-line curve.
r = radius of branch-line curve.
s = interval between the centres of the contiguous inner rails of a double line, commonly called the six-foot.
t = thickness of upper flange of rails.
c = clearance between the rails at the heels of the switches.

Special Symbols for Single-gauge Roads.

g = interval between the centres of the rails, which regulates the gauge, hereafter called the gauge.

Special Symbols for Double-gauge Roads, in which one Rail is common to both Roads.

g = interval between the centres of the rails, which regulates the gauge of the wider road, hereafter called the gauge.
d = interval between the centres of the two rails which are not common to both roads.

Other symbols occur, which refer to special cases, and are explained in the sections, which discuss those special cases.

CHAPTER II.

SWITCHES.

SECTION I.—LENGTH OF SWITCHES AND POSITION OF HEELS OF SWITCHES.

IT is in practice impossible to use switches of the length theoretically required to make both roads perfect, unless the curve of the branch line is very sharp, or is leaving a main-line road which curves in the opposite direction. A limit of deviation from correct curvature must therefore be fixed upon, and the condition which determines the minimum length of switch is simply that this amount of deviation is not to be exceeded.

Whatever length of switch may be adopted, the heel of the switch must be in its proper position; that is, at the point where

the distance between the two rails, termed the clearance, is equal to the arbitrary amount fixed upon by the engineer. Consequently, at this point the distance between the centres of the two rails must be equal to this clearance, plus the thickness of the top flange of the rail; and the distance of the heel of the switch from the springing of the curve is determined by the radii of the curve, and the magnitude of these two quantities. The formulæ, then, will give not only the positions and circular measures of the angles of the crossings, but also the distances of the heels of the switches from the springing. The distance of the point of the switch from the correct springing of the curve will be equal to the excess of the distance of the heel of the switch from the springing over the actual length of switch adopted.

Fig. 1 shows the position of the switches in the case of a branch road leaving a straight main-line road, where the switches are theoretically of correct length. Even in this case the point of the switch is at some little distance from the exact springing of the curve. For strength's sake the fine end of the switch is necessarily made greater than the indent in the stock rail usually allows; it will therefore project a little in front of the rail. The greater the amount of projection which may safely be allowed, the less will be the theoretically correct length of the switch required to make both roads perfect. This projection is rounded off towards the point of the switch, so as to appear almost flush at its upper edge, with the face of the stock rail at the indent. Let this projection be denoted by the letter *e*.

Fig. 2 shows the position of the switches when they are too short, in the case of a curve leaving a straight line. The hard lines show the actual positions of the faces of the rails; the dotted lines the correct positions. The heels of the switches H_1 H_2 are in the same position in both figures, and after passing the heels, both roads are laid to correct lines. The whole of the deviation in this case must of necessity take place in the curved branch line, and will be equal to the tangential offset due to the distance of the point of the switch from the exact springing of the curve, less the quantity, *e*, already ex-

plained. If this deviation, therefore, be denoted by e', the tangential offset at the point of the switch will be equal to $e + e'$.

When one curve leaves another, whether the curvature be in the same or in opposite directions, this deviation may be divided between both roads. The difference, therefore, between the theoretically correct length and that actually adopted, may be much greater, without injuring the roads to a greater extent than in the case referred to (Fig. 2), where the deviation takes place wholly on one road.

From the preceding remarks, we see:—

That the distance from the springing to the heel of the switch is equal, in the case of a curve leaving a straight line, to the length of the tangent, of which the offset is $c + t$; in the case of a curve leaving a curve, curvature in the same direction, to the length of the tangent, of which the difference between the offsets is equal to $c + t$; in the case of a curve leaving a curve, curvature in the opposite direction, to the length of the tangent, of which the sum of the offsets is equal to $c + t$.

Also, that the theoretically correct length of the switch is equal to this distance, less the length of the tangent, whose offset is equal to e, in the case of a curve leaving a straight line; less the length of the tangent, of which the difference between the offsets is equal to e, in the case of a curve leaving a curve, curvature in the same direction; and less the length of the tangent, of which the sum of the offsets is equal to e, in the case of a curve leaving a curve, curvature in opposite directions.

Similarly, that the minimum length is equal to this distance, less the length of a tangent of which the offset, the difference between the offsets, or the sum of the offsets, is equal to $e + e'$.

The radii R, r, are those of the staked-out centre lines. In the case of a single line, these may either be in the centres of the road, or in the same positions as those of a double line, which are always in the middle of the six-foot. In the following investigations the exact radii of the curves of each rail are necessarily used, but the results are simplified by leaving out all products of s, g, c, t, which do not contain other factors of such magnitude as to make the term of importance, and all powers of these quantities higher than the first.

Distance of the Heels of the Switches from the Springing.

a. Curve leaving a Straight Line.

The radius of the curve of the first rail of the branch line is $r + h$, where h cannot exceed $g + \frac{s}{2}$; consequently we get

$$S_1 H_1 = \sqrt{2 (r + h) (c + t)},$$

or neglecting $h (c + t)$,

$$S_1 H_1 = S_2 H_2 = S_3 H_3 = \&c. = \sqrt{2 r (c + t)}.$$

β. Curve leaving Curve, Curvature in the same Direction.

The radii of the curves of the first rails of the main and branch lines are $R + h$ and $r + h$ respectively; therefore we have for determining $S_1 H_1$ the equation

$$c + t = \frac{S_1 H_1^2}{2 (r + h)} - \frac{S_1 H_1^2}{2 (R + h)},$$

whence

$$S_1 H_1 = \sqrt{\frac{2 R r}{R - r} (c + t) + \frac{2 (R + r) h + 2 h^2}{R - r} (c + t)}.$$

The quantity $\frac{2 (R + r) h}{R - r} (c + t)$ is always so small as not to affect the result appreciably; hence, neglecting this and $\frac{2 h^2 (c + t)}{R - r}$, we get

$$S_1 H_1 = S_2 H_2 = S_3 H_3 = \&c. = \sqrt{\frac{2 R r (c + t)}{R - r}}.$$

γ. Curve leaving Curve, Curvature in opposite Directions.

The radii of the curves of the first rail of the main line and of the branch line are $R - h$ and $r + h$ respectively; therefore we have for determining $S_1 H_1$ the equation

$$c + t = \frac{S_1 H_1^2}{2 (R - h)} + \frac{S_1 H_1^2}{2 (r + h)},$$

whence

$$S_1 H_1 = \sqrt{\frac{2 R r}{R + r} (c + t) - \frac{2 (R - r) h + h^2}{R + r} (c + t)}.$$

The quantity $\frac{2\,(R-r)\,h}{R+r}\,(c+t)$ is always so small as not to affect the result appreciably: hence, neglecting this and the quantity $\frac{h^2\,(c+t)}{R+r}$, we get

$$S_1 H_1 = S_2 H_2 = S_3 H_3 = \&c. = \sqrt{\frac{2Rr}{R+r}\,(c+t)}.$$

Lengths of the Switches.

Following the method adopted for ascertaining the distance of the heel of the switch from the springing, we get

α. Curve leaving Straight Line.

Maximum length of switch = $\sqrt{2r}\,(\sqrt{c+t}-\sqrt{e})$.

Minimum length of switch = $\sqrt{2r}\,(\sqrt{c+t}-\sqrt{e+e'})$.

β. Curve leaving Curve, Curvature in the same Direction.

Maximum length of switch = $\sqrt{\frac{2Rr}{R-r}}\,(\sqrt{c+t}-\sqrt{e})$.

Minimum length of switch = $\sqrt{\frac{2Rr}{R-r}}\,(\sqrt{c+t}-\sqrt{e+e'})$.

γ. Curve leaving Curve, Curvature in opposite Directions.

Maximum length of switch = $\sqrt{\frac{2Rr}{R+r}}\,(\sqrt{c+t}-\sqrt{e})$.

Minimum length of switch = $\sqrt{\frac{2Rr}{R+r}}\,(\sqrt{c+t}-\sqrt{e+e'})$.

SECTION II.—SPLAY OF THE SWITCHES.

Switches of the ordinary type in use are made from the rails used in the permanent way. In the case of bridge rails of the Great Western type, the face which rests against the stock rail is the only one dressed off. If the rails be double-headed, or of the Vignoles section, the upper flange not in contact with the stock rail must be dressed off, until it is flush with the web at the point of the switch, the dressing-off commencing at the heel of the switch. We have then only to ascertain at what

distance from the heel of the switch the splaying-off of the face, which rests against the stock rail ought to commence.

This distance is readily determined when the switches are of the correct length, since the splaying-off must commence at a point, whose distance from the springing is such, that the tangential offset, the difference between the tangential offsets, or the sum of the tangential offsets, respectively, is equal to the thickness of the top of the rail at that point, according to which of the three cases it refers, *viz.* curve leaving straight line; curve leaving curve, curvature in the same direction; curve leaving curve, curvature in opposite directions.

From this we see that the ratio of the splay to the whole length of the switch will not be the same in the case of rails with top flanges, as in that of rails of the Great Western type, since in the latter case the thickness of the top of the rail at the commencement of the splay will be equal to its full section width, but in the former this will be diminished by the splaying-off of the flange of the rail not in contact with the stock rail.

When the switch is of the maximum, or theoretically correct, length, therefore the length of the splayed portion will be, when one face only is splayed,

$$C\,(\sqrt{t} - \sqrt{e}),$$

where the factor C has the value appropriate to one of the three cases already described. Now, since C appears also as a simple factor in the expression for the full length of the switch, in each of the three cases, whatever may be the radii of the curves, the ratio

$$\frac{\text{length of splayed part}}{\text{whole length of switch}} = \frac{\sqrt{t} - \sqrt{e}}{\sqrt{c+t} - \sqrt{e}}.$$

In the case of rails with top flanges this ratio assumes a more complicated form. If w be the width of one flange, x the distance from the heel of the switch, at which the splay commences, and w' the diminution in width of the flange at distance x from the heel, owing to the splaying-off of the flange not in contact with the stock rail, we have for determining x and w' the equations

$$\frac{w}{w'} = \frac{C(\sqrt{c+t} - \sqrt{e})}{x}$$

$$C\sqrt{c+t} - C\sqrt{t-w'} = x,$$

whence

$$w' = w\,\frac{\sqrt{c+t} - \sqrt{t-w'}}{\sqrt{c+t} - \sqrt{e}} \qquad = w\,\frac{\sqrt{c+t} - \sqrt{t}}{\sqrt{c+t} - \sqrt{e}}$$

if we neglect w' in the expression $\sqrt{t-w'}$—an omission which cannot appreciably affect the final result. Hence the length of the splayed portion of the switch is

$$C(\sqrt{t-w'} - \sqrt{e}) =$$

$$C\,\frac{\sqrt{t(\sqrt{c+t}-\sqrt{e}) - w(\sqrt{c+t}-\sqrt{t}) - \sqrt{e}\sqrt{\sqrt{c+t}-\sqrt{e}}}}{\sqrt{\sqrt{c+t}-\sqrt{e}}}$$

and the ratio

$$\frac{\text{length of splayed part}}{\text{whole length of switch}} =$$

$$\frac{\sqrt{t(\sqrt{c+t}-\sqrt{e}) - w(\sqrt{c+t}-\sqrt{t}) - \sqrt{e}\sqrt{\sqrt{c+t}-\sqrt{e}}}}{(\sqrt{c+t}-\sqrt{e})^{\frac{1}{2}}}$$

The width of the point of the switch is equal to the quantity e plus the depth of the indent in the stock rail.

If the flanges of the rails be very narrow, the values of the two ratios will not be appreciably different; but if the flanges of the rails be made as wide as some engineers now recommend, the difference in the values will be of material consequence.

When the switches are not of the maximum length theoretically required to make the roads perfect, we may arrive at a value of the ratio of the splayed part to the whole length of the switch by assuming that the switch and the length of stock rail in contact with the switch are practically straight. On this assumption, if E be the thickness of the end of the switch, in the case of rails with no top flanges, we have the ratio

$$\frac{\text{length of splayed part}}{\text{whole length of switch}} = \frac{t - E}{c + t - E} \qquad = \frac{t}{c+t}$$

within the prescribed limits of exactness. And in the case of rails with top flanges,

$$\frac{\text{length of splayed part}}{\text{whole length of switch}} = \frac{t - w' - E}{c + t - E}$$

$$= \frac{t - w'}{c + t} \text{ nearly}$$

$$= \frac{t(\sqrt{c+t} - \sqrt{e}) - w(\sqrt{c+t} - \sqrt{t})}{(c+t)(\sqrt{c+t} - \sqrt{e})}.$$

These ratios are less than those first given. If the ratio of the splayed part to the whole length of the switch be less than either of these, the switches will not close freely at the point, and will be bent towards the middle of the switch in a direction opposite to that of the curve, if force be used. In no case, therefore, can these ratios be less than those last given. If they are greater when the points are closed, the switches, if straight on the inner edge, will not touch the stock rails at the points where the splaying-off commences. Now one switch and one stock rail belong to each road; consequently, if the main line be straight, the switch and stock rail belonging to the main line must be straight too. In order, therefore, that the switches and stock rails may be in contact with each other throughout the whole length of the splayed parts, the switch and the stock rail of the branch road must be bent, the switch being concave and the stock rail convex towards the inner edge. The switches and stock rails will thus consist of a short piece of straight towards the heel meeting tangentially the curves of the branch rails at the heels of the switches, and a short curve sharper than that of the branch meeting tangentially the main-line rails at the points. If both roads be curved, this bending of the switch and stock rail may be divided between each pair.

In all cases, then, it is best to use the larger ratio first given.

Section III.—Fast Points.

It is sometimes convenient to use a fast point instead of a single switch. When a fast point is used, it must be protected by a check rail in the same way as an ordinary crossing.

In Fig. 2ᵃ, P is the fast point, H the point where the splaying-off of the rail commences, and S the springing of the curve. If e, in this case also represent the thickness of the point, we have:

α. Curve leaving Straight Road.

Distance of point from springing

$$= SP = \sqrt{2r(c+e)}.$$

Distance of end of splay from springing

$$= SH = \sqrt{2r(c+t)}.$$

Length of splayed part

$$= PH = \sqrt{2r}\left(\sqrt{c+t} - \sqrt{c+e}\right).$$

β. Curve leaving Curve, Curvature in same Direction.

Distance of point from springing

$$= SP = \sqrt{\frac{2Rr(c+e)}{R-r}}.$$

Distance of end of splay from springing

$$= SH = \sqrt{\frac{2Rr(c+t)}{R-r}}.$$

Length of splayed part

$$= PH = \sqrt{\frac{2Rr}{R-r}}\left\{\sqrt{c+t} - \sqrt{c+e}\right\}.$$

γ. Curve leaving Curve, Curvature in opposite Directions.

Distance of point from springing

$$= SP = \sqrt{\frac{2Rr(c+e)}{R+r}}.$$

Distance of end of splay from springing

$$= SH = \sqrt{\frac{2Rr(c+t)}{R+r}}.$$

Length of splayed part

$$= PH = \sqrt{\frac{2Rr}{R+r}}\left\{\sqrt{c+t} - \sqrt{c+e}\right\}.$$

If the rail be of the Great Western type it will be splayed off wholly on one side; if it have upper flanges both sides will be equally splayed off, and the rail must be bent at the point H till the two faces next the flange of the wheel are brought into the same line.

Section IV.—Values of the Constants.

The clearance c is usually 2″; it may with safety be reduced to 1½″. In the case of cross-sleeper roads, if a double-heel chair be used, so that this amount of clearance is the same in every case, switches of various lengths must be used in order to make perfect work. If a separate chair be used for the branch road at the heel of the switch, the interval may be varied at pleasure, and the same length of switch made to suit a wide range of curves.

The maximum value of e ought not to exceed ⅜″, and once fixed remains the same for all cases.

The same remark does not apply to e'. A deviation from correct lines is less easily perceived in an easy than in a sharp curve, and therefore the greater the radius of the curve the greater will be the permissible value of e'. It has already been pointed out that a greater value may be given to e' in the case of a curve leaving a curve than in the case of a curve leaving a straight line. Practically we may suppose the deviation equally divided between the two curves, and therefore the value of e' in the former case may be double its value in the latter.

It is only in the case of easy curves that any difficulty exists in the matter of switches. If made very long, they would bend in the middle, and be liable to shut and open imperfectly. Wide-section rails require and admit of a greater length than narrow-section rails.

The following examples have been worked out to show what extent of deviation is involved by adopting switches 22 feet, 16 feet, and 12 feet long respectively, in the four cases of a curve of 60 chains, 40 chains, 20 chains, and 10 chains radius leaving a straight road. The maximum length in each case is also given. The values assigned to the arbitrary constants are $c = 2''$ and $1\frac{1}{2}''$, $t = 2\frac{1}{2}''$, $e = \frac{3}{8}''$.

Case I.—Radius, 60 chains.

α. $c = 1\frac{1}{2}''$.

Maximum length = 35·75 feet.

Value of e':

22-feet switch	=	·08	„
16 „	=	·13	„
12 „	=	·16	„

β. $c = 2''$.

Maximum length = 38·85 feet.

Value of e':

22-feet switch	=	·12	„
16 „	=	·16	„
12 „	=	·20	„

Case II.—Radius, 40 chains.

α. $c = 1\frac{1}{2}''$.

Maximum length = 29·09 feet.

Value of e':

22-feet switch	=	·04	„
16 „	=	·09	„
12 „	=	·14	„

β. $c = 2''$.

Maximum length = 31·63 feet.

Value of e':

22-feet switch	=	·06	„
16 „	=	·12	„
12 „	=	·17	„

Case III.—Radius, 20 chains.

α. $c = 1\frac{1}{2}''$.

Maximum length = 20·61 feet.

Value of e':

16-feet switch	=	·04	„
12 „	=	·09	„

β. $c = 2''$.

Maximum length = 21·38 feet.

Value of e':

16-feet switch	=	·06	„
12 „	=	·11	„

Case IV.—Radius, 10 chains.

α. $c = 1\frac{1}{2}''$.

Maximum length = 14·55 feet.

Value of e':

12-feet switch = ·03 „

β. $c = 2''$.

Maximum length = 15·73 feet.

Value of e':

12-feet switch = ·05 „

CHAPTER III.

SINGLE-GAUGE ROADS: ANGLES OF CROSSINGS AND THEIR POSITION.

Section I.—Single-line Junctions.

The staked-out centre line referred to in the following formulæ is in the middle of the single road. When the staked-out centre line is in the same position as that for a double line, the formulæ for double-road junctions must be used.

Case I.—*Curve leaving Straight Road.*

Let O (Fig. 3) be the centre of the curve of the branch line; $O S_2 S_1$ the radius passing through the springing of the curve, and $O c_1^2$ the radius through the point of crossing c_1^2.

Take O for the origin of polar co-ordinates, and $O S_2 S_1$ for the initial line. If θ be the angle which the radius vector at any point of the straight line $S_2 c_1^2$ makes with the initial line, the equation to that line will be

$$\rho = \left(r - \frac{g}{2}\right) \text{sec. } \theta.$$

Also the equation to the curved line $S_1 c_1^2$ is

$$\rho = r + \frac{g}{2}.$$

Consequently, at the point where these intersect,

$$\cos.\ \theta = \frac{2r - g}{2r + g}$$

and

$$\sin.\ \theta = \frac{2\sqrt{2rg}}{2r + g}.$$

Now, since $O S_2 S_1$ and $O c_1^2$ are perpendicular respectively to $S_2 c_1^2$ and the tangent at c_1^2, the angle $S_1 O c_1^2$ is equal to the angle of the crossing. Since, within the prescribed limits of exactness, the sine of the angle is equal to its circular measure, we have

$$c_1^2 = \frac{2\sqrt{2rg}}{2r + g};$$

also

$$S_2 c_1^2 = O c_1^2 \sin.\ \theta = \sqrt{2rg}.$$

CASE II.—*Curve leaving Curve, Curvature in the same Direction.*

Let O (Fig. 4) be the centre of the curve of the main line, O′ of the branch; $O O' S_2 S_1$ the radius passing through the springing of the curves; $O c_1^2$ and $O' c_1^2$ radii passing through the point of crossing c_1^2. Since these radii are perpendicular, respectively, to the tangents to the two curves at the point c_1^2, the angle $O c_1^2 O'$ is equal to the angle of the crossing. Hence if θ be the angle of the crossing,

$$O O'^2 = O c_1^{2^2} + O' c_1^{2^2} - 2\, O c_1^2 . O' c_1^2 \cos.\ \theta,$$

or

$$\overline{R - r}^2 = \overline{R - \frac{g}{2}}^2 + \overline{r + \frac{g}{2}}^2 - 2\left(R - \frac{g}{2}\right)\left(r + \frac{g}{2}\right)\cos.\ \theta.$$

Leaving g^2 out of consideration, we get

$$\cos.\ \theta = \frac{2Rr - (R - r)\, g}{2Rr + (R - r)\, g}$$

and

$$c_1^2 = \sin.\ \theta = \frac{2\sqrt{2Rr(R - r)\, g}}{2Rr + (R - r)\, g};$$

also within the prescribed limits,

$$S_2 c_1^2 = O c_1^2 \text{ sin. } S_2 O c_1^2$$
$$= \frac{O c_1^2 . O' c_1^2}{O O'} \text{ sin. } O c_1^2 O'$$
$$= \frac{\left(R - \frac{g}{2}\right)\left(r + \frac{g}{2}\right)}{R - r} \text{ sin. } \theta$$
$$= \sqrt{\frac{2 R r g}{R - r}}.$$

CASE III.—*Curve leaving Curve, Curvature in opposite Directions.*

Let O (Fig. 5) be the centre of the main line, and O′ of the branch; $O S_2 S_1 O'$ the radii through the springing; $O c_1^2$ and $O' c_1^2$ the radii through the point of crossing c_1^2. Since $O c_1^2$, $O' c_1^2$ are perpendicular respectively to the tangents to the two curves at the point c_1^2, the angle $O' c_1^2 O$ is equal to the obtuse angle between the tangents, or is equal to $\Pi - \theta$, if θ be the angle of the crossing, consequently we have

$$O O'^2 = O c_1^{2^2} + O' c_1^{2^2} + 2 O c_1^2 . O' c_1^2 \cos. \theta,$$

or

$$(R + r)^2 = \overline{R + \frac{g}{2}}^2 + \overline{r + \frac{g}{2}}^2 + 2\left(R + \frac{g}{2}\right)\left(r + \frac{g}{2}\right) \cos. \theta;$$

leaving g^2 out of consideration, we get

$$\cos. \theta = \frac{2 R r - (R + r) g}{2 R r + (R + r) g}$$

and

$$c_1^2 = \text{sin. } \theta = \frac{2\sqrt{2 R r (R + r) g}}{2 R r + (R + r) g};$$

also within the prescribed limits of exactness,

$$S_2 c_1^2 = O c_1^2 \text{ sin. } S_2 O c_1^2,$$
$$= \frac{O c_1^2 O' c_1^2}{O O'} \text{ sin. } O c_1^2 O'$$
$$= \frac{\left(R + \frac{g}{2}\right)\left(r + \frac{g}{2}\right)}{R + r} \text{ sin. } \theta$$
$$= \sqrt{\frac{2 R r g}{R + r}}$$

Section II.—Double-line Junctions.

Case I.—*Curve leaving a Straight Line.*

Following the method adopted, Section I., Case I., the equations to the straight lines $S_2 c_1^2$, $S_3 c_1^3$, $S_4 c_1^4$ (Fig. 6), referred to the centre of the curve as origin, and the normal through $S_1 S_2 S_3 S_4$, as initial line, are

$$\text{To } S_2 c_1^2,\ \rho = \left(r + \frac{s}{2}\right) \text{sec. } \theta. \qquad (1)$$

$$\text{To } S_3 c_1^3,\ \rho = \left(r - \frac{s}{2}\right) \text{sec. } \theta. \qquad (2)$$

$$\text{To } S_4 c_1^4,\ \rho = \left(r - \frac{s}{2} - g\right) \text{sec. } \theta. \qquad (3)$$

Also the equations to the curve lines $S_1 c_1^4$, $S_2 c_2^4$, $S_3 c_3^4$, are

$$\text{To } S_1 c_1^4,\ \rho = r + \frac{s}{2} + g. \qquad (4)$$

$$\text{To } S_2 c_2^4,\ \rho = r + \frac{s}{2}. \qquad (5)$$

$$\text{To } S_3 c_3^4,\ \rho = r - \frac{s}{2}. \qquad (6)$$

The angle θ, between the radius vector passing through any crossing and the initial line, is equal to the angle of the crossing; consequently to obtain the values of c_1^2, c_1^3, c_1^4, we must combine equation (4) with equations (1), (2), and (3), successively.

From (1) and (4)

$$\text{cos. } \theta = \frac{2r + s}{2r + 2g + s},$$

therefore, leaving out squares and products of s and g,

$$c_1^2 = \text{sin. } \theta = \frac{2\sqrt{2rg}}{2r + 2g + s}.$$

From (2) and (4)

$$\text{cos. } \theta = \frac{2r - s}{2r + 2g + s},$$

therefore

$$c_1^3 = \text{sin. } \theta = \frac{2\sqrt{2r(s+g)}}{2r + 2g + s}.$$

From (3) and (4)

$$\cos.\,\theta = \frac{2r - 2g - s}{2r + 2g + s},$$

therefore

$$c_1^4 = \sin.\,\theta = \frac{2\sqrt{2r(2g+s)}}{2r + 2g + s}.$$

To obtain the values of c_2^3, c_2^4, we must combine equation (5) with equations (2) and (3) successively.

From (2) and (5)

$$\cos.\,\theta = \frac{2r - s}{2r + s},$$

therefore

$$c_2^3 = \sin.\,\theta = \frac{2\sqrt{2rs}}{2r + s}.$$

From (3) and (5)

$$\cos.\,\theta = \frac{2r - 2g - s}{2r + s},$$

therefore

$$c_2^4 = \sin.\,\theta = \frac{2\sqrt{2r(s+g)}}{2r + s}.$$

To obtain the value of c_3^4 we must combine equation (3) with equation (6), and we get

$$\cos.\,\theta = \frac{2r - 2g - s}{2r - s},$$

therefore

$$c_3^4 = \sin.\,\theta = \frac{2\sqrt{2rg}}{2r - s};$$

also

$$S_2 c_1^2 = \left(r + g + \frac{s}{2}\right) c_1^2 = \sqrt{2rg}.$$

Similarly,

$$S_3 c_1^3 = \sqrt{2r(g+s)},$$

$$S_4 c_1^4 = \sqrt{2r(2g+s)},$$

$$S_3 c_2^3 = \sqrt{2rs},$$

$$S_4 c_2^4 = \sqrt{2r(s+g)},$$

$$S_4 c_3^4 = \sqrt{2rg}.$$

CASE II.—*Curve leaving Curve, Curvature in the same Direction.*

Following the method adopted in Section I., Case II., we have for determining c_1^2 (Fig. 7) the equation

$$(R - r)^2 = \left(R + \frac{s}{2}\right)^2 + \left(r + \frac{s}{2} + g\right)^2 - 2\left(R + \frac{s}{2}\right)\left(r + \frac{s}{2} + g\right) \cos.\ \theta,$$

whence, neglecting squares and products of s and g, and the products of these squares and products by factors less than unity, we get

$$\cos.\ \theta = \frac{2Rr + (R + r)\, s + 2rg}{2Rr + (R + r)\, s + 2Rg}$$

$$c_1^2 = \sin.\ \theta = \frac{2\sqrt{(R - r)\, g\, \{2Rr + (R + r)\, (s + g)\}}}{2Rr + (R + r)\, s + 2Rg}$$

$$= \frac{2\sqrt{2Rr\,(R - r)\, g}}{2Rr + (R + r)\, s + 2Rg}.$$

For determining c_1^3, the equation

$$(R - r)^2 = \left(R - \frac{s}{2}\right)^2 + \left(r + \frac{s}{2} + g\right)^2 - 2\left(R - \frac{s}{2}\right)\left(r + \frac{s}{2} + g\right) \cos.\ \theta,$$

whence

$$\cos.\ \theta = \frac{2Rr - (R - r)\, s + 2rg}{2Rr + (R - r)\, s + 2Rg}$$

$$c_1^3 = \sin.\ \theta = \frac{2\sqrt{(R - r)\, (s + g)\, \{2Rr + (R + r)\, g\}}}{2Rr + (R - r)\, s + 2Rg}$$

$$= \frac{2\sqrt{2Rr\,(R - r)\, (s + g)}}{2Rr + (R - r)\, s + 2Rg}.$$

For determining c_1^4, the equation

$$(R - r)^2 = \left(R - \frac{s}{2} - g\right)^2 + \left(r + \frac{s}{2} + g\right)^2 - 2\left(R - \frac{s}{2} - g\right)\left(r + \frac{s}{2} + g\right) \cos.\ \theta,$$

whence

$$\cos.\,\theta = \frac{2\,\mathrm{R}r - (\mathrm{R}-r)(s+2g)}{2\,\mathrm{R}r + (\mathrm{R}-r)(s+2g)}$$

$$c_1^4 = \sin.\,\theta = \frac{2\sqrt{2(\mathrm{R}-r)\,\mathrm{R}r\,(s+2g)}}{2\,\mathrm{R}r + (\mathrm{R}-r)(s+2g)}.$$

For determining c_2^3, the equation

$$(\mathrm{R}-r)^2 = \left(\mathrm{R}-\frac{s}{2}\right)^2 + \left(r+\frac{s}{2}\right)^2 - 2\left(\mathrm{R}-\frac{s}{2}\right)\left(r+\frac{s}{2}\right)\cos.\,\theta,$$

whence

$$\cos.\,\theta = \frac{2\,\mathrm{R}r - (\mathrm{R}-r)\,s}{2\,\mathrm{R}r + (\mathrm{R}-r)\,s}$$

$$c_2^3 = \sin.\,\theta = \frac{2\sqrt{2\,\mathrm{R}r\,(\mathrm{R}-r)\,s}}{2\,\mathrm{R}r + (\mathrm{R}-r)\,s}.$$

For determining c_2^4, the equation

$$(\mathrm{R}-r)^2 = \left(\mathrm{R}-\frac{s}{2}-g\right)^2 + \left(r+\frac{s}{2}\right)^2 - 2\left(\mathrm{R}-\frac{s}{2}-g\right)\left(r+\frac{s}{2}\right)\cos.\,\theta,$$

whence

$$\cos.\,\theta = \frac{2\,\mathrm{R}r - (\mathrm{R}-r)\,s - 2rg}{2\,\mathrm{R}r + (\mathrm{R}-r)\,s - 2rg}$$

$$c_2^4 = \sin.\,\theta = \frac{2\sqrt{(\mathrm{R}-r)(s+g)\{2\,\mathrm{R}r - (\mathrm{R}+r)g\}}}{2\,\mathrm{R}r + (\mathrm{R}-r)\,s - 2rg}$$

$$= \frac{2\sqrt{2\,\mathrm{R}r\,(\mathrm{R}-r)(s+g)}}{2\,\mathrm{R}r + (\mathrm{R}-r)\,s - 2rg}.$$

For determining c_3^4, the equation

$$(\mathrm{R}-r)^2 = \left(\mathrm{R}-\frac{s}{2}-g\right)^2 + \left(r-\frac{s}{2}\right)^2 - 2\left(\mathrm{R}-\frac{s}{2}-g\right)\left(r-\frac{s}{2}\right)\cos.\,\theta,$$

whence

$$\cos.\,\theta = \frac{2\,\mathrm{R}r - (\mathrm{R}+r)\,s - 2\,\mathrm{R}g}{2\,\mathrm{R}r - (\mathrm{R}+r)\,s - 2rg}$$

$$c_3^4 = \sin.\,\theta = \frac{2\sqrt{(\mathrm{R}-r)g\{2\,\mathrm{R}r - (\mathrm{R}+r)(s+g)\}}}{2\,\mathrm{R}r - (\mathrm{R}+r)\,s - 2rg}$$

$$= \frac{2\sqrt{2\,\mathrm{R}r\,(\mathrm{R}-r)g}}{2\,\mathrm{R}r - (\mathrm{R}+r)\,s - 2rg};$$

also

$$S_2 c_1^2 = \frac{\left(R + \frac{s}{2}\right)\left(r + \frac{s}{2} + g\right)}{R - r} \cdot c_1^2$$

$$= \sqrt{\frac{\{2Rr + (R+r)(s+g)\}\, g}{R - r}}.$$

Similarly,

$$S_3 c_1^3 = \sqrt{\frac{\{2Rr + (R+r)\, g\}\,(s+g)}{R - r}},$$

$$S_4 c_1^4 = \sqrt{\frac{2Rr\,(s+2g)}{R - r}},$$

$$S_3 c_2^3 = \sqrt{\frac{2Rrs}{R - r}},$$

$$S_4 c_2^4 = \sqrt{\frac{\{2Rr - (R+r)\, g\}\,(s+g)}{R - r}},$$

$$S_4 c_3^4 = \sqrt{\frac{\{2Rr - (R+r)(s+g)\}\, g}{R - r}}.$$

CASE III.—*Curve leaving Curve, Curvature in opposite Directions.*

Following the method adopted in Section I., Case III., we have for determining c_1^2 (Fig. 8) the following equation:

$$(R+r)^2 = \left(R - \frac{s}{2}\right)^2 + \left(r + \frac{s}{2} + g\right)^2$$

$$+ 2\left(R - \frac{s}{2}\right)\left(r + \frac{s}{2} + g\right) \cos.\, \theta,$$

whence, neglecting terms of the same magnitude, as in Case II.,

$$\cos.\, \theta = \frac{2Rr + (R-r)\, s - 2rg}{2Rr + (R-r)\, s + 2Rg}$$

$$c_1^2 = \sin.\, \theta = \frac{2\sqrt{(R+r)\, g\,\{2Rr + (R-r)\,(s+g)\}}}{2Rr + (R-r)\, s + 2Rg}$$

$$= \frac{2\sqrt{2Rr\,(R+r)\, g}}{2Rr + (R-r)\, s + 2Rg};$$

and for determining c_1^3, the equation

$$(R+r)^2 = \left(R + \frac{s}{2}\right)^2 + \left(r + \frac{s}{2} + g\right)^2$$

$$+ 2\left(R + \frac{s}{2}\right)\left(r + \frac{s}{2} + g\right) \cos.\, \theta,$$

whence

$$\cos.\,\theta = \frac{2\,R\,r - (R+r)\,s - 2\,r\,g}{2\,R\,r + (R+r)\,s + 2\,R\,g}$$

$$c_1^{\,3} = \sin.\,\theta = \frac{2\sqrt{(R+r)\,(s+g)\,\{2\,R\,r + (R-r)\,g\}}}{2\,R\,r + (R+r)\,s + 2\,R\,g}$$

$$= \frac{2\sqrt{2\,R\,r\,(R+r)\,(s+g)}}{2\,R\,r + (R+r)\,s + 2\,R\,g}.$$

For determining $c_1^{\,4}$, the equation

$$(R+r)^2 = \left(R + \frac{s}{2} + g\right)^2 + \left(r + \frac{s}{2} + g\right)^2$$

$$+ 2\left(R + \frac{s}{2} + g\right)\left(r + \frac{s}{2} + g\right)\cos.\,\theta,$$

whence

$$\cos.\,\theta = \frac{2\,R\,r - (R+r)\,(s+2\,g)}{2\,R\,r + (R+r)\,(s+2\,g)}$$

$$c_1^{\,4} = \sin.\,\theta = \frac{2\sqrt{2\,R\,r\,(R+r)\,(s+2\,g)}}{2\,R\,r + (R+r)\,(s+2\,g)}.$$

For determining $c_2^{\,3}$, the equation

$$(R+r)^2 = \left(R + \frac{s}{2}\right)^2 + \left(r + \frac{s}{2}\right)^2 + 2\left(R + \frac{s}{2}\right)\left(r + \frac{s}{2}\right)\cos.\,\theta,$$

whence

$$\cos.\,\theta = \frac{2\,R\,r - (R+r)\,s}{2\,R\,r + (R+r)\,s}$$

$$c_2^{\,3} = \sin.\,\theta = \frac{2\sqrt{2\,R\,r\,(R+r)\,s}}{2\,R\,r + (R+r)\,s}.$$

For determining $c_2^{\,4}$, the equation

$$(R+r)^2 = \left(R + \frac{s}{2} + g\right)^2 + \left(r + \frac{s}{2}\right)^2$$

$$+ 2\left(R + \frac{s}{2} + g\right)\left(r + \frac{s}{2}\right)\cos.\,\theta,$$

whence

$$\cos.\,\theta = \frac{2\,R\,r - (R+r)\,s - 2\,R\,g}{2\,R\,r + (R+r)\,s + 2\,r\,g}$$

$$c_2^{\,4} = \sin.\,\theta = \frac{2\sqrt{(R+r)\,(s+g)\,\{2\,R\,r - (R-r)\,g\}}}{2\,R\,r + (R+r)\,s + 2\,r\,g}$$

$$= \frac{2\sqrt{2\,R\,r\,(R+r)\,(s+g)}}{2\,R\,r + (R+r)\,s + 2\,r\,g}.$$

To determine c_3^4, the equation

$$(R+r)^2 = \left(R+\frac{s}{2}+g\right)^2 + \left(r-\frac{s}{2}\right)^2 + 2\left(R+\frac{s}{2}+g\right)\left(r-\frac{s}{2}\right)\cos.\,\theta,$$

whence

$$\cos.\,\theta = \frac{2Rr-(R-r)s-2Rg}{2Rr-(R-r)s+2rg}$$

$$c_3^4 = \sin.\,\theta = \frac{2\sqrt{(R+r)g\,\{2Rr-(R-r)(s+g)\}}}{2Rr-(R-r)s+2rg}$$

$$= \frac{2\sqrt{2Rr(R+r)g}}{2Rr-(R-r)s+2rg},$$

also

$$S_3 c_1^3 = \frac{\left(R-\frac{s}{2}\right)\left(r+\frac{s}{2}+g\right)}{R+r} c_1^3$$

$$= \sqrt{\frac{\{2Rr+(R-r)(s+g)\}g}{R+r}}$$

$$= \sqrt{\frac{2Rrg}{R+r}}.$$

Similarly,

$$S_2 c_1^2 = \sqrt{\frac{\{2Rr+(R-r)g\}(s+g)}{R+r}}$$

$$= \sqrt{\frac{2Rr(s+g)}{R+r}},$$

$$S_4 c_1^4 = \sqrt{\frac{2Rr(s+2g)}{R+r}},$$

$$S_3 c_2^3 = \sqrt{\frac{2Rrs}{R+r}},$$

$$S_4 c_2^4 = \sqrt{\frac{\{2Rr-(R-r)g\}(s+g)}{R+r}}$$

$$= \sqrt{\frac{2Rr(s+g)}{R+r}},$$

$$S_4 c_3^4 = \sqrt{\frac{\{2Rr-(R-r)(s+g)\}g}{R+r}}$$

$$= \sqrt{\frac{2Rrg}{R+r}}.$$

The formulæ in Section I., Case III., and Section II., Case III., may be derived at once from those in Section I., Case II., and Section II., Case II., by putting $R = -R$.

Section III.—Three Throws.

Case I.—*Main Line Straight, Branch Roads Curving in opposite Directions.*

Let r_1, r_2 be the radii of the roads leaving the main line, r_1 to the left, r_2 to the right; c_{r_1}, c_{r_2}, c_m the points, where the rails of these roads cut those of the main line and each other respectively (Fig. 9); S_1, S_2 being the springing common to the two curves.

From Section I., Cases I. and III., we get

$$c_{r_1} = \frac{2\sqrt{2r_1 g}}{2r_1 + g},$$

$$c_{r_2} = \frac{2\sqrt{2r_2 g}}{2r_2 + g},$$

$$c_m = \frac{2\sqrt{2r_1 r_2 (r_1 + r_2) g}}{2r_1 r_2 + (r_1 + r_2) g};$$

and

$$S_1 c_{r_1} = \sqrt{2r_1 g},$$

$$S_2 c_{r_2} = \sqrt{2r_2 g},$$

$$S_1 c_m = S_2 c_m = \sqrt{\frac{2r_1 r_2 g}{r_1 + r_2}}.$$

Case II.—*Main Line Straight, Branch Roads Curving in the same Direction.*

With the same notation as in Case I., we have (Fig. 10) from Section I., Cases I. and II., r_1 being $> r_2$,

$$c_{r_1} = \frac{2\sqrt{2r_1 g}}{2r_1 + g},$$

$$c_{r_2} = \frac{2\sqrt{2r_2 g}}{2r_2 + g},$$

$$c_m = \frac{2\sqrt{2r_1 r_2 (r_1 - r_2) g}}{2r_1 r_2 + (r_1 - r_2) g};$$

and

$$S_2 c_{r_1} = \sqrt{2 r_1 g},$$

$$S_2 c_{r_2} = \sqrt{2 r_2 g},$$

$$S_1 c_m = S_2 c_m = \sqrt{\frac{2 r_1 r_2 g}{r_1 - r_2}}.$$

CASE III.—*Main Line Curved, Branch Roads Curving in opposite Directions.*

Let r_2 be the radius of the curve, whose curvature is in the same direction as that of the main line (Fig. 11).

From Section I., Cases II. and III., we get

$$c_{r_1} = \frac{2\sqrt{2 R r_1 (R + r_1) g}}{2 R r_1 + (R + r_1) g},$$

$$c_{r_2} = \frac{2\sqrt{2 R r_2 (R - r_2) g}}{2 R r_2 + (R - r_2) g},$$

$$c_m = \frac{2\sqrt{2 r_1 r_2 (r_1 + r_2) g}}{2 r_1 r_2 + (r_1 + r_2) g},$$

$$S_1 c_{r_1} = \sqrt{\frac{2 R r_1 g}{R + r_1}},$$

$$S_2 c_{r_2} = \sqrt{\frac{2 R r_2 g}{R - r_2}},$$

$$S_1 c_m = S_2 c_m = \sqrt{\frac{2 r_1 r_2 g}{r_1 + r_2}}.$$

CASE IV.—*Main Line Curved, both Branch Roads Curving in the same Direction as the Main Line.*

Let r_1 be $> r_2$, we have (Fig. 12) from Section I., Case II.,

$$c_{r_1} = \frac{2\sqrt{2 R r_1 (R - r_1) g}}{2 R r_1 + (R - r_1) g},$$

$$c_{r_2} = \frac{2\sqrt{2 R r_2 (R - r_2) g}}{2 R r_2 + (R - r_2) g},$$

$$c_m = \frac{2\sqrt{2 r_1 r_2 (r_1 - r_2) g}}{2 r_1 r_2 + (r_1 - r_2) g};$$

and

$$S_2 c_{r_1} = \sqrt{\frac{2 R r_1 g}{R - r_1}},$$

$$S_2 c_{r_2} = \sqrt{\frac{2 R r_2 g}{R - r_2}},$$

$$S_1 c_m = S_2 c_m = \sqrt{\frac{2 r_1 r_2 g}{r_1 - r_2}}.$$

CASE V.—*Main Line Curved, both Branch Roads Curving in opposite direction to Main Line.*

Let r_1 be $> r_2$, we have (Fig. 13) from Section I., Cases II. and III.,

$$c_{r_1} = \frac{2\sqrt{2 R r_1 (R + r_1) g}}{2 R r_1 + (R + r_1) g},$$

$$c_{r_2} = \frac{2\sqrt{2 R r_2 (R + r_2) g}}{2 R r_2 + (R + r_2) g},$$

$$c_m = \frac{2\sqrt{2 r_1 r_2 (r_1 - r_2) g}}{2 r_1 r_2 + (r_1 - r_2) g};$$

and

$$S_2 c_{r_1} = \sqrt{\frac{2 R r_1 g}{R + r_1}},$$

$$S_2 c_{r_2} = \sqrt{\frac{2 R r_2 g}{R + r_2}},$$

$$S_1 c_m = S_2 c_m = \sqrt{\frac{2 r_1 r_2 g}{r_1 - r_2}}.$$

In each of these cases the expression for the distance $S_1 c_m$, $S_2 c_m$, gives strictly the distance of the crossing c_m from the springing, measured parallel to the tangent at the springing. In practice, therefore, the distance ought to be measured from the nearest of the two points S_1, S_2 to the crossing c_m.

SECTION IV.—CROSS-OVER ROADS.

Cross-over roads form a communication between two contiguous roads of a double line. This communication may be established by two reversing curves, which meet at a common

tangent, or are separated by a short interval of straight. Since an unlimited number of combinations of curves separated by an interval of straight can be adopted for securing a communication between two contiguous roads, it is evident that no general formulæ applicable to every case can be obtained, unless one particular combination be fixed upon: for instance, this one might be adopted as the simplest, *viz.* that the interval of straight should extend between the two normals to the cross-over road, which pass through the points of the crossings. The interval of straight afforded by this, the simplest possible combination, would be so short as not practically to improve the road, and the formulæ obtained would not be nearly so simple as those obtained on these two assumptions:—

1st. That the reverse curves meet at a common tangent.

2nd. That the point of contrary flexure is midway between the two roads.

These two assumptions have therefore been adopted.

Case I.—*Main Line Straight.*

In order that assumption (2) may be satisfied, both curves must have the same radius. Let r be this radius; $S_1 S_2 S_3 S_4$, $S'_1 S'_2 S'_3 S'_4$ (Fig. 14) the points where radii from the centres O O′ respectively cut the rails of the main line at right angles. Let the rails $S_1 S'_3$, $S_2 S'_4$ cross the rails $S_2 S'_2$, $S_3 S'_3$, in the points c_1^2 and c_2^3 respectively.

Since the offset to the tangent to the curve of the centre line of the cross-over (shown dotted in the Figure) at the springing, which passes through the point, where the centre line of the cross-over crosses the centre line of the two main roads (shown dotted in the Figure), is equal to $\frac{s+g}{2}$, the distance between the springings of the reverse curves, which is equal to the sum of the length of these tangents, is equal to

$$2\sqrt{r(s+g)}.$$

Since the distance of the heels of either set of switches from the springing of the curve is equal to $\sqrt{2r(c+t)}$, if l be the actual lengths of the switches, and D the distance between the

points of the switches measured parallel to the main line, we have

$$D = 2\sqrt{r(s+g)} - 2\{\sqrt{2r(c+t)} - l\}.$$

If the radius r be given, this equation will give the distance between the points of the switches. If the distance D be given, we have conversely,

$$\text{Radius } r = \frac{(D-2l)^2}{4\{\sqrt{s+g} - \sqrt{2(c+t)}\}^2}.$$

Before we can determine the values of c_1^2, c_2^3 and their position, we must ascertain whether the rails $S_2S'_2$, $S_3S'_3$ are cut by the curves, whose centres are O, O′ respectively, or *vice versâ*. This will depend upon the relative values of s and g.

Join $O\,c_1^2$, $O'\,c_2^3$. Let t, t' be the points where $O\,c_1^2$, $O'\,c_2^3$ meet $S_2S'_4$, $S_1S'_3$ respectively. If the rail $S_2S'_2$ be cut by the curve whose centre is O, the point t, where the radius through c_1^2 cuts $S_2S'_4$, will lie between the line $S_2S'_2$ and $S_3S'_3$. Hence the condition that the rail $S_2S'_2$ should be cut by the curve, whose centre is O, is simply that the offset to the tangent to the rail of the cross-over at S_2, which passes through t, must not be greater than s. If it be equal to s, c_1^2, c_2^3 are points of contrary flexure.

The length of the tangent at S_2, whose offset passes through t, is to that of the tangent at S_1, whose offset passes through c_1^2, as $r - \frac{g}{2} : r + \frac{g}{2}$. Since the length of the latter is equal to

$$\sqrt{(2r+g)g},$$

the length of the former is

$$\frac{2r-g}{2r+g} \cdot \sqrt{(2r+g)g},$$

and therefore the length of the offset which passes through t is

$$\frac{(2r-g)^2(2r+g)g}{(2r+g)^2(2r-g)} = \frac{2r-g}{2r+g} \cdot g.$$

Since s must not be less than this, we get the following condition:

$$\frac{s}{g} \text{ must be } \geqq \frac{2r-g}{2r+g}.$$

For all admissible values of r and g, the value of the fraction $\frac{2r-g}{2r+g}$ is practically unity; therefore the condition may be simply stated as follows:—

If s be greater than g, the inside rail of either road will be cut by the outside rail of the reverse curve, which starts from that road.

If s be equal to g, the points, where the rails of the cross-over cut the rails of the main line, are practically points of contrary flexure.

If s be less than g, the inside rail of either road will be cut by the inside rail of the reverse curve, which starts from the opposite road.

We have then different values for c_1^2 and c_2^3, dependent on this condition.

$\alpha.$ $s \overline{=} g.$

From Section I., Case II.,

$$c_1^2 = c_2^3 = \frac{2\sqrt{2rg}}{2r+g},$$

$$S_2 c_1^2 = S'_3 c_2^3 = \sqrt{2rg},$$

$$S'_3 c_1^2 = S_2 c_2^3 = 2\sqrt{r(s+g)} - \sqrt{2rg}.$$

$\beta.$ $s < g.$

Following the method adopted in Section I., Case I.,

$$c_1^2 = c_2^3 = \frac{2\sqrt{2rs}}{2r-g},$$

$$S_2 c_1^2 = S'_3 c_2^3 = 2\sqrt{r(s+g)} - \sqrt{2rs},$$

$$S_3 c_2^3 = S'_2 c_1^2 = \sqrt{2rs}.$$

Case II.—*Main Line Curved.*

Let O (Fig. 15) be the centre of the main-line curve; O′, O″ the centres of the reverse curves.

Let the normals O O″, O O′ cut the rails of the main line at the points $S_1 S_2 S_3 S_4$ and $S'_1 S'_2 S'_3 S'_4$ respectively, and let P be the point of intersection of the centre lines of the cross-over

and the main road: join O P, O″ P, and O′ P: the points O′, O″, and P all lie in the same straight line.

Let the radii O′ P, O″ P be denoted by r_1, r_2, and the angles O′ O O″, O O′ O″, O O″ O′ by the letters O, O′, O″ respectively.

Within the assigned limits of exactness, the arcual distance of P from either springing will be the same, whether measured along the centre line of the cross-over or of the main road, therefore,

$$\angle \text{P O O}' = \frac{r_1}{\text{R}} \text{O}',$$

$$\angle \text{P O O}'' = \frac{r_2}{\text{R}} \text{O}'';$$

but

$$\angle \text{P O O}' + \angle \text{P O O}'' = \text{O}' \text{O O}'';$$

therefore

$$r_1 \text{O}' + r_2 \text{O}'' = \text{R O}; \qquad (1)$$

also

$$\text{O} = \text{O}'' - \text{O}'. \qquad (2)$$

From (1) and (2) we get

$$\text{O}' = \frac{\text{R} - r_2}{r_1 + r_2} \text{O}.$$

$$\text{O}'' = \frac{\text{R} + r_1}{r_1 + r^2} \text{O}.$$

Within the prescribed limits of exactness, the lengths of the tangents to the main-line curves and the reverse curve of the cross-over, at the points where the normal O O″ meets those curves, whose offsets pass through P, are equal to each other and to r_2 O″. Therefore the tangential offset to the main line, which passes through P, will be $\frac{r_2^2 \text{O}''^2}{2 \text{R}}$, and that to the reverse curve $\frac{r_2 \text{O}''^2}{2}$. Similarly, the lengths of the tangents to the main-line curves and the reverse curve of the cross-over, at the points, where the normal O O′ meets those curves, will be equal to each other and to r_1 O′. Therefore the tangential offset to the main line which passes through P, will be $\frac{r_1^2 \text{O}'^2}{2 \text{R}}$, and that to the reverse curve $\frac{r_1 \text{O}'^2}{2}$. Since the reverse curves meet at the

point P, we have these two relations between their offsets and those to the main line:

Offset to curve radius $r_1 = \frac{s+g}{2}$ − offset to main-line curve;

Offset to curve radius $r_2 = \frac{s+g}{2}$ + offset to main-line curve;

therefore

$$r_1 O'^2 = s + g - \frac{r_1^2 O'^2}{R}, \tag{3}$$

$$r_2 O'^2 = (s+g) + \frac{r_2^2 O'^2}{R}. \tag{4}$$

From (3)

$$r_1 (R + r_1) O'^2 = R (s+g),$$

substituting for O'^2 its value,

$$r_1 (R + r_1) (R - r_2)^2 O^2 = R (r_1 + r_2)^2 (s+g). \tag{5}$$

From (4)

$$r_2 (R - r_2) O'^2 = R (s+g),$$

substituting for O'^2 its value,

$$r_2 (R_2 - r_2) (R + r_1)^2 O^2 = R (r_1 + r_2)^2 (s+g). \tag{6}$$

From (5) and (6)

$$O^2 = \frac{R (r_1 + r_2)^2 (s+g)}{r_1 (R + r_1) (R - r_2)^2}$$

$$= \frac{R (r_1 + r_2)^2 (s+g)}{r_2 (R - r_2) (R + r_1)^2}.$$

Therefore

$$r_1 (R + r_1) (R - r_2)^2 = r_2 (R - r_2) (R + r_1)^2$$

$$\frac{r_1}{R + r_1} = \frac{r_2}{R - r_2}$$

$$r_1 = \frac{R r_2}{R - 2 r_2}, \qquad r_2 = \frac{R r_1}{R + 2 r_1} \tag{a}$$

$$O = \frac{2\sqrt{r_2 (s+g)}}{\sqrt{R (R - r_2)}} = \frac{2\sqrt{r_1 (s+g)}}{\sqrt{R (R + r_1)}} \tag{b}$$

$$r_2 = \frac{R^2 O^2}{4 (s+g) + R O^2} \tag{c}$$

$$r_1 = \frac{R^2 O^2}{4 (s+g) - R O^2}. \tag{d}$$

When either radius is given, formulæ (*a*) and (*b*) will give the other, and the angular intervals between the springings of the curves; when this angular interval is given, formulæ (*c*) and (*d*) will give the radii. It must be carefully noted that r_2 is the radius of the reverse curve, which curves in the same direction as the main line; r_1 that of the reverse curve, whose curvature is in the opposite direction.

The circular measure of the angular interval O, in terms of the linear distance D between the actual points of the switches, is given by the equation

$$\mathrm{RO} = \mathrm{D} + \sqrt{\frac{2\,\mathrm{R}\,r_2}{\mathrm{R}-r_2}(c+t)} + \sqrt{\frac{2\,\mathrm{R}\,r_1}{\mathrm{R}+r_1}(c+t)} - 2l$$

$$= \mathrm{D} + 2\,\mathrm{RO}\sqrt{\frac{c+t}{2(s+g)}} - 2l,$$

since

$$\frac{r_2}{\mathrm{R}-r_2} = \frac{r_1}{\mathrm{R}+r_1} = \frac{\mathrm{RO}^2}{4(s+g)},$$

$$\therefore \mathrm{O} = \frac{(\mathrm{D}-2l)\sqrt{(s+g)}}{\mathrm{R}\left\{\sqrt{s+g} - \sqrt{2(c+t)}\right\}}.$$

Let c_1^2, c_2^3 be the points where the rails S_2, S'_2, S_3 S'_3 of the main line are cut by the rails S_1 S'_3, S_2 S'_4 of the reverse curves respectively. Before the values of c_1^2, c_2^3 can be obtained, it is necessary, as in Case I., to ascertain by which of the reverse curves the rails $S_2 S'_2$ and $S_3 S'_3$ are cut.

Let $O'' c_1^2$, $O' c_2^3$ cut the rails $S_2 S'_4$, $S_1 S'_3$ in the points t'', t' respectively. The conditions, that t'', t' should lie between the rails $S_2 S'_2$ and $S_3 S'_3$, are manifestly, that the offset to the tangent to the reverse-curve rail at S_2, which passes through t'', should not be greater than s, plus the offset to the corresponding length of main-line curve, and that the offset to the tangent to the reverse curve at S'_3, which passes through t', should not be greater than s, minus the offset to the corresponding length of main-line curve. Now the lengths of the tangents at S_2, S'_3 whose offsets pass through t'', t' are to the lengths of the tan-

gents at $S_1 S'_4$ whose offsets pass through c_1^2 and c_2^3 respectively, as $r_2 - \frac{g}{2} : r_2 + \frac{g}{2}$, and as $r_1 - \frac{g}{2} : r_1 + \frac{g}{2}$; and the lengths of the latter are, Section I., Cases II. and III.,

$$\sqrt{\frac{2Rr_1g}{R+r_1}} \quad \text{and} \quad \sqrt{\frac{2Rr_2g}{R-r_2}}.$$

Now, within the limits of exactness specified, the tangential offsets to the curves, radii $r_2 \pm \frac{g}{2}$ and $r_1 \pm \frac{g}{2}$, will be the same as those to the curve, radius r_2, the length of the tangent being the same in each case. We may also use R, the radius of the centre line of the space between the two roads, for $R \pm \frac{s+g}{2}$, the radii of the centre lines of the two roads. Therefore the tangential offsets to the reverse curves, which pass through t'' and t', are respectively

$$\left(\frac{2r_2 - g}{2r_2 + g}\right)^2 \frac{Rg}{R - r_2} \quad \text{and} \quad \left(\frac{2r_1 - g}{2r_1 + g}\right)^2 \frac{Rg}{R + r_1};$$

and the tangential offsets to the main-line curve, which pass through t'' t', are respectively

$$\left(\frac{2r_2 - g}{2r_2 + g}\right)^2 \frac{r_2 g}{R - r_2} \quad \text{and} \quad \left(\frac{2r_1 - g}{2r_1 + g}\right)^2 \frac{r_1 g}{R + r_1}.$$

Consequently we must have

$$s \gtreqless \left(\frac{2r_2 - g}{2r_2 + g}\right)^2 \left(\frac{R}{R - r_2} - \frac{r_2}{R - r_2}\right) g$$

and

$$s \gtreqless \left(\frac{2r_1 - g}{2r_1 + g}\right)^2 \left(\frac{R}{R + r_1} + \frac{r_1}{R + r_1}\right) g,$$

or

$$\frac{s}{g} \gtreqless \left(\frac{2r_2 - g}{2r_2 + g}\right)^2 \gtreqless \left(\frac{2r_1 - g}{2r_1 + g}\right)^2.$$

Hence the same remarks apply in Case II. as in Case I. We have therefore

$\alpha.$ $s \gtreqless g.$

$$c_1^2 = \frac{2\sqrt{2Rr_2(R-r_2)g}}{2Rr_2+(R-r_2)g},$$

$$c_2^2 = \frac{2\sqrt{2Rr_1(R+r_1)g}}{2Rr_1+(R+r_1)g},$$

$$S_2 c_1^2 = \sqrt{\frac{2Rr_2 g}{R-r_2}},$$

$$S'_2 c_2^2 = \sqrt{\frac{2Rr_1 g}{R+r_1}},$$

$$S'_2 c_1^2 = \sqrt{\frac{Rr_2}{R-r_2}}\,(2\sqrt{s+g}-\sqrt{2g}),$$

$$S_2 c_2^2 = \sqrt{\frac{Rr_1}{R+r_1}}\,(2\sqrt{s+g}-\sqrt{2g}).$$

$\beta.$ $s < g.$

$$c_1^2 = \frac{2\sqrt{2Rr_1(R+r_1)s}}{2Rr_1+(R+r_1)s},$$

$$c_2^2 = \frac{2\sqrt{2Rr_2(R-r_2)s}}{2Rr_2+(R-r_2)s},$$

$$S_2 c_2^2 = \sqrt{\frac{2Rr_2 s}{R-r_2}},$$

$$S'_2 c_1^2 = \sqrt{\frac{2Rr_1 s}{R+r_1}},$$

$$S_2 c_1^2 = \sqrt{\frac{Rr_1}{R+r_1}}\,\{2\sqrt{s+g}-\sqrt{2s}\},$$

$$S_2 c_2^2 = \sqrt{\frac{Rr_2}{R-r_2}}\,\{2\sqrt{s+g}-\sqrt{2s}\}.$$

Since $\frac{r_1}{R+r_1} = \frac{r_2}{R-r_2}$, the values of c_1^2, c_2^2 given in (α) and (β), show that

$$c_1^2 = c_2^2,$$

and also

$$S_2 c_1^2 = S'_2 c_2^2$$

$$S'_2 c_1^2 = S_2 c_2^2.$$

Section V.—Through Roads.

Through roads are those, which form a communication between two roads separated by one or more intervening roads.

Both the roads connected, and those crossed, are supposed to be parallel to one another, and are for distinction's sake called main roads.

As in the case of simple cross-over roads, an endless combination of reverse curves meeting at a common tangent, and of reverse curves separated by an interval of straight may be formed. Only two cases are investigated:

1st. Where the interval of straight commences at the point, where a rail of one reverse curve first cuts a main-road rail, and continues, until it reaches the point, where a rail of the other reverse curve first cuts a main-road rail. If s be not less than g, these points will be where the through road cuts the rails of the roads, between which the communication is established. If s be less than g, where the through road cuts the nearest rails of the intervening roads.

2nd. Where the reverse curves meet at a common tangent, the point of contrary flexure being midway between the two roads.

Case I.—*Main Roads Straight: Reverse Curves separated by an interval of Straight, which extends from extreme crossing to extreme crossing.*

Let there be n intermediate roads: let (Fig. 16)

$$S_1\, S_2\, S_3\, S_4 \;\ldots\; S_{2m+1}\, S_{2m+2} \;\ldots\; S_{2n+1}\, S_{2n+2}\, S_{2n+3}\, S_{2n+4}$$

and

$$S'_1\, S'_2\, S'_3\, S'_4 \;\ldots\; S'_{2m+1}\, S'_{2m+2} \;\ldots\; S'_{2n+1}\, S'_{2n+2}\, S'_{2n+3}\, S'_{2n+4}$$

be the points where the normals through the springing of the reverse curves cut the main-road rails: also let the points, where the rails of the main road are cut by the rails of the through road and the circular measures of the angles of the respective crossings be denoted by

$$\begin{array}{cccccc} c_1^2\, c_1^3\, c_1^4 & \ldots & c_1^{2m+1}\, c_1^{2m+2} & \ldots & c_1^{2n+1}\, c_1^{2n+2} \\ c_2^2\, c_2^4 & \ldots & c_2^{2m+1}\, c_2^{2m+2} & \ldots & c_2^{2n+1}\, c_2^{2n+2}\, c_2^{2n+3} \end{array}$$

Since these angles are equal to each other, the radii of the reverse curves are equal, and therefore the distances of the points, where the rail $S_1 S'_{2n+3}$ cuts the rails of any intermediate road from the line $S_1 S_{2n+4}$ will be equal to the distances of the points where the rail $S_2 S'_{2n+4}$ cuts the rails of another intermediate road, similarly situated with respect to the roads, between which communication is established, from the line $S'_1 S'_{2n+4}$. Consequently

$$S_{2m+2}\, c_1^{2m+2} = S'_{2(n-m+1)+1}\, c_2^{2(n-m+1)+1}$$

$$S_{2m+2}\, c_2^{2m+2} = S'_{2(n-m+1)+1}\, c_1^{2(n-m+1)+1}$$

$$S_{2m+1}\, c_1^{2m+1} = S'_{2(n-m+1)+2}\, c_2^{2(n-m+1)+2}$$

$$S_{2m+1}\, c_2^{2m+1} = S'_{2(n-m+1)+2}\, c_1^{2(n-m+1)+2}$$

and manifestly

$$S_1\, c_1^2 = S'_{2n+3}\, c_2^{2n+3}$$

$$S_{2n+3}\, c_2^2 = S'_2\, c_1^2$$

The interval between two crossings of any the same main-road rail is equal to the gauge multiplied by the cosecant of the angle of the crossing, and is therefore the same in every case; also the distance, measured parallel to the main roads between the points, where either rail of the through road cuts any two rails of the main roads, is equal to the distance between those rails multiplied by the cotangent of the angle of the crossing.

Let r, θ denote the radius and circular measure of the angle of the crossing.

$\alpha.$ $s \overline{\overline{>}} g.$

$$\text{sin. } \theta = \theta = \frac{2\sqrt{2rg}}{2r+g},$$

$$\text{cosec. } \theta = \frac{1}{\text{sin. } \theta} = \frac{2r+g}{2\sqrt{2rg}},$$

$$\text{cot. } \theta = \frac{\sqrt{1-\text{sin.}^2\theta}}{\text{sin. } \theta} = \frac{2r-g}{2\sqrt{2rg}}.$$

$$S_2 c_1^2 = S'_{2n+2} c_2^{2n+2}$$
$$= \sqrt{2rg}.$$
$$S_{2m+1} c_1^{2m+1} = S'_{2(n-m+1)+2} c_2^{2(n-m+1)+2}$$
$$= \sqrt{2rg} + \{ms + (m-1)g\} \cot. \theta. \quad (1)$$
$$S_{2m+1} c_2^{2m+1} = S'_{2(n-m+1)+2} c_1^{2(n-m+1)+2}$$
$$= \sqrt{2rg} + \{ms + (m-1)g\} \cot. \theta - g \operatorname{cosec}. \theta. \quad (2)$$
$$S_{2m+2} c_1^{2m+2} = S'_{2(n-m+1)+1} c_2^{2(n-m+1)+1}$$
$$= \sqrt{2rg} + m(s+g) \cot. \theta. \quad (3)$$
$$S_{2m+2} c_2^{2m+2} = S'_{2(n-m+1)+1} c_1^{2(n-m+1)+1}$$
$$= \sqrt{2rg} + m(s+g) \cot. \theta - g \operatorname{cosec}. \theta. \quad (4)$$

The total distance between the springings of the reverse curves is equal to

$$S_{2n+2} c_2^{2n+2} + S'_{2n+2} c_2^{2n+2} = S_{2n+2} c_1^{2n+2}$$
$$+ s \cot. \theta - g \operatorname{cosec}. \theta + S_2 c_1^2,$$

or expressed in terms of the radius this interval is

$$2\sqrt{2rg} + \frac{\{(n+1)s + ng\}(2r-g)}{2\sqrt{2rg}} - \frac{g(2r+g)}{2\sqrt{2rg}},$$

which is equal to

$$\frac{\{(n+1)s + (n+3)g\}\sqrt{r}}{\sqrt{2g}} - \frac{(n+1)(s+g)g}{2\sqrt{2rg}}.$$

The term $\frac{(n+1)(s+g)g}{2\sqrt{2rg}}$ is less than g, whatever the gauge may be, unless n exceeds 10 (a case which can hardly occur in practice) for minimum admissible values of r. A deviation from exactness of this amount, in the expression for the total distance between the springings of the reverse curves, will not appreciably affect the value of r when determined from the known interval between the points of the switches, consequently we have

$$r = \frac{2g(D-2l)^2}{\{(n+1)s + (n+3)g - 4\sqrt{g(c+l)}\}^2}.$$

$\beta.$ $s < g.$

$$\text{sin. } \theta = \theta = \frac{2\sqrt{2rs}}{2r-g},$$

$$\text{cosec. } \theta = \frac{1}{\text{sin. } \theta} = \frac{2r-g}{2\sqrt{2rs}},$$

$$\text{cot. } \theta = \frac{\sqrt{1 - \text{sin.}^2 \theta}}{\text{sin. } \theta} = \frac{2r-g-2s}{2\sqrt{2rs}}.$$

$$S_2 c_1^2 = S_{2n+2} c_2^{2n+2}$$
$$= S_2 c_1^2 - s \text{ cot. } \theta$$
$$= S_2 c_2^2 + g \text{ cosec. } \theta - s \text{ cot. } \theta$$
$$= \sqrt{2rs} + g \text{ cosec. } \theta - s \text{ cot. } \theta.$$

$$S_{2m+1} c_1^{2m+1} = S'_{2(n-m+1)+2} c_2^{2(n-m+1)+2}$$
$$= \sqrt{2rs} + (m-1)(s+g) \text{ cot. } \theta + g \text{ cosec. } \theta. \quad (1)$$

$$S_{2m+1} c_2^{2m+1} = S'_{2(n-m+1)+2} c_1^{2(n-m+1)+2}$$
$$= \sqrt{2rs} + (m-1)(s+g) \text{ cot. } \theta. \quad (2)$$

$$S_{2m+2} c_1^{2m+2} = S'_{2(n-m+1)+1} c_2^{2(n-m+1)+1}$$
$$= \sqrt{2rs} + \{(m-1)s + mg\} \text{ cot. } \theta + g \text{ cosec. } \theta. \quad (3)$$

$$S_{2m+2} c_2^{2m+2} = S'_{2(n-m+1)+1} c_1^{2(n-m+1)+1}$$
$$= \sqrt{2rs} + \{\overline{m-1} \,.\, s + mg\} \text{ cot. } \theta. \quad (4)$$

The total distance between the springings of the reverse curves is equal to

$$S_{2n+2} c_1^{2n+2} + S'_{2n+2} c_1^{2n+2} = S_{2n+2} c_2^{2n+2} + g \text{ cosec. } \theta + S_2 c_2^2,$$

or expressed in terms of the radius this interval is

$$2\sqrt{2rs} + \frac{\{(n-1)s + ng\}\{2r - 2s - g\}}{2\sqrt{2rs}} + \frac{g(2r-g)}{2\sqrt{2rs}},$$

which is equal to

$$\frac{\{(n+3)s + (n+1)g\}\sqrt{r}}{\sqrt{2s}} - \frac{2(n-1)s^2 + (3n-1)sg + (n+1)g^2}{2\sqrt{2rs}}.$$

Neglecting the fraction, whose numerator is the sum of squares and products of s and g, we get

$$r = \frac{2s(D - 2l)^2}{\{(n+3)s + (n+1)g - 4\sqrt{s(c+t)}\}^2}.$$

If in equations (1), (2), (3), and (4), in (α) and (β), we substitute for m successive values 1, 2, 3 ... &c. .... n, we shall obtain the distances of the crossings of the 1st, 2nd ... &c. .... $\overline{2n-1}^{th}$ and $2n^{th}$ rails of the n intermediate roads from either normal line through the springing. And if in equation (3), in (α) and (β), we substitute $(n+1)$ for (n), we shall obtain the distance of the crossings c_2^{2n+2} and c_1^{2} from the lines $S_1 S_{2n+4}$ and $S'_1 S'_{2n+4}$ respectively.

CASE II.—*Main Roads Straight: Reverse Curves of Through Road meeting at a common point of contrary flexure, midway between the two roads, between which communication is established.*

Since the reverse curves meet at a common tangent, midway between the outside rails, their radii are equal; consequently, the distances from either normal of the points, where the inner and outer rails of one reverse curve cut the rails of the main road, are equal to the distances from the other normal of the points, where the corresponding rails of the other reverse curve cut the main-road rails similarly situated.

Since the total interval between the centres of the two outside roads is

$$(n+1)(s+g),$$

the total distance between the normals through the springings is

$$2\sqrt{(n+1)\,r\,(s+g)},$$

and therefore

$$r = \frac{(D-2l)^2}{4\left\{\sqrt{(n+1)(s+g)} - \sqrt{2(c+t)}\right\}^2}.$$

Let the symbols in Fig. 17 have the same value as in Fig. 16. Whatever may be the relative values of s and g, except as regards the two middle rails, we shall obtain by following the method pointed out, Section I., Case I.,

$$\begin{aligned} c_1^2 &= c_2^{2n+2} \\ &= \frac{2\sqrt{2rg}}{2r+g}. \\ c_1^{2m+1} &= c_2^{2(n-m+1)+2} \\ &= \frac{2\sqrt{2rm(s+g)}}{2r+g}. \end{aligned} \qquad (1)$$

$$c_2^{2m+1} = c_1^{2(n-m+1)+2}$$

$$= \frac{2\sqrt{2r\{ms+(m-1)g\}}}{2r-g}. \qquad (2)$$

$$c_1^{2m+2} = c_2^{2(n-m+1)+1}$$

$$= \frac{2\sqrt{2r\{ms+(m+1)g\}}}{2r-g}. \qquad (3)$$

$$c_2^{2m+2} = c_1^{2(n-m+1)+1}$$

$$= \frac{2\sqrt{2rm(s+g)}}{2r-g}. \qquad (4)$$

Also

$$S_2 c_1^2 = S_{2n+2} c_2^{2n+2}$$

$$= \sqrt{2rg}.$$

$$S_{2n+2} c_2^{2n+2} = S'_2 c_1^2$$

$$= 2\sqrt{r(n+1)(s+g)} - \sqrt{2rg}.$$

For the distance of the crossings of the first $n-1$ or n rails of the intervening roads from the normal line $S_1 \ldots S_{2n+4}$, and of the last $n-1$ or n rails from the normal line $S'_1 \ldots S'_{2n+4}$

$$S_{2m+1} c_1^{2m+1} = S'_{2(n-m+1)+2} c_2^{2(n-m+1)+2}$$

$$= \sqrt{2rm(s+g)}. \qquad (5)$$

$$S_{2m+1} c_2^{2m+1} = S'_{2(n-m+1)+2} c_1^{2(n-m+1)+2}$$

$$= \sqrt{2r\{ms+(m-1)g\}}. \qquad (6)$$

$$S_{2m+2} c_1^{2m+2} = S'_{2(n-m+1)+1} c_2^{2(n-m+1)+1}$$

$$= \sqrt{2r\{ms+(m+1)g\}}. \qquad (7)$$

$$S_{2m+2} c_2^{2m+2} = S'_{2(n-m+1)+1} c_1^{2(n-m+1)+1}$$

$$= \sqrt{2rm(s+g)}. \qquad (8)$$

For the distances of the crossings of the last n or $n-1$ rails of the intervening roads from the normal line $S_1 \ldots S_{2n+4}$, and of the first n or $n-1$ rails from the normal line $S'_1 \ldots S'_{2n+4}$

$$S_{2m+1} c_1^{2m+1} = S'_{2(n-m+1)+2} c_2^{2(n-m+1)+2}$$

$$= 2\sqrt{r(n+1)(s+g)} - \sqrt{2r(n-m+1)(s+g)}. \qquad (5^a)$$

$$S_{2m+1}c_2^{2m+1} = S'_{2(n-m+1)+2}c_1^{2(n-m+1)+2}$$
$$= 2\sqrt{r(n+1)(s+g)} - \sqrt{2r\{(n-m+1)s+(n-m+2)g\}}. \quad (6^a)$$

$$S_{2m+2}c_1^{2m+2} = S'_{2(n-m+1)+1}c_2^{2(n-m+1)+1}$$
$$= 2\sqrt{r(n+1)(s+g)} - \sqrt{2r\{(n-m+1)s+(n-m)g\}}. \quad (7^a)$$

$$S_{2m+2}c_2^{2m+2} = S'_{2(n-m+1)+1}c_1^{2(n-m+1)+1}$$
$$= 2\sqrt{r(n+1)(s+g)} - \sqrt{2r(n-m+1)(s+g)}. \quad (8^a)$$

a. $s \gtreqless g$.

If n be even, the reverse curves meet in the centre of the space between the two middle roads, and the rails of the $\frac{n}{2}$ intervening roads, on each side of the centre, are all cut by the rails of the reverse curve, which is on the same side of the centre line; consequently we obtain the circular measures of the angles of the crossings by substituting for m in equations (1), (2), (3), and (4), the successive values

$$1, 2, 3 \ldots \&c. \ldots \frac{n}{2},$$

and the distances of the points of the crossings of the rails from the normals, by substituting for m, in equations (5), (6), (7), and (8), the successive values

$$1, 2, 3 \ldots \&c. \ldots \frac{n}{2};$$

and in equations (5^a), (6^a), (7^a), and (8^a), the successive values

$$\frac{n+2}{2}, \frac{n+4}{2} \ldots \&c. \ldots n.$$

If n be odd, the reverse curves meet in the centre of the middle road, one rail of which is therefore cut by the rails of one reverse curve, and the other rail by the other; consequently we obtain the circular measures of the crossings of the first $n-1$ rails on either side of the middle road, by substituting for m the successive values

$$1, 2, 3 \ldots \&c. \ldots \frac{n-1}{2}$$

in equations (1), (2), (3), and (4); and the circular measures of the angles of the crossings of the middle road, by putting

$$m = \frac{n+1}{2}$$

in equations (1) and (2).

The distances of the crossings of the $n-1$ rails, on either side of the middle road from the normals, are obtained by substituting for m, in equations (5), (6), (7), and (8), the successive values

$$1, 2, 3 \ldots \&c. \ldots \frac{n-1}{2},$$

and in equations (5^a), (6^a), (7^a), and (8^a), the successive values

$$\frac{n+3}{2}, \frac{n+5}{2} \ldots \&c. \ldots n.$$

The distances of the crossings of the middle road from the normals are obtained by putting

$$m = \frac{n+1}{2}$$

in equations (5), (6), (7^a), and (8^a).

β. $s < g$.

If n be even, since the reverse curves meet in the middle of the space s, between the two middle roads, the $(n+2)^{th}$ and $(n+3)^{th}$ main-road rails may be cut by the rails of the reverse curves, whose springings do not lie on the same side of the centre line of the n intervening roads as the rails in question. To ascertain this, we must follow the method described in the case of cross-over roads.

The offset to the tangent to the reverse-curve rail $S_1 S'_{2n+3}$ at S_1, which passes through the point c_1^{n+2} is equal to

$$\frac{(n+2)g + ns}{2},$$

therefore the length of the tangent is equal to

$$\sqrt{\frac{(2r+g)\{(n+2)g+ns\}}{2}}.$$

Now, the length of the tangent to the reverse-curve rail $S_2 S'_{2n+4}$ at S_2, whose offset passes through the point, where the

normal $O c_1^{2n+2}$ cuts the rail $S_2 S'_{2n+4}$, is to the length of the tangent at S_1 as $2r - g : 2r + g$; consequently the length of the offset, which passes the point where the normal $O c_1^{n+2}$ cuts $S_2 S'_{2n+4}$, is

$$\left(\frac{2r-g}{2r+g}\right)^2 \frac{(2r+g)}{2(2r-g)} \frac{\{(n+2)g+ns\}}{} = \frac{2r-g}{2r+g} \left\{\frac{(n+2)g+ns}{2}\right\}.$$

If therefore the $(n+3)^{rd}$ rail be not cut by the curve, which springs from the points S_1 and S_2, the interval between the rails $S_2 S'_2$ and $S_{n+2} S'_{n+2}$ must not be less than this offset, or we must have

$$\frac{(n+2)s+ng}{2} \geqq \frac{2r-g}{2r+g} \left\{\frac{(n+2)g+ns}{2}\right\},$$

which is equivalent to

$$s \gtreqless g.$$

Consequently, when s is less than g, the rail $S_{n+2} S'_{n+2}$ is cut by the reverse curve, which springs from the points S_1, S_2; and the rail $S_{n+2} S'_{n+2}$ by the reverse curve, which springs from the points S'_{2n+2}, S'_{2n+4}.

In order therefore to get the circular measures and positions of the angles of the crossings of the $n-2$ intermediate rails on either side of the two middle roads, we must substitute for m, in equations (1), (2), (3), and (4), the successive values

$$1, 2, 3 \ldots \&c. \ldots \frac{n-2}{2},$$

and in equations (5), (6), (7), and (8), the successive values

$$1, 2, 3 \ldots \&c. \ldots \frac{n-2}{2},$$

and in equations (5^a), (6^a), (7^a), and (8^a), the successive values

$$\frac{n+4}{2}, \frac{n+6}{2} \ldots \&c. \ldots n.$$

To find the circular measures and positions of the angles c_1^{n+1}, c_2^{n+1}, c_2^{n+2}, c_1^{n+3}, c_1^{n+4}, and c_2^{n+4}, we must put $m = \frac{n}{2}$ in

equations (1), (2), and (4); also $m = \frac{n}{2}$ in equations (5), (6), and (8); and $m = \frac{n}{2} + 1$ in equations (5^a), (7^a), and (8^a).

The value and positions of the remaining angles, c_1^{n+2} and c_2^{n+2}, are given by the equations

$$c_1^{n+2} = c_2^{n+2} = \frac{2\sqrt{r\{(n+2)s + ng\}}}{2r - g},$$

$$S_{n+2}\, c_2^{n+2} = S'_{n+2}\, c_1^{n+2} = \sqrt{r\{(n+2)s + ng\}},$$

$$S_{n+2}\, c_1^{n+2} = S'_{n+2}\, c_2^{n+2} = 2\sqrt{r(n+1)(s+g)} - \sqrt{r\{(n+2)s + ng\}}.$$

If n be odd, since the curves reverse in the centre of the middle road, the formulæ must be used as explained in (*a*).

CASE III.—*Main Roads Curved: Reverse Curves separated by an interval of Straight, which extends from extreme crossing to extreme crossing.*

Let O, O′, O″ (Fig. 18) be the centres of the main-line and reverse curves; r_1 the radius of the curve centre O′; r_2 that of curve centre O″. If R be the radius of the centre line of the outside road $S_1 S'_1$, the radius of the centre line of the m^{th} intermediate road will be

$$R - m(s + g),$$

and of the inside road $S_{2n+4} S'_{2n+4}$

$$R - (n+1)(s+g).$$

Let the normals O O′, O O″ cut the main-line rails at the points $S'_1 S'_2 \ldots S'_{2m+1} S'_{2m+2} \ldots S'_{2n+3} S'_{2n+4}$ and $S_1 S_2 \ldots S_{2m+1} S_{2m+2} \ldots S_{2m+3} S_{2m+4}$ respectively; and let the rest of the letters have the same signification as before.

Join $O'' c_1^2$, $O' c_2^{2n+3}$. Let these meet the centre line of the through road in the points p'' p', and let the straight lines

$c_1^2 c_1^{2n+2} p'' p'$, and $c_2^3 c_2^{2n+2}$ meet a line through O, parallel to the radii $O'' c_1^2$, $O' c_2^{2n+2}$, in the points P″, P′, and P respectively. Draw O″ Q″, O′ Q′ at right angles to the radii $O'' c_1^2$, $O' c_2^{2n+2}$, meeting O P″ and O P″ produced in Q″ and Q′.

The perpendicular from O, upon the centre line $p' p''$ produced, is equal to

$$\begin{aligned} O Q' - P' Q' &= O O' \cos. Q' O O' - O' p' \\ &= \{R + r_1 - (n+1)(s+g)\} \cos. O' - r_1 \\ &= R - (n+1)(s+g) - \{R + r_1 - (n+1)(s+g)\} \frac{O'^2}{2} \end{aligned}$$

neglecting the fourth and higher powers of O′ in the expansion of cos. O′. This perpendicular is likewise equal to

$$\begin{aligned} O Q'' + Q'' P' &= O O'' \cos. Q'' O O'' + O'' p' \\ &= (R - r_2) \cos. O'' + r_2 \\ &= R - (R - r_2) \frac{O''^2}{2}. \end{aligned}$$

Equating these two values, we get

$$(R - r_2) O''^2 = \{R + r_1 - (n+1)(s+g)\} O''^2 + 2(n+1)(s+g).$$

a. $s \gtrless g$.

The rails of the reverse curve will cut the two inner rails of the outside roads.

The offset to the tangent to the curve, radius $r_2 + \frac{g}{2}$, at S_1, which passes through c_1^2, is equal to the gauge plus the offset to the tangent to the curve, radius $R - \frac{g}{2}$, at S_2, which likewise passes through c_1^2. Since the length of the tangent is $O''\left(r_2 + \frac{g}{2}\right)$, we have

$$\frac{O''^2\left(r_2 + \frac{g}{2}\right)}{2} = g + \frac{O''^2\left(r_2 + \frac{g}{2}\right)^2}{2\left(R - \frac{g}{2}\right)}.$$

If we neglect the terms $O''^2 g$, $\frac{O''^2 r_2 g}{R}$, $O''^2 g^2$, and use the radius R, instead of $R - \frac{g}{2}$, we get

$$\frac{O''^2 r_2}{2} = g + \frac{O''^2 r_2^2}{R}.$$

Similarly,

$$\frac{O''^2 r_1}{2} = g - \frac{O''^2 r_1^2}{R};$$

therefore

$$O''^2 = \frac{2Rg}{r_2(R - r_2)},$$

$$O'^2 = \frac{2\{R - (n+1)(s+g)\}g}{r_1\{R + r_1 - (n+1)(s+g)\}}$$

$$= \frac{2Rg}{r_1\{R + r_1 - (n+1)(s+g)\}}.$$

Substituting these values of O'^2, O''^2 in the equation of relation between r_1, r_2, O', O'', we get

$$\frac{R}{r_2} = \frac{R - (n+1)(s+g)}{r_1} + \frac{(n+1)(s+g)}{g},$$

whence

$$r_1 = \frac{\{R - (n+1)(s+g)\} r_2 g}{Rg - (n+1)(s+g) r_2},$$

$$O = O'' - O'$$

$$= \sqrt{\frac{2Rg}{r_2(R - r_2)}} - \sqrt{\frac{2Rg}{r_1\{R + r_1 - (n+1)(s+g)\}}}.$$

If we substitute for O''^2 and O'^2 their values in terms of the radii r_2 and r_1 in the expressions for the length of the perpendicular from the origin upon the line $p'p''$, we get

$$O P' = \frac{R(r_2 - g)}{r_2} = \frac{\{R - (n+1)(s+g)\}(r_1 - g)}{r_1}.$$

The length of the perpendicular from O upon $c_1^2 P''$ is therefore equal to

$$\frac{R(r_2 - g)}{r_2} + \frac{g}{2} = \frac{2Rr_2 - g(2R - r_2)}{2r_2},$$

and upon c_2^3 P

$$\frac{R(r_2 - g)}{r_2} - \frac{g}{2} = \frac{2Rr_2 - g(2R + r_2)}{2r_2}.$$

If $\rho\,\theta$ be the polar co-ordinates of any point in either of these lines referred to origin O and initial line O O'', the equation to c_1^2 P'' will be

$$\rho \cos.(O'' - \theta) = \frac{2Rr_2 - g(R - r_2)}{2r_2},$$

and to c_2^3 P

$$\rho \cos.(O'' - \theta) = \frac{2Rr_2 - g(R + r_2)}{2r_2}.$$

At the points, where either of these lines cuts the rails of the main road, the angle $O'' - \theta$ is the angle between the normal to the curve of the main-line rail at the crossing, and the perpendicular to the straight part of the through road, and is therefore equal to the angle of the crossing.

For the m^{th} intervening road the values of ρ are

$$\rho = R - m(s + g) + \frac{g}{2}$$

and

$$\rho = R - m(s + g) - \frac{g}{2}.$$

Therefore, putting these values of ρ successively in the equations to the lines c_1^2 P'' and c_2^3 P, we get

$$c_1^{2m+1} = \sqrt{1 - \frac{\{2Rr_2 - g(2R - r_2)\}^2}{4r_2^2\left\{R - m(s + g) + \frac{g}{2}\right\}^2}}$$

$$= \frac{2\sqrt{2Rr_2\{Rg - r_2 m(s+g)\}}}{r_2\{2R - 2m(s + g) + g\}}. \qquad (4)$$

$$c_2^{2m+1} = \sqrt{1 - \frac{\{2Rr_2 - g(2R + r_2)\}^2}{4r_2^2\left\{R - m(s + g) + \frac{g}{2}\right\}^2}}$$

$$= \frac{2\sqrt{2Rr_2\{(R + r_2)g - r_2 m(s+g)\}}}{r_2\{2R - 2m(s + g) + g\}}. \qquad (5)$$

$$c_1^{2m+2} = \sqrt{1 - \frac{\{2\,R\,r_2 - g\,(2\,R - r_2)\}^2}{4\,r_2^2\left\{R - m\,(s+g) - \frac{g}{2}\right\}^2}}$$

$$= \frac{2\sqrt{2\,R\,r_2\,\{(R - r_2)\,g - r_2\,m\,(s+g)\}}}{r_2\,\{2\,R - 2\,m\,(s+g) - g\}}. \qquad (6)$$

$$c_2^{2m+2} = \sqrt{1 - \frac{\{2\,R\,r_2 - g\,(2\,R + r_2)\}^2}{4\,r_2^2\left\{R - m\,(s+g) - \frac{g}{2}\right\}^2}}$$

$$= \frac{2\sqrt{2\,R\,r_2\,\{R\,g - r_2\,m\,(s+g)\}}}{r_2\,\{2\,R - 2\,m\,(s+g) - g\}}. \qquad (7)$$

If we substitute for m in each of these equations successive values 1, 2, 3 ... &c.... n we shall obtain the circular measures of the angles of the crossings of the n intermediate roads.

Also,

$$c_1^2 = \sqrt{1 - \frac{\{2\,R\,r_2 - g\,(2\,R - r_2)\}^2}{4\,r_2^2\left(R - \frac{g}{2}\right)^2}}$$

$$= \frac{2\sqrt{2\,R\,r_2\,(R - r_2)\,g}}{r_2\,(2\,R - g)}.$$

$$c_2^{2n+2} = \sqrt{1 - \frac{\{2\,R\,r_2 - g\,(2\,R + r_2)\}^2}{4\,r_2^2\left\{R - (n+1)\,(s+g) + \frac{g}{2}\right\}^2}}$$

$$= \frac{2\sqrt{2R\,r_2\{R\,g - r_2\,(n+1)\,(s+g) + r_2\,g\}}}{r_2\,\{2\,R - 2\,(n+1)\,(s+g) + g\}}.$$

The linear distance of any of the crossings from the line $S_1\,S_{2n+4}$ is equal to the radius of the curve of the rail of the main line on which the crossing is situated, or, with sufficient exactness, to the radius of the centre line of the intermediate road referred to, multiplied by the circular measure of its angular co-ordinate, or by θ, consequently

$$S_{2m+1}\,c_1^{2m+1} = \{R - m\,(s+g)\}\,\theta$$

$$= \{R - m\,(s+g)\}\,\{O'' - c_1^{2m+1}\}. \qquad (8)$$

$$S_{2m+1}\,c_2^{2m+1} = \{R - m\,(s+g)\}\,\{O'' - c_2^{2m+1}\}. \qquad (9)$$

$$S_{2m+2}\,c_1^{2m+2} = \{R - m\,(s+g)\}\,\{O'' - c_1^{2m+2}\}. \qquad (10)$$

$$S_{2m+2}\,c_2^{2m+2} = \{R - m\,(s+g)\}\,\{O'' - c_2^{2m+2}\}. \qquad (11)$$

If we substitute for m in each of these equations the successive values

$$1, 2, 3 \ldots \&c. \ldots n$$

we shall obtain the distances of every crossing of the n intermediate rails from the normal line $S_1 S_{2n+4}$. The distance of each of the crossings of the m^{th} road from the other normal is equal to $\{R - m(s+g)\}O$, minus its distance from $S_1 S_{2n+4}$. This distance may, however, be found directly from the following equations:

$$S_{2m+1}\, c_1^{2m+1} = \{R - m(s+g)\}\,(c_1^{2m+1} - O'), \qquad (8^a)$$

$$S_{2m+1}\, c_2^{2m+1} = \{R - m(s+g)\}\,(c_2^{2m+1} - O'), \qquad (9^a)$$

$$S_{2m+2}\, c_1^{2m+2} = \{R - m(s+g)\}\,(c_1^{2m+2} - O'), \qquad (10^a)$$

$$S_{2m+2}\, c_2^{2m+2} = \{R - m(s+g)\}\,(c_2^{2m+2} - O'), \qquad (11^a)$$

because they are equal to the radii of the curves to which they belong, multiplied by the circular measure of $(O - \theta)$, which is equal to

$$O'' - O' - (O'' - c) = c - O',$$

c being the circular measure of the angle of the crossing in question.

Also,

$$S_2\, c_1^2 = R\,(O'' - c_1^2), \qquad (12)$$

$$S'_2\, c_1^2 = R\,O - R\,(O'' - c_1^2) = R\,(c_1^2 - O'), \qquad (13)$$

$$S_{2n+3}\, c_2^{2n+3} = \{R - (n+1)(s+g)\}\,(O'' - c_2^{2n+3}), \qquad (14)$$

$$S'_{2n+3} c_2^{2n+3} = \{R-(n+1)(s+g)\}O - \{R-(n+1)(s+g)\}(O'' - c_2^{2n+3})$$

$$= \{R - (n+1)(s+g)\}\,(c_2^{2n+3} - O'). \qquad (15)$$

$s < g$.

The rails of the reverse curves will cut the rails of the intermediate roads. In this case the offset to the tangent to the curve, radius $r_2 - \frac{g}{2}$, at the point S_2, which passes through the point c_2^3, is equal to s plus the offset to the tangent to the curve, radius $R - s - \frac{g}{2}$, at the point S_3, which passes through the point c_2^3. Therefore by the same steps as in (a)

$$\frac{O''^3 r_2}{2} = s + \frac{O''^3 r_2^2}{2R},$$

E

and

$$\frac{O''_2 r_1}{2} = s - \frac{O''_2 r_1^2}{2\{R-(n+1)(s+g)\}};$$

therefore

$$O'_2 = \frac{2Rs}{(R-r_2)r_2},$$

$$O''_2 = \frac{2\{R-(n+1)(s+g)\}s}{r_1\{R+r_1-(n+1)(s+g)\}}$$

$$= \frac{2Rs}{r_1\{R+r_1-\overline{n+1}.(s+g)\}}.$$

Substituting these values in the equation of relation between r_1, r_2, O', O'', we get

$$\frac{R}{r_2} = \frac{R-(n+1)(s+g)}{r_1} + \frac{(n+1)(s+g)}{s};$$

whence

$$r_1 = \frac{\{R-(n+1)(s+g)\}r_2 s}{Rs - r_2(n+1)(s+g)},$$

$$r_2 = \frac{Rr_1 s}{\{R-(n+1)(s+g)\}s + r_1(n+1)(s+g)},$$

$$O = O'' - O'$$

$$= \sqrt{\frac{2Rs}{(R-r_2)r_2}} - \sqrt{\frac{2Rs}{r_1\{R+r_1-(n+1)(s+g)\}}}.$$

The length of the perpendicular from O on $p''p'P'$ is

$$\frac{R(r_2-s)}{r_2} = \frac{\{R-(n+1)(s+g)\}(r_1-s)}{r_1},$$

and on $c_1^2 P''$

$$\frac{2Rr_2 - 2Rs + r_2 g}{2r_2},$$

and on $c_1^3 P$

$$\frac{2Rr_2 - 2Rs - r_2 g}{2r_2};$$

therefore

$$c_1^{2m+1} = \sqrt{1 - \frac{\{2R(r_2-s)+r_2 g\}^2}{4r_2^2\left\{R-m(s+g)+\frac{g}{2}\right\}^2}}$$

$$= \frac{2\sqrt{2Rr_2\{Rs - r_2 m(s+g)\}}}{r_2\{2R-2m(s+g)+g\}},$$

$$c_2^{2m+1} = \sqrt{1 - \frac{\{2R(r_2 - s) - r_2 g\}^2}{4r_2^2\left\{R - m(s+g) + \frac{g}{2}\right\}^2}}$$

$$= \frac{2\sqrt{2Rr_2\{Rs - r_2 m(s+g) + r_2 g\}}}{r_2\{2R - 2m(s+g) + g\}},$$

$$c_1^{2m+2} = \sqrt{1 - \frac{\{2R(r_2 - s) + r_2 g\}^2}{4r_2^2\left\{R - m(s+g) - \frac{g}{2}\right\}^2}}$$

$$= \frac{2\sqrt{2Rr_2\{Rs - r_2 m(s+g) - r_2 g\}}}{r_2\{2R - 2m(s+g) - g\}}; \qquad (6)$$

$$c_2^{2m+2} = \sqrt{1 - \frac{\{2R(r_2 - s) - r_2 g\}^2}{4r_2^2\left\{R - m(s+g) - \frac{g}{2}\right\}^2}}$$

$$= \frac{2\sqrt{2Rr_2\{Rs - r_2 m(s+g)\}}}{r_2\{2R - 2m(s+g) - g\}}. \qquad (7)$$

If we substitute for m in each of these equations successive values 1, 2, 3 ... &c. .... n we shall obtain the circular measures of the angles of the crossings of the n intermediate roads.

Also

$$c_1^2 = \sqrt{1 - \frac{\{2R(r_2 - s) + r_2 g\}^2}{4r_2^2\left(R - \frac{g}{2}\right)^2}}$$

$$= \frac{2\sqrt{2Rr_2(Rs - r_2 g)}}{r_2(2R - g)},$$

$$c_2^{2n+2} = \sqrt{1 - \frac{\{2R(r_2 - s) - r_2 g\}^2}{4r_2^2\left\{R - (n+1)(s+g) + \frac{g}{2}\right\}^2}}$$

$$= \frac{2\sqrt{2Rr_2\{Rs - r_2(n+1)(s+g) + r_2 g\}}}{r_2\{2R - 2(n+1)(s+g) + g\}}.$$

If in equations (8), (9), (10), (11), (8^a), (9^a), (10^a), and (11^a) we give O′ O″ and the symbols c_1^2 c_1^{m+1}, &c., their proper values, and give to m successive values 1, 2, 3 ... &c. .... n we shall then obtain the positions of all the crossings of the intermediate roads.

Equations (12), (13), (14), and (15) will in like manner give the positions of c_1^2, c_2^{2n+3}.

If one of the radii r_1, r_2 be given, one of the equations (1) (2) in (α) and (β) will give the other: equation (3) in (α) and (β) gives the angular interval between the springings of the reverse curves. It is impossible to get a simple solution of the converse of the proposition, *viz.* given the angular interval to find the radii without transgressing the limits of deviation from exactness to which the formulæ have been restricted.

When $s \gtrless g$ the equations which give the values of r_1, r_2 are

$$r_1 = \frac{\{R - (n+1)(s+g)\} r_2 g}{R g - (n+1)(s+g) r_2}, \quad (1)$$

$$r_2 = \frac{R r_1 g}{\{R - (n+1)(s+g)\} g + (n+1)(s+g) r_1}. \quad (2)$$

If in (1) and (2) we write r_1 for r_2, r_2 for r_1, and $-R$ for R_1, we get

$$r_2 = \frac{\{R + (n+1)(s+g)\} r_1 g}{R g + (n+1)(s+g) r_1} \quad (1^a)$$

$$r_1 = \frac{R r_2 g}{\{R + (n+1)(s+g)\} g - (n+1)(s+g) r_2}. \quad (2^b)$$

If R, in equations (1^a) and (2^a), be the radius of the innermost road, we see that the value of r_2 given by (1^a) is identical with the value of r_2 given by (2) in terms of r_1; and the value of r_1 given by (2^a) is identical with the value of r_1 given by (1) in terms of r_2. Hence from the formulæ giving the positions and circular measures of the angles of the crossings in terms of r_2 and the radius of the outside road, we can obtain formulæ in terms of r_1 and the radius of the innermost road by simply writing r_1 for r_2 and $-R$ for R. The numbering of the rails in this latter case of course dating from the springing of the reverse curve radius r_1.

We may show in the same way that the same rule holds when $s < g$.

CASE IV.—*Main Roads Curved: Reverse Curves meeting at common point of contrary flexure midway between the two Roads, between which communication is established.*

Let the symbols in Fig. 19 have the same meaning as in Fig. 18: join O′ O″, cutting the centre line of the space between the two outside roads in the point P. If n be odd, P will be in the centre of the middle road. If n be even, P will be in the centre of the space between the two middle roads. The radius of the centre line of the system of roads is

$$R - \frac{(n+1)(s+g)}{2}$$

whether n be even or odd. Let it be denoted by R_0. Following the method of Section IV., Case II., we get

$$O' = \frac{R_0 - r_2}{r_1 + r_2} O = \frac{R_0 O}{2 r_1},$$

$$O'' = \frac{R_0 + r_1}{r_1 + r_2} O = \frac{R_0 O}{2 r_2},$$

$$r_1 = \frac{R_0 r_2}{R - 2 r_2}, \qquad r_2 = \frac{R_0 r_1}{R_0 + 2 r_1}; \tag{a}$$

$$O = \frac{2\sqrt{r_2 (n+1)(s+g)}}{\sqrt{R_0 (R_0 - r_2)}} = \frac{2\sqrt{r_1 (n+1)(s+g)}}{\sqrt{R_0 (R_0 + r_1)}}; \tag{b}$$

$$r_2 = \frac{R_0^2 O^2}{4(n+1)(s+g) + R_0 O^2}, \quad r_1 = \frac{R_0^2 O^2}{4(n+1)(s+g) - R_0 O^2}. \tag{d}$$

When either radius is given, formulæ (*a*) and (*b*) will give the other and the angular interval O between the springings of the reverse curves. When this angular interval is given, formula (*d*) will give the radii.

The circular measure of this angular interval in terms of the linear distance D, between the normals through the actual points of the switches, measured along the centre line, radius R_0, is given by the equation

$$R_0 O = D + \frac{\sqrt{2 R_0 r_2 (c+t)}}{\sqrt{R_0 - r_2}} + \frac{\sqrt{2 R_0 r_1 (c+t)}}{\sqrt{R + r_1}} - 2l$$

$$= D + \frac{2 R O \sqrt{c+t}}{\sqrt{2(n+1)(s+g)}} - 2l,$$

since

$$\frac{r_2}{R_0 - r_2} = \frac{r_1}{R_0 + r_1} = \frac{R_0 O^2}{4(n+1)(s+g)};$$

therefore

$$O = \frac{(D - 2l)\sqrt{2(c+t)}}{R_0\{\sqrt{2(n+1)(s+g)} - 2\sqrt{c+t}\}}.$$

Whatever may be the relative values of s and g, except as regards the two middle rails, we shall obtain by following the methods pointed out in Section I., Cases II. and III.:—

For the circular measures of the angles, at which the main-line rails are cut by the rails of the reverse curve, radius r_2,

$$c_1^2 = \frac{2\sqrt{2Rr_2(R - r_2)g}}{2Rr_2 + (R - r_2)g},$$

$$c_1^{2m+1} = \frac{2\sqrt{2Rr_2(R - r_2)m(s+g)}}{2Rr_2 + g(R + r_2) - 2r_2 m(s+g)}, \quad (1)$$

$$c_2^{2m+1} = \frac{2\sqrt{2Rr_2(R - r_2)\{m(s+g) - g\}}}{2Rr_2 - g(R - r_2) - 2r_2 m(s+g)}, \quad (2)$$

$$c_1^{2m+2} = \frac{2\sqrt{2Rr_2(R - r_2)\{m(s+g) + g\}}}{2Rr_2 + g(R - r_2) - 2r_2 m(s+g)}, \quad (3)$$

$$c_2^{2m+2} = \frac{2\sqrt{2Rr_2(R - r_2)m(s+g)}}{2Rr_2 - g(R + r_2) - 2r_2 m(s+g)}. \quad (4)$$

For the circular measures of the angles, at which the rails of the intervening roads are cut by the rails of the reverse curve, radius r_1, if $R_1 = R - (n+1)(s+g)$,

$$c_2^{2n+3} = \frac{2\sqrt{2R_1 r_1(R_1 + r_1)g}}{2R_1 r_1 + (R_1 + r_1)g},$$

$$c_1^{2m+1} = \frac{2\sqrt{2R_1 r_1(R_1 + r_1)(n - m + 1)(s+g)}}{2R_1 r_1 - (R_1 - r_1)g + 2r_1(n - m + 1)(s+g)}, \quad (1^a)$$

$$c_2^{2m+1} = \frac{2\sqrt{2R_1 r_1(R_1 + r_1)\{(n - m + 1)(s+g) + g\}}}{2R_1 r_1 + (R_1 + r_1)g + 2r_1(n - m + 1)(s+g)}, \quad (2^a)$$

$$c_1^{2m+2} = \frac{2\sqrt{2R_1 r_1(R_1 + r_1)\{(n - m + 1)(s+g) - g\}}}{2R_1 r_1 - (R_1 + r_1)g + 2r_1(n - m + 1)(s+g)}, \quad (3^a)$$

$$c_2^{2m+2} = \frac{2\sqrt{2R_1 r_1(R_1 + r_1)(n - m + 1)(s+g)}}{2R_1 r_1 + (R_1 - r_1)g + 2r_1(n - m + 1)(s+g)}, \quad (4^a)$$

since the m^{th} intervening road from one of the outside roads is equal to the $(n - m + 1)^{th}$ from the other.

Also the distances of the crossings of the first $n - 1$ rails of the intervening roads from the normal $S_1 S_{2m+4}$ are given by the equations

$$S_{2m+1} c_1^{2m+1} = \sqrt{\frac{2 R r_2 m (s+g)}{R - r_2}}, \tag{5}$$

$$S_{2m+1} c_2^{2m+1} = \sqrt{\frac{2 R r_2 \{m (s+g) - g\}}{R - r_2}}, \tag{6}$$

$$S_{2m+2} c_1^{2m+2} = \sqrt{\frac{2 R r_2 \{m (s+g) + g\}}{R - r_2}}, \tag{7}$$

$$S_{2m+2} c_2^{2m+2} = \sqrt{\frac{2 R r_2 m (s+g)}{R - r_2}}. \tag{8}$$

The distances of the crossings of the last $(n - 1)$ rails from the same normal are given by the equations

$$S_{2m+1} c_1^{2m+1} = \{R - m (s+g)\} O - \sqrt{\frac{2 R_1 r_1 (n - m + 1) (s+g)}{R_1 + r_1}}, \tag{5a}$$

$$S_{2m+1} c_2^{2m+1} = \{R - m (s+g)\} O - \sqrt{\frac{2 R_1 r_1 \{(n - m + 1) (s+g) + g\}}{R_1 + r_1}}, \tag{6a}$$

$$S_{2m+2} c_1^{2m+2} = \{R - m (s+g)\} O - \sqrt{\frac{2 R_1 r_1 \{(n - m + 1) (s+g) - g\}}{R_1 + r_1}}, \tag{7a}$$

$$S_{2m+2} c_2^{2m+2} = \{R - m (s+g)\} O - \sqrt{\frac{2 R_1 r_1 (n - m + 1) (s+g)}{R_1 + r_1}}. \tag{8a}$$

The distance of each crossing of either set from the other normal is equal to $\{R - m (s+g)\} O$, minus its distance from the first normal.

The circular measures and positions of the crossings of the two middle rails are given by one or other of the above equations, in the manner explained in the following clauses (α) and (β).

a. $s \gtreqless g$.

If n be even, the reverse curves meet in the centre of the space between the two middle roads; and we shall obtain the circular measures of the angles of the crossings of the main-road rails, cut by the curve radius r_2, and the distances of those crossings from the normal $S_1 S_{2n+4}$, by giving m the successive values

$$1, 2, 3 \ldots \frac{n}{2},$$

in equations (1), (2), (3), (4), (5), (6), (7), and (8).

The circular measures of the angles of the crossings of the main-road rails cut by the curve radius r_1, and the distances of those crossings from the same normal, by giving m the successive values

$$\frac{n}{2}+1, \frac{n}{2}+2 \ldots n,$$

in equations (1^a), (2^a), (3^a), (4^a), (5^a), (6^a), (7^a), and (8^a).

If n be odd, the reverse curve will intersect in the centre of the middle road, and we shall obtain the circular measures of the angles of the crossings of the main-road rails cut by the curve radius r_2, and the distances of those crossings from the normal $S_1 S_{2n+4}$, by giving m the successive values

$$1, 2, 3 \ldots \frac{n-1}{2},$$

in equations (1), (2), (3), (4), (5), (6), (7), and (8), and the value $\frac{n+1}{2}$ in equations (1), (2), (5), and (6).

The circular measures of the angles of the crossings of the main-road rails, cut by the curve radius r_1, and the distances of those crossings from the same normal are obtained by giving m the successive values

$$\frac{n+3}{2}, \frac{n+5}{2} \ldots n,$$

in equations (1^a), (2^a), (3^a), (4^a), (5^a), (6^a), (7^a), and (8^a), and the value $\frac{n+1}{2}$ in equations (1^a), (2^a), (5^a), and (6^a).

β. $s < g$.

If n be even, since the curves reverse in the centre of the space between the two middle roads, the n^{th} and $(n+1)^{th}$ rails may be cut by the rails of the reverse curves, whose springings do not lie on the same side of the centre line of the n intervening roads as the rails in question.

The length of the tangent at S_{n+2} to the curve, radius $R - \frac{n(s+g)}{2}$, whose offset passes through c_1^{n+2}, is equal to the distance of c_1^{n+2} from S_{n+2}, if $S_{n+2}S'_{n+2}$ be cut by the curve radius r_2, and is therefore

$$\sqrt{\frac{R r_2 \{n(s+g)+2g\}}{R - r_2}}.$$

This is likewise equal to the tangent at S_1 to the curve radius $r_2 + \frac{g}{2}$, whose offset passes through c_1^{n+2}. Since the length of the tangent at S_2, whose offset passes through the point, where the normal $O'' c_1^{n+2}$ meets the rail $S_2 S'_{2n+4}$, is to this in the ratio $r_2 - \frac{g}{2} : r_2 + \frac{g}{2}$, the length of the offset to the tangent at S_2, just described, is equal to

$$\left(\frac{2r_2 - g}{2r_2 + g}\right)^2 \frac{R\{n(s+g)+2g\}}{2(R - r_2)}, \qquad (a)$$

and the length of the offset to the curve radius $R - \frac{n(s+g)}{2}$ at S_{2n+2}, for the same length of tangent, is

$$\left(\frac{2r_2 - g}{2r_2 + g}\right) \frac{r_2\{n(s+g)+2g\}}{2(R - r_2)}, \qquad (b)$$

since for this purpose we may use R instead of $R - \frac{n(s+g)}{2}$.

Now, the offset (a) must not be greater than the sum of space between the rails $S_2 S'_2$ and $S_{n+2} S'_{n+2}$, plus the offset (b), or, transposing, we must have

$$\frac{n+2}{2} s + \frac{n}{2} g \gtrless \left(\frac{2r_2 - g}{2r_2 + g}\right)^2 \left\{\frac{R}{2(R - r_2)} - \frac{r_2}{2(R - r_2)}\right\} \times \{n(s+g)+2g\} \gtrless \frac{n(s+g)+2g}{2},$$

or

$$s \gtrless g.$$

If this ratio of inequality hold, we must use the formulæ as explained in (*a*); if it do not, we must find the circular measures of the angles of the crossings of the first $(n-2)$ rails of the intervening roads, cut by the rails of the curve radius r_2, by giving m the successive values

$$1, 2, 3 \ldots \frac{n}{2} - 1$$

in equations (1), (2), (3), (4), (5), (6), (7), and (8): of the crossings c_1^{n+1}, c_2^{n+1}, c_2^{n+2}, by putting $m = \frac{n}{2}$ in equations (1), (2), (4), (5), (6), and (8); and of the crossing c_1^{n+2} by putting $m = \frac{n}{2}$ in equations (3^a) and (7^a).

Similarly, we must find the circular measures and distances of the crossing c_2^{n+2} from the normal $S_1 S_{2n+4}$ by putting $m = \frac{n}{2} + 1$ in equations (2) and (6): of the crossings c_1^{n+3}, c_1^{n+4}, c_2^{n+4}, by putting $m = \frac{n}{2} + 1$ in equations (1^a), (3^a), (4^a), (5^a), (7^a), and (8^a); and of the crossings of the last $n-2$ rails of the intervening roads, cut by the rails of the curve radius r, by giving m the successive values

$$\frac{n}{2} + 2, \frac{n}{2} + 3 \ldots n,$$

in equations (1^a), (2^a), (3^a), (4^a), (5^a), (6^a), (7^a), and (8^a).

If n be odd, since the curves intersect in the centre of the middle road, the formulæ must be used as explained in (*a*).

As in Case III., formulæ in terms of r_2 may be converted into formulæ in terms of r_1 by putting r_1 for r_2 and $-R$ for R; R in this latter case being the radius of the innermost road, and *vice versâ*.

The formulæ in the preceding four cases give the position of every crossing. In practice, however, the simplest plan is to set out the centre line of the through road on the ground.

In Cases I. and III., if the position of the crossings c_1^2, c_1^{2n+2}, c_2^3, and c_2^{2n+3}, be marked on the ground by means of the distances from the springing given by the formulæ, straight lines

joining c_1^2 with c_1^{2n+2} and c_2^3 with c_2^{2n+2} will show the positions of the crossings of the intervening rails.

In Cases II. and IV. the formulæ give the intervals between the springings of the curves in each case. The positions of the springings can thus be marked out on the ground, and the centre lines of the reverse curves set out.

To simplify this, and to ensure the correctness of the setting out, the point where the reverse curves meet may be marked out on the ground. The distance of this point from the normal line through either springing, measured along a line, midway between the two outside roads, is given by the following formulæ:

Case II.,

$$\sqrt{(n+1)(s+g)r};$$

Case IV.,

$$\sqrt{\frac{R_0 r_2 (n+1)(s+g)}{R_0 - r_2}} = \sqrt{\frac{R_0 r_1 (n+1)(s+g)}{R_0 + r_1}}.$$

Section VI.—Crossings on the Level.

The data on which the calculations are founded are the radii of the main roads and the angle at which the centre lines of the two roads cut each other. Let this angle be denoted by φ. Following the principle of the notation explained in Chapter I., the positions and circular measures of the angles at which the first, second, third, and fourth rails of the crossing road cut the corresponding rails of the road crossed, will be $c_1^1 c_1^2 c_1^3 c_1^4$, $c_2^1 c_2^2 c_2^3 c_2^4$, $c_3^1 c_3^2 c_3^3 c_3^4$, and $c_4^1 c_4^2 c_4^3 c_4^4$. The two outside rails, which cut one another externally at an obtuse angle, have the same denomination, that is, are either both first or both fourth rails. When a single line crosses a double line, there will only occur the two first groups of crossings; and when a single line crosses a single line, only the crossings $c_1^1 c_1^2 c_2^1 c_2^2$. The staked-out centre lines in these two last cases may not occupy the same position relatively to the rails as in the case of a double road crossing a double road; consequently three sets of formulæ are apparently required. It will, however, be necessary to give only one set, *viz.* that for a double road crossing a double road,

since the angles at which the centre lines of the roads, if doubled, would intersect, may be obtained by calculation from the observed angle of intersection of the staked-out centre lines, and the formulæ will give the required circular measures in terms of this calculated angle.

Case I.—*Both Roads Straight.*

All the angles are equal each to each and to the observed angle.

Case II.—*Crossed Roads Straight; Crossing Road Curved.*

The length of the perpendicular from the centre of the crossing road upon the centre line of the road crossed will be

$$r \cos. \varphi$$

if r be the radius of the centre line of the crossing road, and therefore the lengths of the perpendiculars from the same centre upon the first, second, third, and fourth rails of the crossed road will be

$$r \cos. \varphi + \frac{s}{2} + g,$$

$$r \cos. \varphi + \frac{s}{2},$$

$$r \cos. \varphi - \frac{s}{2},$$

$$r \cos. \varphi - \frac{s}{2} - g.$$

Now, the cosine of the angle of any crossing is equal to the ratio of the perpendicular from the centre of the crossing road upon the rail crossed to the radius of the crossing rail; therefore

$$\cos. c_1^1 = \frac{2r \cos. \varphi + s + 2g}{2r + s + 2},$$

$$\cos. c_1^2 = \frac{2r \cos. \varphi + s}{2r + s + 2g},$$

$$\cos. c_1^3 = \frac{2r \cos. \varphi - s}{2r + s + 2g},$$

$$\text{cos. } c_1^4 = \frac{2r \text{ cos. } \varphi - s - 2g}{2r + s + 2g},$$

$$\text{cos. } c_2^1 = \frac{2r \text{ cos. } \varphi + s + 2g}{2r + s},$$

$$\text{cos. } c_2^2 = \frac{2r \text{ cos. } \varphi + s}{2r + s},$$

$$\text{cos. } c_2^3 = \frac{2r \text{ cos. } \varphi - s}{2r + s},$$

$$\text{cos. } c_2^4 = \frac{2r \text{ cos. } \varphi - s - 2g}{2r + s},$$

$$\text{cos. } c_3^1 = \frac{2r \text{ cos. } \varphi + s + 2g}{2r - s},$$

$$\text{cos. } c_3^2 = \frac{2r \text{ cos. } \varphi + s}{2r - s},$$

$$\text{cos. } c_3^3 = \frac{2r \text{ cos. } \varphi - s}{2r - s},$$

$$\text{cos. } c_3^4 = \frac{2r \text{ cos. } \varphi - s - 2g}{2r - s},$$

$$\text{cos. } c_4^1 = \frac{2r \text{ cos. } \varphi + s + 2g}{2r - s - 2g},$$

$$\text{cos. } c_4^2 = \frac{2r \text{ cos. } \varphi + s}{2r - s - 2g},$$

$$\text{cos. } c_4^3 = \frac{2r \text{ cos. } \varphi - s}{2r - s - 2g},$$

$$\text{cos. } c_4^4 = \frac{2r \text{ cos. } \varphi - s - 2g}{2r - s - 2g}.$$

The calculated values of the angle to be substituted for the observed angle ϕ in the above formulæ are

α. Single Line crossing Single Line.

$$\text{cos.}^{-1} \frac{(2r + s + g) \text{ cos. } \varphi - s - g}{2r}.$$

β. Single Line crossing Double Line.

$$\text{cos.}^{-1} \frac{(2r + s + g) \text{ cos. } \varphi}{2r}.$$

γ. Double Line crossing Single Line.

$$\text{cos.}^{-1} \frac{2r \text{ cos. } \varphi - s - g}{2r}.$$

CASE III.—*Crossed Line Curved; Crossing Line Curved: Centres of both Curves on the same side of the Line.*

Let R, r be the radii of the crossed and crossing roads respectively, δ the distance between their centres. Then

$$\delta^2 = R^2 + r^2 - 2Rr\cos.\,\varphi,$$

since the angle between the radii is equal to the observed angle ϕ. For the same reason

$$\cos.\,c_1^1 = \frac{\left(R+\frac{s}{2}+g\right)^2+\left(r+\frac{s}{2}+g\right)^2-\delta^2}{2\left(R+\frac{s}{2}+g\right)\left(r+\frac{s}{2}+g\right)}$$

$$= \frac{2Rr\cos.\,\varphi+(R+r)(s+2g)}{2Rr+(R+r)(s+2g)}.$$

Similarly,

$$\cos.\,c_1^2 = \frac{2Rr\cos.\,\varphi+(R+r)s+2rg}{2Rr+(R+r)s+2Rg},$$

$$\cos.\,c_1^3 = \frac{2Rr\cos.\,\varphi-(R-r)s+2rg}{2Rr+(R-r)s+2Rg},$$

$$\cos.\,c_1^4 = \frac{2Rr\cos.\,\varphi-(R-r)(s+2g)}{2Rr+(R-r)(s+2g)},$$

$$\cos.\,c_2^1 = \frac{2Rr\cos.\,\varphi+(R+r)s+2Rg}{2Rr+(R+r)s+2rg},$$

$$\cos.\,c_2^2 = \frac{2Rr\cos.\,\varphi+(R+r)s}{2Rr+(R+r)s},$$

$$\cos.\,c_2^3 = \frac{2Rr\cos.\,\varphi-(R-r)s}{2Rr+(R-r)s},$$

$$\cos.\,c_2^4 = \frac{2Rr\cos.\,\varphi-(R-r)s-2Rg}{2Rr+(R-r)s-2rg},$$

$$\cos.\,c_3^1 = \frac{2Rr\cos.\,\varphi+(R-r)s+2Rg}{2Rr-(R-r)s+2rg},$$

$$\cos.\,c_3^2 = \frac{2Rr\cos.\,\varphi+(R-r)s}{2Rr-(R-r)s},$$

$$\cos.\,c_3^3 = \frac{2Rr\cos.\,\varphi-(R+r)s}{2Rr-(R+r)s},$$

$$\text{cos. } c_3^4 = \frac{2\,\mathrm{R}r\,\text{cos. }\varphi - (\mathrm{R}+r)\,s - 2\,\mathrm{R}g}{2\,\mathrm{R}r - (\mathrm{R}+r)\,s - 2rg},$$

$$\text{cos. } c_4^1 = \frac{2\,\mathrm{R}r\,\text{cos. }\varphi + (\mathrm{R}-r)\,(s+2g)}{2\,\mathrm{R}r - (\mathrm{R}-r)\,(s+2g)},$$

$$\text{cos. } c_4^2 = \frac{2\,\mathrm{R}r\,\text{cos. }\varphi + (\mathrm{R}-r)\,s - 2rg}{2\,\mathrm{R}r - (\mathrm{R}-r)\,s - 2\,\mathrm{R}g},$$

$$\text{cos. } c_4^3 = \frac{2\,\mathrm{R}r\,\text{cos. }\varphi - (\mathrm{R}+r)\,s - 2rg}{2\,\mathrm{R}r - (\mathrm{R}+r)\,s - 2\,\mathrm{R}g},$$

$$\text{cos. } c_4^4 = \frac{2\,\mathrm{R}r\,\text{cos. }\varphi - (\mathrm{R}+r)\,(s+2g)}{2\,\mathrm{R}r - (\mathrm{R}+r)\,(s+2g)}.$$

The calculated values of the angle to be substituted for the observed angle ϕ in the above formulæ are:

α. Single Line crossing Single Line.

$$\text{cos.}^{-1}\frac{2\,\mathrm{R}r\,\text{cos. }\varphi - (\mathrm{R}+r)\,(s+2g)\,(1-\text{cos. }\varphi)}{2\,\mathrm{R}r}.$$

β. Single Line crossing Double Line.

$$\text{cos.}^{-1}\frac{2\,\mathrm{R}r\,\text{cos. }\varphi + (\mathrm{R}\,\text{cos. }\varphi - r)\,(s+2g)}{2\,\mathrm{R}r}.$$

γ. Double Line crossing Single Line.

$$\text{cos.}^{-1}\frac{2\,\mathrm{R}r\,\text{cos. }\varphi + (r\,\text{cos. }\varphi - \mathrm{R})\,(s+2g)}{2\,\mathrm{R}r}.$$

CASE IV.—*Crossed Line Curved; Crossing Line Curved: Centres of Curves on opposite sides of Line.*

In this case

$$\delta^2 = \mathrm{R}^2 + r^2 + 2\,\mathrm{R}r\,\text{cos. }\varphi,$$

and

$$\text{cos. } c_1^1 = \frac{\delta^2 - \left(\mathrm{R}+\frac{s}{2}+g\right)^2 - \left(r+\frac{s}{2}+g\right)^2}{2\left(\mathrm{R}+\frac{s}{2}+g\right)\left(r+\frac{s}{2}+g\right)}$$

$$= \frac{2\,\mathrm{R}r\,\text{cos. }\varphi - (\mathrm{R}+r)\,(s+2g)}{2\,\mathrm{R}r + (\mathrm{R}+r)\,(s+2g)}.$$

Similarly,

$$\cos.\ c_1^{\,2} = \frac{2\,R\,r\cos.\,\varphi - (R+r)\,s - 2\,r\,g}{2\,R\,r + (R+r)\,s + 2\,R\,g},$$

$$\cos.\ c_1^{\,3} = \frac{2\,R\,r\cos.\,\varphi + (R-r)\,s - 2\,r\,g}{2\,R\,r + (R-r)\,s + 2\,R\,g},$$

$$\cos.\ c_1^{\,4} = \frac{2\,R\,r\cos.\,\varphi + (R-r)\,(s+2\,g)}{2\,R\,r + (R-r)\,(s+2\,g)},$$

$$\cos.\ c_2^{\,1} = \frac{2\,R\,r\cos.\,\varphi - (R+r)\,s - 2\,R\,g}{2\,R\,r + (R+r)\,s + 2\,r\,g},$$

$$\cos.\ c_2^{\,2} = \frac{2\,R\,r\cos.\,\varphi - (R+r)\,s}{2\,R\,r + (R+r)\,s},$$

$$\cos.\ c_2^{\,3} = \frac{2\,R\,r\cos.\,\varphi + (R-r)\,s}{2\,R\,r + (R-r)\,s},$$

$$\cos.\ c_2^{\,4} = \frac{2\,R\,r\cos.\,\varphi + (R-r)\,s + 2\,R\,g}{2\,R\,r + (R-r)\,s - 2\,r\,g},$$

$$\cos.\ c_2^{\,1} = \frac{2\,R\,r\cos.\,\varphi - (R-r)\,s - 2\,R\,g}{2\,R\,r - (R-r)\,s + 2\,r\,g},$$

$$\cos.\ c_3^{\,2} = \frac{2\,R\,r\cos.\,\varphi - (R-r)\,s}{2\,R\,r + (R-r)\,s},$$

$$\cos.\ c_3^{\,3} = \frac{2\,R\,r\cos.\,\varphi + (R+r)\,s}{2\,R\,r - (R+r)\,s},$$

$$\cos.\ c_3^{\,4} = \frac{2\,R\,r\cos.\,\varphi + (R+r)\,s + 2\,R\,g}{2\,R\,r - (R+r)\,s - 2\,r\,g},$$

$$\cos.\ c_4^{\,1} = \frac{2\,R\,r\cos.\,\varphi - (R-r)\,(s+2\,g)}{2\,R\,r - (R-r)\,(s+2\,g)},$$

$$\cos.\ c_4^{\,2} = \frac{2\,R\,r\cos.\,\varphi - (R-r)\,s + 2\,r\,g}{2\,R\,r - (R-r)\,s - 2\,R\,g},$$

$$\cos.\ c_4^{\,3} = \frac{2\,R\,r\cos.\,\varphi + (R+r)\,s + 2\,r\,g}{2\,R\,r - (R+r)\,s - 2\,R\,g},$$

$$\cos.\ c_4^{\,4} = \frac{2\,R\,r\cos.\,\varphi + (R+r)\,(s+2\,g)}{2\,R\,r - (R+r)\,(s+2\,g)}.$$

The calculated values of the angle to be substituted for the observed angle ϕ in the above formulæ are:

a. Single Line crossing Single Line.

$$\cos.^{-1}\frac{2\,R\,r\cos.\,\varphi + (R+r)\,(s+2\,g)\,(1+\cos.\,\varphi)}{2\,R\,r}.$$

β. Single Line crossing Double Line.

$$\cos.^{-1} \frac{2\,R\,r\cos.\varphi - (R\cos.\varphi + r)(s + 2g)}{2\,R\,r}.$$

γ. Double Line crossing Single Line.

$$\cos.^{-1} \frac{2\,R\,r\cos.\varphi - (R + r\cos.\varphi)(s + 2g)}{2\,R\,r}.$$

The formulæ give the values of the cosines of the angles, and not of the sines as in the previous examples, because the former are more simple. Since the angles of the crossings will rarely be so small, that we may put the circular measure of the angle equal to its sine, it will be necessary to ascertain the value of the angle in degrees and parts of a degree, which may be obtained from the cosine as readily as from the sine, and with as much exactness, unless φ be nearly a right angle, in which case all the angles will be practically equal, whatever may be the radii of the curves.

The circular measure of each angle will be obtained by multiplying its value in degrees and decimals of a degree by the factor

$$\frac{\pi}{180} = \cdot 017.$$

CHAPTER IV.

DOUBLE-GAUGE ROADS: ANGLES OF CROSSINGS AND THEIR POSITION.

ONLY those double-gauge roads are treated of which have one rail common to both roads, *viz.* the outside rail next the platform.

When the projection of the width of the carriages beyond the rails is not the same for both gauges, they cannot have a rail common to both roads. Since the platform must be sufficiently near to the rail to suit the carriages, which have the less projection, and the outside rail of the other road must be

at such a distance from the platform as to allow the carriages to clear it in passing, therefore the outside rail cannot be common to the two roads. Since also the interval between the two inner rails of a double road, called the six-foot, is fixed by the Regulations of the Board of Trade to suit carriages whose excess of width above that of the gauge is not greater than a certain amount, therefore the inside rail cannot be common to the two roads if the projection of the width of the carriages which project least be equal to this fixed amount of excess. In this case, therefore, there will be four rails. If the narrower-gauge road carriages project more than those of the broad gauge, the rails of the narrow-gauge road will be wholly within those of the wider-gauge road; if they project less, one will be within and the other outside, next the platform. If such a double-gauge road should ever exist, the formulæ given in Chapter III. can be readily adapted to suit the case, by giving proper values to R, r, s and g in the manner explained in the following sections for ordinary double-gauge roads.

Section I.—Single-line Junctions.

In the case of single lines, the centre line of the narrow-gauge road is separated from that of the wider road by an interval equal to half the difference between the gauges, *viz.* $\frac{d}{2}$; and the centre line of the space between the two rails, which are not common to both roads, from the centre line of the wider-gauge road, by an interval equal to half the width of the narrow gauge, *viz.* $\frac{g-d}{2}$. So that, if R, r be the radii of the broad-gauge roads, the radii of the narrow-gauge roads will be

$$R \pm \frac{d}{2}, \text{ and } r \pm \frac{d}{2},$$

$\pm$ according as the curvature is from or towards the rail common to both roads; and the radii of the centre line of the space between the two rails which are not common to both roads will be

$$R \mp \frac{g-d}{2}, \text{ and } r \mp \frac{g-d}{2},$$

$\mp$ according as the curvature is from or towards the rail common to both roads.

The formulæ given in Chapter III., Section I., will give the circular measures and positions of the crossings of the wider-gauge road treated as a single-gauge road, the radii being R and r and the gauge g. Consequently, if we substitute for R r and g the values proper to the narrower-gauge road, and to the interval between the two rails, not common to both roads, treated as a road whose gauge is d, we shall obtain the circular measures and positions of the crossings of those roads.

CASE I.—*Curve leaving Straight Road.*

α. Branch Line turning from the Rail common to both Roads (*Fig.* 20).

Putting $r = r + \frac{d}{2}$ and $g = g - d$, we get for c_1^2

$$c_1^2 = \frac{2\sqrt{2\left(r+\frac{d}{2}\right)(g-d)}}{2\left(r+\frac{d}{2}\right)+g-d}$$

$$= \frac{2\sqrt{2r(g-d)}}{2r+g},$$

and

$$c_1^3 = \frac{2\sqrt{2rg}}{2r+g}.$$

Putting $r = r - \frac{g-d}{2}$ and $g = d$, we get for c_2^3

$$c_2^3 = \frac{2\sqrt{2\left(r-\frac{g-d}{2}\right)d}}{2\left(r-\frac{g-d}{2}\right)+d}$$

$$= \frac{2\sqrt{2rd}}{2r+2d-g}.$$

Similarly,

$$S_2 c_1^2 = \sqrt{2r(g-d)},$$

$$S_2 c_1^3 = \sqrt{2rg},$$

$$S_2 c_2^3 = \sqrt{2rd}.$$

β. Branch Road turning towards the Rail common to both Roads (Fig. 21).

Putting $r = r + \frac{g-d}{2}$, and $g = d$, we get for c_1^2

$$c_1^2 = \frac{2\sqrt{2\left(r+\frac{g-d}{2}\right)d}}{2\left(r+\frac{g-d}{2}\right)+d}$$

$$= \frac{2\sqrt{2rd}}{2r+g},$$

and

$$c_1^3 = \frac{2\sqrt{2rg}}{2r+g}.$$

Putting $r = r - \frac{d}{2}$, and $g = g - d$, we get for c_2^3

$$c_2^3 = \frac{2\sqrt{2\left(r-\frac{d}{2}\right)(g-d)}}{2\left(r-\frac{d}{2}\right)+g-d}$$

$$= \frac{2\sqrt{2r(g-d)}}{2r+g-2d}.$$

Similarly,

$$S_2 c_1^2 = 2\sqrt{2rd},$$
$$S_3 c_1^3 = 2\sqrt{2rg},$$
$$S_3 c_2^3 = 2\sqrt{2r(g-d)}.$$

CASE II.—*Curve leaving Curve, Curvature in the same direction.*

α. Both Roads turning from the Rail common to the two Roads (Fig. 22).

Putting $R = R + \frac{d}{2}$, $r = r + \frac{d}{2}$, and $g = g - d$, we get for c_1^2

$$c_1^2 = \frac{2\sqrt{2\left(R+\frac{d}{2}\right)\left(r+\frac{d}{2}\right)(R-r)(g-d)}}{2\left(R+\frac{d}{2}\right)\left(r+\frac{d}{2}\right)+(R-r)(g-d)}$$

$$= \frac{2\sqrt{2Rr(R-r)(g-d)}}{2Rr+(R+r)d+(R-r)(g-d)},$$

also

$$c_1^2 = \frac{2\sqrt{2Rr(R-r)g}}{2Rr+(R-r)g}.$$

Putting $R = R - \frac{g-d}{2}$, $r = r - \frac{g-d}{2}$, and $g = d$, we get for c_2^2

$$c_2^2 = \frac{2\sqrt{2\left(R-\frac{g-d}{2}\right)\left(r-\frac{g-d}{2}\right)(R-r)d}}{2\left(R-\frac{g-d}{2}\right)\left(r-\frac{g-d}{2}\right)+(R-r)d}$$

$$= \frac{2\sqrt{2Rr(R-r)d}}{2Rr-(R+r)(g-d)+(R-r)d}.$$

Similarly,

$$S_2\, c_1^2 = \sqrt{\frac{2Rr(g-d)}{R-r}},$$

$$S_2\, c_1^2 = \sqrt{\frac{2Rrg}{R-r}},$$

$$S_2\, c_2^2 = \sqrt{\frac{2Rrd}{R-r}}.$$

β. Both Roads turning towards the Rail common to both Roads (Fig. 23).

Putting $R = R + \frac{g-d}{2}$, $r = r + \frac{g-d}{2}$, and $g = d$, we get for c_1^2

$$c_1^2 = \frac{2\sqrt{\left(R+\frac{g-d}{2}\right)\left(r+\frac{g-d}{2}\right)(R-r)d}}{2\left(R+\frac{g-d}{2}\right)\left(r+\frac{g-d}{2}\right)+(R-r)d}$$

$$= \frac{2\sqrt{2Rr(R-r)d}}{2Rr+(R+r)(g-d)+(R-r)d},$$

also

$$c_1^2 = \frac{2\sqrt{2Rr(R-r)g}}{2Rr+(R-r)g}.$$

Putting $R = R - \frac{d}{2}$, $r = r - \frac{d}{2}$, and $g = g - d$, we get for c_2^3

$$c_2^3 = \frac{2\sqrt{2\left(R-\frac{d}{2}\right)\left(r-\frac{d}{2}\right)(R-r)(g-d)}}{2\left(R-\frac{d}{2}\right)\left(r-\frac{d}{2}\right)+(R-r)(g-d)}$$

$$= \frac{2\sqrt{2Rr(R-r)(g-d)}}{2Rr-(R+r)d+(R-r)(g-d)}.$$

Similarly,

$$S_2\, c_1^2 = \sqrt{\frac{2Rrd}{R-r}},$$

$$S_2\, c_1^3 = \sqrt{\frac{2Rrg}{R-r}},$$

$$S_2\, c_2^3 = \sqrt{\frac{2Rr(g-d)}{R-r}}.$$

CASE III.—*Curve leaving Curve, Curvature in opposite directions.*

a. Branch Road curving from the Rail common to both Roads (Fig. 24).

Putting $R = R - \frac{d}{2}$, $r = r + \frac{d}{2}$, and $g = g - d$, we get for c_1^2

$$c_1^2 = \frac{2\sqrt{2\left(R-\frac{d}{2}\right)\left(r+\frac{d}{2}\right)(R+r)(g-d)}}{2\left(R-\frac{d}{2}\right)\left(r+\frac{d}{2}\right)+(R+r)(g-d)}$$

$$= \frac{2\sqrt{2Rr(R+r)(g-d)}}{2Rr+(R-r)d+(R+r)(g-d)},$$

also

$$c_1^3 = \frac{2\sqrt{2Rr(R+r)g}}{2Rr+(R+r)g}.$$

Putting $R = R + \frac{g-d}{2}$, $r = r - \frac{g-d}{2}$, and $g = d$, we get for c_2^3

$$c_2^3 = \frac{2\sqrt{2\left(R+\frac{g-d}{2}\right)\left(r-\frac{g-d}{2}\right)(R+r)d}}{2\left(R+\frac{g-d}{2}\right)\left(r-\frac{g-d}{2}\right)+(R+r)d}$$

$$= \frac{2\sqrt{2Rr(R+r)d}}{2Rr-(R-r)(g-d)+(R+r)d}.$$

Similarly,

$$S_2 c_1^2 = \sqrt{\frac{2Rr(g-d)}{R+r}},$$

$$S_2 c_1^3 = \sqrt{\frac{2Rrg}{R+r}},$$

$$S_2 c_2^2 = \sqrt{\frac{2Rrd}{R+r}}.$$

β. Branch Road Curving towards the Rail common to both Roads (*Fig.* 25).

Putting $R = R - \frac{g-d}{2}$, $r = r + \frac{g-d}{2}$, $g = d$, we get for c_1^2

$$c_1^2 = \frac{2\sqrt{2\left(R-\frac{g-d}{2}\right)\left(r+\frac{g-d}{2}\right)(R+r)d}}{2\left(R-\frac{g-d}{2}\right)\left(r+\frac{g-d}{2}\right)+(R+r)d}$$

$$= \frac{2\sqrt{2Rr(R+r)d}}{2Rr+(R-r)(g-d)+(R+r)d};$$

also

$$c_1^3 = \frac{2\sqrt{2Rr(R+r)g}}{2Rr+(R+r)g}.$$

Putting $R = R + \frac{d}{2}$, $r = r - \frac{d}{2}$, $g = g - d$, we get for c_2^3

$$c_2^3 = \frac{2\sqrt{2\left(R+\frac{d}{2}\right)\left(r-\frac{d}{2}\right)(R+r)(g-d)}}{2\left(R+\frac{d}{2}\right)\left(r-\frac{d}{2}\right)+(R+r)(g-d)}$$

$$= \frac{2\sqrt{2Rr(R+r)(g-d)}}{2Rr-(R-r)d+(R+r)(g-d)}.$$

Similarly,

$$S_2 c_1^2 = \sqrt{\frac{2Rrd}{R+r}},$$

$$S_2 c_1^3 = \sqrt{\frac{2Rrg}{R+r}},$$

$$S_2 c_2^3 = \sqrt{\frac{2Rr(g-d)}{R+r}},$$

If we compare the values of c_1^2, c_2^3 in (α) and (β) in each case, we see that c_1^2 in (α) $= c_2^3$ in (β), and c_2^3 in (α) $= c_1^2$ in (β), &c. A simple inspection of the figures suffices to show that these relations must hold. The formulæ have been worked out independently, in order to more fully illustrate the method of obtaining the formulæ for double-gauge, from those for single-gauge, roads.

Section II.—Double-line Junctions.

In the case of mixed-gauge double lines the centre line of the broad-gauge coincides with that of the narrow-gauge road, since each is in the middle of the space between the two inner rails of the road to which it refers, and the two outside rails are common to both roads.

For the same reason this centre line coincides with that of the two spaces contained between the two inner rails of the up and down roads. Hence, if we substitute, in the formula given in Chapter III., Section II., for s and g the values appropriate to the three cases, *viz.* the broad-gauge road, the narrow-gauge, and the space between the two inner rails of each road, we shall obtain the circular measures and positions of each of the crossing angles.

For the sake of more fully illustrating the method of deriving the formulæ for mixed-gauge from those for single-gauge double-line junctions, each road is taken by itself. This method will not give the angles in consecutive order, but it will show how easily the formulæ in Chapter III. can be directly adapted to any system of double-gauge road.

Case I.—*Curve leaving a Straight Line* (*Fig.* 26).

1st. Broad-gauge Road Crossings.

$$c_1^3 = c_1^2 \text{ in single-gauge formulæ}$$

$$= \frac{2\sqrt{2rg}}{2r+2g+s},$$

$$c_1^4 = c_1^3 \text{ in single-gauge formulæ}$$

$$= \frac{2\sqrt{2r(s+g)}}{2r+2g+s},$$

$c_1^6 = c_1^4$ in single-gauge formulæ

$$= \frac{2\sqrt{2r(s+2g)}}{2r+2g+s},$$

$c_2^4 = c_2^3$ in single-gauge formulæ

$$= \frac{2\sqrt{2rs}}{2r+s},$$

$c_2^6 = c_2^4$ in single-gauge formulæ

$$= \frac{2\sqrt{2r(s+g)}}{2r+s},$$

$c_4^6 = c_3^4$ in single-gauge formulæ

$$= \frac{2\sqrt{2rg}}{2r-s}.$$

2nd. Narrow-gauge Crossings.

Putting $g = g - d$, $s = s + 2d$ in the formulæ for c_1^2, c_1^3, c_2^3, c_2^4, and c_3^4, we get

$$c_1^2 = \frac{2\sqrt{2r(g-d)}}{2r+2g+s},$$

$$c_1^5 = \frac{2\sqrt{2r(s+g+d)}}{2r+2g+s},$$

$$c_2^5 = \frac{2\sqrt{2r(s+2d)}}{2r+s+2d},$$

$$c_2^6 = \frac{2\sqrt{2r(s+g+d)}}{2r+s+2d},$$

$$c_5^6 = \frac{2\sqrt{2r(g-d)}}{2r-s-2d}.$$

3rd. Space between two Inner Rails of each Road.

Putting $g = d$ in the formulæ for c_1^2, c_1^3, c_2^4, and c_3^4, we obtain

$$c_2^3 = \frac{2\sqrt{2rd}}{2r+2d+s},$$

$$c_2^4 = \frac{2\sqrt{2r(s+d)}}{2r+2d+s},$$

$$c_3^5 = \frac{2\sqrt{2r(s+d)}}{2r+s},$$

$$c_4^5 = \frac{2\sqrt{2rd}}{2r-s}.$$

Hence also

$$S_2 c_1^2 = \sqrt{2r(g-d)},$$

$$S_3 c_1^3 = \sqrt{2rg},$$

$$S_4 c_1^4 = \sqrt{2r(g+s)},$$

$$S_5 c_1^5 = \sqrt{2r(g+s+d)},$$

$$S_6 c_1^6 = \sqrt{2r(2g+s)},$$

$$S_3 c_2^3 = \sqrt{2rd},$$

$$S_4 c_2^4 = \sqrt{2r(s+d)},$$

$$S_5 c_2^5 = \sqrt{2r(s+2d)},$$

$$S_6 c_2^6 = \sqrt{2r(s+g+d)},$$

$$S_4 c_3^4 = \sqrt{2rs},$$

$$S_5 c_3^5 = \sqrt{2r(s+d)},$$

$$S_6 c_3^6 = \sqrt{2r(s+g)},$$

$$S_5 c_4^5 = \sqrt{2rd},$$

$$S_6 c_4^6 = \sqrt{2rg},$$

$$S_6 c_5^6 = \sqrt{2r(g-d)}.$$

CASE II.—*Curve leaving Curve, Curvature in the same direction (Fig. 27).*

1st. Broad-gauge Road Crossings.

$c_1^3 = c_1^2$ in single-gauge roads

$$= \frac{2\sqrt{2\mathrm{R}r(\mathrm{R}-r)g}}{2\mathrm{R}r+(\mathrm{R}+r)s+2\mathrm{R}g},$$

$c_1^4 = c_1^3$ in single-gauge formulæ

$$= \frac{2\sqrt{2\mathrm{R}r(\mathrm{R}-r)(s+g)}}{2\mathrm{R}r+(\mathrm{R}-r)s+2\mathrm{R}g},$$

$$c_1^5 = c_1^4 \text{ in single-gauge formulæ}$$
$$= \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,(s+2\,g)}}{2\,\mathrm{R}\,r+(\mathrm{R}-r)\,(s+2\,g)},$$

$$c_2^4 = c_2^3 \text{ in single-gauge formulæ}$$
$$= \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,s}}{2\,\mathrm{R}\,r+(\mathrm{R}-r)\,s},$$

$$c_2^6 = c_2^4 \text{ in single-gauge formulæ}$$
$$= \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,(s+g)}}{2\,\mathrm{R}\,r+(\mathrm{R}-r)\,s-2\,r\,g},$$

$$c_4^6 = c_3^4 \text{ in single-gauge formulæ}$$
$$= \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,g}}{2\,\mathrm{R}\,r-(\mathrm{R}+r)\,s-2\,r\,g}.$$

2nd. Narrow-gauge Roads.

Putting $g = g - d$ and $s = s + 2\,d$ in the formulæ for c_1^2, c_1^3, c_2^3, c_2^4, and c_3^4, we get

$$c_1^2 = \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,(g-d)}}{2\,\mathrm{R}\,r+(\mathrm{R}+r)\,(s+2\,d)+2\,\mathrm{R}\,(g-d)}$$
$$= \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,(g-d)}}{2\,\mathrm{R}\,r+(\mathrm{R}+r)\,s+2\,\mathrm{R}\,g+2\,r\,d},$$

$$c_1^3 = \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,(s+g+d)}}{2\,\mathrm{R}\,r+(\mathrm{R}-r)\,(s+2\,d)+2\,\mathrm{R}\,(g-d)}$$
$$= \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,(s+g+d)}}{2\,\mathrm{R}\,r+(\mathrm{R}-r)\,s+2\,\mathrm{R}\,g-2\,r\,d},$$

$$c_2^3 = \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,(s+2\,d)}}{2\,\mathrm{R}\,r+(\mathrm{R}-r)\,(s+2\,d)},$$

$$c_2^4 = \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,(s+g+d)}}{2\,\mathrm{R}\,r+(\mathrm{R}-r)\,(s+2\,d)-2\,r\,(g-d)}$$
$$= \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,(s+g+d)}}{2\,\mathrm{R}\,r+(\mathrm{R}-r)\,s+2\,\mathrm{R}\,d-2\,r\,g},$$

$$c_3^4 = \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,(g-d)}}{2\,\mathrm{R}\,r-(\mathrm{R}+r)\,(s+2\,d)-2\,r\,(g-d)}$$
$$= \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,(g-d)}}{2\,\mathrm{R}\,r-(\mathrm{R}+r)\,s-2\,\mathrm{R}\,d-2\,r\,g}.$$

3rd. Space between two Inner Rails of each Road.

Putting $g = d$ in the formulæ for c_1^2, c_1^3, c_2^4, and c_3^4, we get

$$c_2^3 = \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,d}}{2\,\mathrm{R}\,r+(\mathrm{R}+r)\,s+2\,\mathrm{R}\,d},$$

$$c_2^4 = \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,(s+d)}}{2\,\mathrm{R}\,r+(\mathrm{R}-r)\,s+2\,\mathrm{R}\,d},$$

$$c_3^5 = \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,(s+d)}}{2\,\mathrm{R}\,r+(\mathrm{R}-r)\,s-2\,r\,d},$$

$$c_4^5 = \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,d}}{2\,\mathrm{R}\,r-(\mathrm{R}+r)\,s-2\,r\,d}.$$

Similarly,

$$\mathrm{S}_2\,c_1^2 = \sqrt{\frac{\{2\,\mathrm{R}\,r+(\mathrm{R}+r)\,(s+g+d)\}\,(g-d)}{\mathrm{R}-r}},$$

$$\mathrm{S}_2\,c_1^3 = \sqrt{\frac{\{2\,\mathrm{R}\,r+(\mathrm{R}+r)\,(s+g)\}\,g}{\mathrm{R}-r}},$$

$$\mathrm{S}_4\,c_1^4 = \sqrt{\frac{\{2\,\mathrm{R}\,r+(\mathrm{R}+r)\,g\}\,(s+g)}{\mathrm{R}-r}},$$

$$\mathrm{S}_5\,c_1^5 = \sqrt{\frac{\{2\,\mathrm{R}\,r+(\mathrm{R}+r)\,(g-d)\}\,(s+g+d)}{\mathrm{R}-r}},$$

$$\mathrm{S}_6\,c_1^6 = \sqrt{\frac{2\,\mathrm{R}\,r\,(s+2\,g)}{\mathrm{R}-r}},$$

$$\mathrm{S}_3\,c_2^3 = \sqrt{\frac{\{2\,\mathrm{R}\,r+(\mathrm{R}+r)\,(s+d)\}\,d}{\mathrm{R}-r}},$$

$$\mathrm{S}_4\,c_2^4 = \sqrt{\frac{\{2\,\mathrm{R}\,r+(\mathrm{R}+r)\,d\}\,(s+d)}{\mathrm{R}-r}},$$

$$\mathrm{S}_5\,c_2^5 = \sqrt{\frac{2\,\mathrm{R}\,r\,(s+2\,d)}{\mathrm{R}-r}},$$

$$\mathrm{S}_6\,c_2^6 = \sqrt{\frac{\{2\,\mathrm{R}\,r-(\mathrm{R}+r)\,(g-d)\}\,(s+g+d)}{\mathrm{R}-r}},$$

$$\mathrm{S}_4\,c_3^4 = \sqrt{\frac{2\,\mathrm{R}\,r\,s}{\mathrm{R}-r}},$$

$$S_5\, c_3^5 = \sqrt{\frac{\{2Rr-(R+r)d\}(s+g+d)}{R-r}},$$

$$S_6\, c_3^6 = \sqrt{\frac{\{2Rr-(R+r)(s+g)\}g}{R-r}},$$

$$S_5\, c_4^5 = \sqrt{\frac{\{2Rr-(R+r)d\}(s+d)}{R-r}},$$

$$S_6\, c_4^6 = \sqrt{\frac{\{2Rr-(R+r)(s+g)\}g}{R-r}},$$

$$S_6\, c_5^6 = \sqrt{\frac{\{2Rr-(R+r)(s+g+d)\}(g-d)}{R-r}}.$$

CASE III.—*Curve leaving Curve, Curvature in opposite directions* (*Fig.* 28).

1*st. Broad-gauge Crossings.*

$c_1^3 = c_1^3$ in single-gauge roads

$$= \frac{2\sqrt{2Rr(R+r)g}}{2Rr+(R-r)s+2Rg},$$

$c_1^4 = c_1^3$ in single-gauge roads

$$= \frac{2\sqrt{2Rr(R+r)(s+g)}}{2Rr+(R+r)s+2Rg},$$

$c_1^6 = c_1^4$ in single-gauge roads

$$= \frac{2\sqrt{2Rr(R+r)(s+2g)}}{2Rr+(R+r)(s+2g)},$$

$c_3^4 = c_3^3$ in single-gauge roads

$$= \frac{2\sqrt{2Rr(R+r)s}}{2Rr+(R+r)s},$$

$c_3^6 = c_3^4$ in single-gauge roads

$$= \frac{2\sqrt{2Rr(R+r)(s+g)}}{2Rr+(R+r)s+2rg},$$

$c_4^6 = c_3^4$ in single-gauge roads

$$= \frac{2\sqrt{2Rr(R+r)g}}{2Rr-(R-r)s+2rg}.$$

2nd. Narrow-gauge Crossings.

Putting $g = g - d$ and $s = s + 2d$ in the formulæ for c_1^2, c_1^3, c_2^3, c_2^4, and c_3^4, we get

$$c_1^2 = \frac{2\sqrt{2Rr(R+r)(g-d)}}{2Rr+(R-r)(s+2d)+2R(g-d)}$$

$$= \frac{2\sqrt{2Rr(R+r)(g-d)}}{2Rr+(R-r)s+2R(g-d)},$$

$$c_1^5 = \frac{2\sqrt{2Rr(R+r)(s+g+d)}}{2Rr+(R+r)(s+2d)+2R(g-d)}$$

$$= \frac{2\sqrt{2Rr(R+r)(s+g+d)}}{2Rr+(R+r)s+2R(g+d)},$$

$$c_2^5 = \frac{2\sqrt{2Rr(R+r)(s+2d)}}{2Rr+(R+r)(s+2d)},$$

$$c_2^6 = \frac{2\sqrt{2Rr(R+r)(s+g+d)}}{2Rr+(R+r)(s+2d)+2r(g-d)}$$

$$= \frac{2\sqrt{2Rr(R+r)(s+g+d)}}{2Rr+(R+r)s+2Rd+2rg},$$

$$c_3^6 = \frac{2\sqrt{2Rr(R+r)(g-d)}}{2Rr-(R-r)(s+2d)+2r(g-d)}$$

$$= \frac{2\sqrt{2Rr(R+r)(g-d)}}{2Rr-(R-r)s-2Rd+2rg}.$$

3rd. Space between two Inner Rails of each Road.

Putting $g = d$ in the formulæ for c_1^2, c_1^3, c_2^4, and c_3^4, we get

$$c_2^3 = \frac{2\sqrt{2Rr(R+r)d}}{2Rr+(R-r)s+2Rd},$$

$$c_2^4 = \frac{2\sqrt{2Rr(R+r)(s+d)}}{2Rr+(R+r)s+2Rd},$$

$$c_3^5 = \frac{2\sqrt{2Rr(R+r)(s+d)}}{2Rr+(R+r)s+2rd},$$

$$c_4^5 = \frac{2\sqrt{2Rr(R+r)d}}{2Rr-(R-r)s+2rd}.$$

Similarly,

$$S_2 c_1^2 = \sqrt{\frac{2Rr(g-d)}{R+r}},$$

$$S_3 c_1^3 = \sqrt{\frac{2Rrg}{R+r}},$$

$$S_4 c_1^4 = \sqrt{\frac{2Rr(s+g)}{R+r}},$$

$$S_5 c_1^5 = \sqrt{\frac{2Rr(s+g+d)}{R+r}},$$

$$S_6 c_1^6 = \sqrt{\frac{2Rr(s+2g)}{R+r}},$$

$$S_3 c_2^3 = \sqrt{\frac{2Rrd}{R+r}},$$

$$S_4 c_2^4 = \sqrt{\frac{2Rr(s+d)}{R+r}},$$

$$S_5 c_2^5 = \sqrt{\frac{2Rr(s+2d)}{R+r}},$$

$$S_6 c_2^6 = \sqrt{\frac{2Rr(s+g+d)}{R+r}},$$

$$S_4 c_3^4 = \sqrt{\frac{2Rrs}{R+r}},$$

$$S_5 c_3^5 = \sqrt{\frac{2Rr(s+d)}{R+r}},$$

$$S_6 c_3^6 = \sqrt{\frac{2Rr(s+g)}{R+r}},$$

$$S_5 c_4^5 = \sqrt{\frac{2Rrd}{R+r}},$$

$$S_6 c_4^6 = \sqrt{\frac{2Rrg}{R+r}},$$

$$S_6 c_5^6 = \sqrt{\frac{2Rr(g-d)}{R+r}}.$$

Section III.—Three Throws.

Let R, r_1, r_2 be the radii of the curves of the centre lines of the broad-gauge main and branch roads. The radii of the curves of the centre lines of the corresponding roads, narrow

gauge, will be $R \pm \frac{d}{2}$, $r_1 \pm \frac{d}{2}$, and $r_2 \pm \frac{d}{2}$; $\pm$ according as the curvature in each case is from, or towards, the rail common to both gauges: and of the space between the two other rails $R \mp \frac{g-d}{2}$, $r_1 \mp \frac{g-d}{2}$, and $r_2 \mp \frac{g-d}{2}$, $\mp$ according as the curvature in each case is from, or towards, the rail common to both gauges.

Let the points where either rail of the broad-gauge main line is cut by the rails of the curves, radii r_1 and $r_1 \pm \frac{d}{2}$, be denoted by the symbols $c_{r_1}^{g}$ and $c_{r_1}^{g-d}$, and where cut by the rails of the curves, radii r_2 and $r_2 \pm \frac{d}{2}$ by the symbols $c_{r_2}^{g}$ and $c_{r_2}^{g-d}$, respectively; and the points, where the middle rail is cut by the broad-gauge rails, by the symbols $c_{r_1}^{d}$ and $c_{r_2}^{d}$. Also let the point where a broad-gauge rail of one branch road cuts the rail common to both branch roads be denoted by the symbol c_g^g; where a narrow-gauge rail of one branch cuts the rail common to both gauges by the symbol c_g^{g-d}; where the broad and narrow-gauge rails, which are not common to both roads, intersect, by the symbol c_g^d. Let these symbols, as usual, denote the circular measures of the angles.

We may obtain the circular measures and positions of these angles, either by substituting successively in the formulæ given in Chapter III., Section III., the values appropriate for each crossing, in the manner explained in the previous sections of this chapter; or write them down at once from the formulæ given in Section I.

Case I.—*Main Road Straight, Branch Roads Curving in opposite directions* (*Fig.* 29).

If r_1 be the radius of the curve of the centre line of the broad-gauge branch road, which curves towards the rail common to both roads, the radii of the centre lines, and the corresponding intervals between the centres of the rails, will be

$$r_1 \text{ and } g;$$

$$r_1 - \frac{d}{2} \text{ and } g - d,$$

$$r_1 + \frac{g-d}{2} \text{ and } d.$$

$$r_2 \text{ and } g;$$

$$r_2 + \frac{d}{2} \text{ and } g - d,$$

$$r_2 - \frac{g-d}{2} \text{ and } d.$$

Therefore

$$c_{r_2}^{g} = \frac{2\sqrt{2 r_2 g}}{2 r_2 + g},$$

$$c_{r_2}^{d} = \frac{2\sqrt{2 r_2 (g-d)}}{2 r_2 + g},$$

$$c_{r_2}^{g-d} = \frac{2\sqrt{2 r_2 d}}{2 r_2 + 2 d - g},$$

$$c_{r_1}^{g} = \frac{2\sqrt{2 r_1 g}}{2 r_1 + g},$$

$$c_{r_1}^{d} = \frac{2\sqrt{2 r_1 d}}{2 r_1 + g},$$

$$c_{r_1}^{g-d} = \frac{2\sqrt{2 r_1 (g-d)}}{2 r - 2 d + g},$$

$$c_{g}^{g} = \frac{2\sqrt{2 r_1 r_2 (r_1 + r_2) g}}{2 r_1 r_2 + (r_1 + r_2) g},$$

$$c_{g}^{g-d} = \frac{2\sqrt{2 r_1 r_2 (r_1 + r_2)(g-d)}}{2 r_1 r_2 + (r_1 - r_2) d + (r_1 + r_2)(g-d)},$$

$$c_{g}^{d} = \frac{2\sqrt{2 r_1 r_2 (r_1 + r_2) d}}{2 r_1 r_2 - (r_1 - r_2)(g-d) + (r_1 + r_2) d};$$

$$S_2\, c_{r_2}^{g} = \sqrt{2 r_2 g},$$

$$S_2\, c_{r_2}^{g-d} = \sqrt{2 r_2 d},$$

$$S_2\, c_{r_2}^{d} = \sqrt{2 r_2 (g-d)},$$

$$S_2\, c_{r_1}^{d} = \sqrt{2\, r_1\, d},$$

$$S_1\, c_{r_1}^{g} = \sqrt{2\, r_1\, g},$$

$$S_1\, c_{r_1}^{g-d} = \sqrt{2\, r_1\, (g-d)},$$

$$S_1\, c_g^{g-d} = S_2\, c_g^{g-d} = \sqrt{\frac{2\, r_1\, r_2\, (g-d)}{r_1 + r_2}},$$

$$S_1\, c_g^{g} = S_2\, c_g^{g} = \sqrt{\frac{2\, r_1\, r_2\, g}{r_1 + r_2}},$$

$$S_2\, c_g^{d} = S_2\, c_g^{d} = \sqrt{\frac{2\, r_1\, r_2\, d}{r_1 + r_2}}.$$

CASE II.—*Main Road Straight, Branch Roads Curving in the same direction from the Rail common to both Roads* (*Fig.* 30).

The radii of the centre lines and the corresponding intervals between the rails will be:

r_1 and g;

$r_1 + \frac{d}{2}$ and $g - d$,

$r_1 - \frac{g-d}{2}$ and d.

r_2 and g;

$r_2 + \frac{d}{2}$ and $g - d$,

$r_2 - \frac{g-d}{2}$ and d.

Therefore

$$c_{r_2}^{g} = \frac{2\sqrt{2\, r_2\, g}}{2\, r_2 + g},$$

$$c_{r_2}^{g-d} = \frac{2\sqrt{2\, r_2\, d}}{2\, r_2 + 2\, d - g},$$

$$c_{r_2}^{d} = \frac{2\sqrt{2\, r_2\, (g-d)}}{2\, r_2 + g},$$

$$c_{r_1}^{g} = \frac{2\sqrt{2\, r_1\, g}}{2\, r_1 + g},$$

$$c_{r_1}^{g-d} = \frac{2\sqrt{2r_1 d}}{2r_1 + 2d - g},$$

$$c_{r_1}^{d} = \frac{2\sqrt{2r_1(g-d)}}{2r_1 + g},$$

$$c_g^{g} = \frac{2\sqrt{r_1 r_2 (r_2 - r_1)\, g}}{2r_1 r_2 + (r_2 - r_1)\, g},$$

$$c_g^{d} = \frac{2\sqrt{2r_1 r_2 (r_2 - r_1)\, d}}{2r_1 r_2 - (r_1 + r_2)(g-d) + (r_2 - r_1)\, d},$$

$$c_g^{g-d} = \frac{2\sqrt{2r_1 r_2 (r_2 - r_1)(g-d)}}{2r_1 r_2 + (r_1 + r_2)\, d + (r_2 - r_1)(g-d)};$$

$$S_2\, c_{r_2}^{g} = \sqrt{2r_2 g},$$

$$S_2\, c_{r_2}^{g-d} = \sqrt{2r_2 d},$$

$$S_2\, c_{r_1}^{g} = \sqrt{2r_1 g},$$

$$S_2\, c_{r_1}^{g-d} = \sqrt{2r_1 d},$$

$$S_2\, c_{r_2}^{d} = \sqrt{2r_2(g-d)},$$

$$S_2\, c_{r_1}^{d} = \sqrt{2r_1(g-d)},$$

$$S_1\, c_g^{g} = S_2\, c_g^{g} = \sqrt{\frac{2r_1 r_2 g}{r_2 - r_1}},$$

$$S_1\, c_g^{g-d} = S_2\, c_g^{g-d} = \sqrt{\frac{2r_1 r_2 (g-d)}{r_2 - r_1}},$$

$$S_2\, c_g^{d} = S_2\, c_g^{d} = \sqrt{\frac{2r_1 r_2 d}{r_2 - r_1}}.$$

CASE III.—*Main Roads Straight, Branch Roads Curving in the same direction towards the Rail common to both Roads* (*Fig.* 31).

The radii of the centre lines and the corresponding intervals between the rails will be:

r_1 and g;

$r_1 - \frac{d}{2}$ and $g - d$,

$r_1 + \frac{g-d}{2}$ and d.

$$r_2 \text{ and } g;$$

$$r_2 - \frac{d}{2} \text{ and } g - d,$$

$$r_2 + \frac{g-d}{2} \text{ and } d.$$

Therefore

$$c_{r_2}^{g} = \frac{2\sqrt{2\,r_2\,g}}{2\,r_2 + g},$$

$$c_{r_2}^{g-d} = \frac{2\sqrt{2\,r_2\,(g-d)}}{2\,r_2 - 2\,d + g},$$

$$c_{r_1}^{g} = \frac{2\sqrt{2\,r_1\,g}}{2\,r_1 + g},$$

$$c_{r_1}^{g-d} = \frac{2\sqrt{2\,r_1\,(g-d)}}{2\,r_1 - 2\,d + g},$$

$$c_{r_2}^{d} = \frac{2\sqrt{2\,r_2\,d}}{2\,r_2 + g},$$

$$c_{r_1}^{d} = \frac{2\sqrt{2\,r_1\,d}}{2\,r_1 + g},$$

$$c_{g}^{g} = \frac{2\sqrt{2\,r_1\,r_2\,(r_2 - r_1)\,g}}{2\,r_1\,r_2 + (r_2 - r_1)\,g},$$

$$c_{g}^{g-d} = \frac{2\sqrt{2\,r_1\,r_2\,(r_2 - r_1)\,(g-d)}}{2\,r_1\,r_2 - (r_1 + r_2)\,d + (r_2 - r_1)\,(g-d)},$$

$$c_{g}^{d} = \frac{2\sqrt{2\,r_1\,r_2\,(r_2 - r_1)\,d}}{2\,r_1\,r_2 + (r_2 + r_1)\,(g-d) + (r_2 - r_1)\,d};$$

$$S_2\,c_{r_2}^{g} = \sqrt{2\,r_2\,g},$$

$$S_2\,c_{r_2}^{g-d} = \sqrt{2\,r_2\,(g-d)},$$

$$S_2\,c_{r_1}^{g} = \sqrt{2\,r_1\,g},$$

$$S_2\,c_{r_1}^{g-d} = \sqrt{2\,r_1\,(g-d)},$$

$$S_2\,c_{r_2}^{d} = \sqrt{2\,r_2\,d},$$

$$S_2\,c_{r_1}^{d} = \sqrt{2\,r_1\,d},$$

$$S_1 c_g^g = S_2 c_g^g = \sqrt{\frac{2 r_1 r_2 g}{r_2 - r_1}},$$

$$S_2 c_g^{g-d} = S_2 c_g^{g-d} = \sqrt{\frac{2 r_1 r_2 (g-d)}{r_2 - r_1}},$$

$$S_1 c_g^d = S_2 c_g^d = \sqrt{\frac{2 r_1 r_2 d}{r_2 - r_1}}.$$

CASE IV.—*Main Line Curved, Branch Roads Curving in opposite directions* (*Fig.* 32).

If r_1 be the radius of the curve of the centre line of the broad-gauge branch road, which curves towards the rail common to both gauges, the radii of the centre lines and the corresponding intervals between the rails will be:

$$\text{R and } g;$$

$$\text{R} \pm \frac{d}{2} \text{ and } g - d,$$

$$\text{R} \mp \frac{g-d}{2} \text{ and } d.$$

$$r_1 \text{ and } g;$$

$$r_1 - \frac{d}{2} \text{ and } g - d,$$

$$r_1 + \frac{g-d}{2} \text{ and } d.$$

$$r_2 \text{ and } g;$$

$$r_2 + \frac{d}{2} \text{ and } g - d.$$

$\text{R} \pm \frac{d}{2}$ and $\text{R} \mp \frac{g-d}{2}$, &c., according as the curvature of the main road is from or towards the rail common to both gauges.

α. Curvature of Main Road from the Rail common to both Roads (*Fig.* 32).

$$c_{r_2}^g = \frac{2\sqrt{2\,\text{R}\,r_2(\text{R}-r_2)\,g}}{2\,\text{R}\,r_2 + (\text{R}-r_2)\,g},$$

$$c_{r_2}^{g-d} = \frac{2\sqrt{2\,\text{R}\,r_2(\text{R}-r_2)\,d}}{2\,\text{R}\,r_2 + (\text{R}+r_2)\,d + (\text{R}-r_2)\,(g-d)},$$

$$S_1 c_g^g = S_2 c_g^g = \sqrt{\frac{2 r_1 r_2 g}{r_1 + r_2}},$$

$$S_1 c_g^d = S_2 c_g^d = \sqrt{\frac{2 r_1 r_2 d}{r_1 + r_2}},$$

$$S_2 c_g^{g-d} = S_2 c_g^{g-d} = \sqrt{\frac{2 r_1 r_2 (g-d)}{r_1 + r_2}}.$$

CASE V.—*Main Line Curved, Branch Roads Curving in the same direction as the Main Roads.*

The radii of the centre lines and the corresponding intervals between the rails will be:

$$\text{R and } g;$$

$$\text{R} \pm \frac{d}{2} \text{ and } g - d,$$

$$\text{R} \mp \frac{g-d}{2} \text{ and } d.$$

$$r_1 \text{ and } g;$$

$$r_1 \pm \frac{d}{2} \text{ and } g - d,$$

$$r_1 \mp \frac{g-d}{2} \text{ and } d.$$

$$r_2 \text{ and } g;$$

$$r_2 \pm \frac{d}{2} \text{ and } g - d,$$

$$r_2 \mp \frac{g-d}{2} \text{ and } d.$$

The upper or lower signs to be taken according as the curvature is from or towards the rail common to both roads.

Let r_2 be greater than r_1.

a. Curvature from the Rail common to both Roads (*Fig.* 34).

$$c_{r_2}^d = \frac{2\sqrt{2\,\text{R}\,r_2\,(\text{R} - r_2)\,(g-d)}}{2\,\text{R}\,r_2 + (\text{R} + r_2)\,d + (\text{R} - r_2)\,(g-d)},$$

$$c_{r_2}^g = \frac{2\sqrt{2\,\text{R}\,r_2\,(\text{R} - r_2)\,g}}{2\,\text{R}\,r_2 + (\text{R} - r_2)\,g},$$

$$c_{r_2}^{g-d} = \frac{2\sqrt{2\,R\,r_2\,(R-r_2)\,d}}{2\,R\,r_2-(R+r_2)\,(g-d)+(R-r_2)\,d},$$

$$c_{r_1}^{d} = \frac{2\sqrt{2\,R\,r_1\,(R-r_1)\,(g-d)}}{2\,R\,r_1+(R+r_1)\,d+(R-r_1)\,(g-d)},$$

$$c_{r_1}^{g} = \frac{2\sqrt{2\,R\,r_1\,(R-r_1)\,g}}{2\,R\,r_1+(R-r_1)\,g},$$

$$c_{r_1}^{g-d} = \frac{2\sqrt{2\,R\,r_1\,(R-r_1)\,d}}{2\,R\,r_1-(R+r_1)\,(g-d)+(R-r_1)\,d},$$

$$c_{g}^{g} = \frac{2\sqrt{2\,r_1\,r_2\,(r_2-r_1)\,g}}{2\,r_1\,r_2+(r_2-r_1)\,g},$$

$$c_{g}^{g-d} = \frac{2\sqrt{2\,r_1\,r_2\,(r_2-r_1)\,(g-d)}}{2\,r_1\,r_2+(r_2+r_1)\,d+(r_2-r_1)\,(g-d)},$$

$$c_{g}^{d} = \frac{2\sqrt{2\,r_1\,r_2\,(r_2-r_1)\,d}}{2\,r_1\,r_2-(r_2+r_1)\,(g-d)+(r_2-r_1)\,d}.$$

Similarly,

$$S_2\,c_{r_1}^{d} = \sqrt{\frac{2\,R\,r_1\,(g-d)}{R-r_1}},$$

$$S_2\,c_{r_2}^{d} = \sqrt{\frac{2\,R\,r_2\,(g-d)}{R-r_2}},$$

$$S_2\,c_{r_1}^{g-d} = \sqrt{\frac{2\,R\,r_1\,d}{R-r_1}},$$

$$S_2\,c_{r_1}^{g} = \sqrt{\frac{2\,R\,r_1\,g}{R-r_1}},$$

$$S_2\,c_{r_2}^{g-d} = \sqrt{\frac{2\,R\,r_2\,d}{R-r_2}},$$

$$S_2\,c_{r_2}^{g} = \sqrt{\frac{2\,R\,r_2\,g}{R-r_2}},$$

$$S_1\,c_{g}^{g} = S_2\,c_{g}^{g} = \sqrt{\frac{2\,r_1\,r_2\,g}{r_2-r_1}},$$

$$S_1\,c_{g}^{g-d} = S_2\,c_{g}^{g-d} = \sqrt{\frac{2\,r_1\,r_2\,(g-d)}{r_2-r_1}},$$

$$S_2\,c_{g}^{d} = S_2\,c_{g}^{d} = \sqrt{\frac{2\,r_1\,r_2\,d}{r_2-r_1}}.$$

β. Curvature towards the Rails common to both Roads (*Fig.* 35).

$$c_{r_1}^{d} = \frac{2\sqrt{2\,R\,r_1\,(R-r_1)\,d}}{2\,R\,r_1+(R+r_1)\,(g-d)+(R-r_1)\,d},$$

$$c_{r_1}^{g} = \frac{2\sqrt{2\,R\,r_1\,(R-r_1)\,g}}{2\,R\,r_1+(R-r_1)\,g},$$

$$c_{r_1}^{g-d} = \frac{2\sqrt{2\,R\,r_1\,(R-r_1)\,(g-d)}}{2\,R\,r_1-(R+r_1)\,d+(R-r_1)\,(g-d)},$$

$$c_{r_2}^{d} = \frac{2\sqrt{2\,R\,r_2\,(R-r_2)\,d}}{2\,R\,r_2+(R+r_2)\,(g-d)+(R-r_2)\,d},$$

$$c_{r_2}^{g} = \frac{2\sqrt{2\,R\,r_2\,(R-r_2)\,g}}{2\,R\,r_2+(R-r_2)\,g},$$

$$c_{r_2}^{g-d} = \frac{2\sqrt{2\,R\,r_2\,(R-r_2)\,(g-d)}}{2\,R\,r_2-(R+r_2)\,d+(R-r_2)\,(g-d)},$$

$$c_{g}^{g-d} = \frac{2\sqrt{2\,r_1\,r_2\,(r_2-r_1)\,(g-d)}}{2\,r_1\,r_2-(r_2+r_1)\,d+(r_2-r_1)\,(g-d)},$$

$$c_{g}^{g} = \frac{2\sqrt{2\,r_1\,r_2\,(r_2-r_1)\,g}}{2\,r_1\,r_2+(r_2-r_1)\,g},$$

$$c_{g}^{d} = \frac{2\sqrt{2\,r_1\,r_2\,(r_2-r_1)\,d}}{2\,r_1\,r_2+(r_2+r_1)\,(g-d)+(r_2-r_1)\,d}.$$

Similarly,

$$S_2\,c_{r_1}^{d} = \sqrt{\frac{2\,R\,r_1\,d}{R-r_1}},$$

$$S_2\,c_{r_2}^{d} = \sqrt{\frac{2\,R\,r_2\,d}{R-r_2}},$$

$$S_2\,c_{r_1}^{g} = \sqrt{\frac{2\,R\,r_1\,g}{R-r_1}},$$

$$S_2\,c_{r_1}^{g-d} = \sqrt{\frac{2\,R\,r_1\,(g-d)}{R-r_1}},$$

$$S_2\,c_{r_2}^{g} = \sqrt{\frac{2\,R\,r_2\,g}{R-r_2}},$$

$$S_2\,c_{r_2}^{g-d} = \sqrt{\frac{2\,R\,r_2\,(g-d)}{R-r_2}},$$

$$S_1 c_g^g = S_2 c_g^g = \sqrt{\frac{2 r_1 r_2 g}{r_2 - r_1}},$$

$$S_2 c_g^{g-d} = S_1 c_g^{g-d} = \sqrt{\frac{2 r_1 r_2 (g-d)}{r_2 - r_1}},$$

$$S_1 c_g^d = S_2 c_g^d = \sqrt{\frac{2 r_1 r_2 d}{r_2 - r_1}}.$$

CASE VI.—*Main Line Curved, Branch Roads Curving in the opposite direction to the Main Line.*

The radii of the centre lines and the corresponding intervals between the rails will be:

$$\text{R and } g;$$

$$\text{R} \mp \frac{d}{2} \text{ and } g - d,$$

$$\text{R} \pm \frac{g-d}{2} \text{ and } d.$$

$$r_1 \text{ and } g;$$

$$r_1 \pm \frac{d}{2} \text{ and } g - d,$$

$$r_1 \mp \frac{g-d}{2} \text{ and } d.$$

$$r_2 \text{ and } g;$$

$$r_2 \pm \frac{d}{2} \text{ and } g - d,$$

$$r_2 \mp \frac{g-d}{2} \text{ and } d.$$

The upper or lower sign to be taken according as the curvature is from or towards the rail common to both roads.

Let r_2 be greater than r_1.

α. Curvature from the Rail common to both Curves (*Fig.* 36).

$$c_{r_2}^d = \frac{2\sqrt{2\,\text{R}\,r_2\,(\text{R}+r_2)\,(g-d)}}{2\,\text{R}\,r_2 + (\text{R}-r_2)\,d + (\text{R}+r_2)\,(g-d)},$$

$$c_{r_2}^g = \frac{2\sqrt{2\,\text{R}\,r_2\,(\text{R}+r_2)\,g}}{2\,\text{R}\,r_2 + (\text{R}+r_2)\,g},$$

$$c_{r_2}^{g-d} = \frac{2\sqrt{2\,\text{R}\,r_2\,(\text{R}+r_2)\,d}}{2\,\text{R}\,r_2 - (\text{R}-r_2)\,(g-d) + (\text{R}+r_2)\,d},$$

$$c_{r_1}^{d} = \frac{2\sqrt{2\,R\,r_1\,(R+r_1)\,(g-d)}}{2\,R\,r_1+(R-r_1)\,d+(R+r_1)\,(g-d)},$$

$$c_{r_1}^{g} = \frac{2\sqrt{2\,R\,r_1\,(R+r_1)\,g}}{2\,R\,r_1+(R+r_1)\,g},$$

$$c_{r_1}^{g-d} = \frac{2\sqrt{2\,R\,r_1\,(R+r_1)\,d}}{2\,R\,r_1-(R-r_1)\,(g-d)+(R+r_1)\,d},$$

$$c_{g}^{g-d} = \frac{2\sqrt{2\,r_2\,r_1\,(r_2-r_1)\,(g-d)}}{2\,r_2\,r_1+(r_2+r_1)\,d+(r_2-r_1)\,(g-d)},$$

$$c_{g}^{g} = \frac{2\sqrt{2\,r_2\,r_1\,(r_2-r_1)\,g}}{2\,r_2\,r_1+(r_2-r_1)\,g},$$

$$c_{g}^{d} = \frac{2\sqrt{2\,r_2\,r_1\,(r_2-r_1)\,d}}{2\,r_2\,r_1-(r_2+r_1)\,(g-d)+(r_2-r_1)\,d}.$$

Similarly,

$$S_2\,c_{r_2}^{d} = \sqrt{\frac{2\,R\,r_2\,(g-d)}{R+r_2}},$$

$$S_2\,c_{r_1}^{d} = \sqrt{\frac{2\,R\,r_1\,(g-d)}{R+r_1}},$$

$$S_2\,c_{r_2}^{g} = \sqrt{\frac{2\,R\,r_2\,g}{R+r_2}},$$

$$S_2\,c_{r_1}^{g} = \sqrt{\frac{2\,R\,r_1\,g}{R+r_1}},$$

$$S_2\,c_{r_2}^{g-d} = \sqrt{\frac{2\,R\,r_2\,d}{R+r_2}},$$

$$S_2\,c_{r_1}^{g-d} = \sqrt{\frac{2\,R\,r_1\,d}{R+r_1}},$$

$$S_1\,c_{g}^{g} = S_2\,c_{g}^{g} = \sqrt{\frac{2\,r_2\,r_1\,g}{r_2-r_1}},$$

$$S_1\,c_{g}^{g-d} = S_2\,c_{g}^{g-d} = \sqrt{\frac{2\,r_2\,r_1\,g}{r_2-r_1}},$$

$$S_2\,c_{g}^{d} = S_2\,c_{g}^{d} = \sqrt{\frac{2\,r_2\,r_1\,d}{r_2-r_1}}.$$

β. Curvature towards the Rail common to both Roads (*Fig.* 37).

$$c_{r_2}^{d} = \frac{2\sqrt{2\,R\,r_2\,(R+r_2)\,d}}{2\,R\,r_2+(R-r_2)\,(g-d)+(R+r_2)\,d},$$

$$c_{r_2}^{g} = \frac{2\sqrt{2\,R\,r_2\,(R+r_2)\,g}}{2\,R\,r_2+(R+r_2)\,g},$$

$$c_{r_2}^{g-d} = \frac{2\sqrt{2\,R\,r_2\,(R+r_2)\,(g-d)}}{2\,R\,r_2-(R-r_2)\,d+(R+r_2)\,(g-d)},$$

$$c_{r_1}^{d} = \frac{2\sqrt{2\,R\,r_1\,(R+r_1)\,d}}{2\,R\,r_1+(R-r_1)\,(g-d)+(R+r_1)\,d},$$

$$c_{r_1}^{g} = \frac{2\sqrt{2\,R\,r_1\,(R+r_1)\,g}}{2\,R\,r_1+(R+r_1)\,g},$$

$$c_{r_1}^{g-d} = \frac{2\sqrt{2\,R\,r_1\,(R+r_1)\,(g-d)}}{2\,R\,r_1-(R-r_1)\,d+(R+r_1)\,(g-d)},$$

$$c_{g}^{d} = \frac{2\sqrt{2\,r_2\,r_1\,(r_2-r_1)\,d}}{2\,r_2\,r_1+(r_2+r_1)\,(g-d)+(r_2-r_1)\,d},$$

$$c_{g}^{g} = \frac{2\sqrt{2\,r_2\,r_1\,(r_2-r_1)\,g}}{2\,r_2\,r_1+(r_2-r_1)\,g},$$

$$c_{g}^{g-d} = \frac{2\sqrt{2\,r_1\,r_2\,(r_2-r_1)\,(g-d)}}{2\,r_2\,r_1-(r_2+r_1)\,d+(r_2-r_1)\,(g-d)}.$$

Similarly,

$$S_2\,c_{r_2}^{d} = \sqrt{\frac{2\,R\,r_2\,d}{R+r_2}},$$

$$S_2\,c_{r_1}^{d} = \sqrt{\frac{2\,R\,r_1\,d}{R+r_1}},$$

$$S_2\,c_{r_2}^{g} = \sqrt{\frac{2\,R\,r_2\,g}{R+r_2}},$$

$$S_2\,c_{r_2}^{g-d} = \sqrt{\frac{2\,R\,r_2\,(g-d)}{R+r_2}},$$

$$S_2\,c_{r_1}^{g} = \sqrt{\frac{2\,R\,r_1\,g}{R+r_1}},$$

$$S_2\,c_{r_1}^{g-d} = \sqrt{\frac{2\,R\,r_1\,(g-d)}{R+r_1}},$$

$$S_1 c_g^d = S_2 c_g^d = \sqrt{\frac{2 r_1 r_2 d}{r_2 - r_1}},$$

$$S_1 c_g^g = S_2 c_g^g = \sqrt{\frac{2 r_1 r_2 g}{r_2 - r_1}},$$

$$S_1 c_g^{g-d} = S_2 c_g^{g-d} = \sqrt{\frac{2 r_1 r_2 (g-d)}{r_2 - r_1}}.$$

Section IV.—Cross-over Roads.

The two roads between which communication is to be established may be either the two roads of the main line, in which the outside rails are common to both gauges, and the centre lines of the spaces between the inner rails of both gauge roads coincident; or a pair of roads on a set of sidings, in which the centre line of the space between the two inner rails of the broader-gauge road may not coincide with that of the space between the inner rails of the narrower-gauge road, but be removed from it by an interval equal to $\frac{d}{2}$. In the former case, the rail of the cross-over which starts from the middle rail at one springing ends at the outside rail at the other, and consequently the cross-over roads have not a rail common to both roads. In the latter case the rail of the narrow-gauge cross-over which starts from the middle rail at one springing ends at the middle rail at the other springing, and the cross-over roads have one rail common to both roads. Hence there are two classes, and three cases in each class, *viz.*,

α. $s > g$.
β. $s < g$, but $> g - d$.
γ. $s < g - d$.

No example exists, or is it likely to exist, in which s is less than $g - \frac{d}{2}$; therefore no further notice will be taken of (γ).

Class I.—*Between two Main-line Roads.*

That form of cross-over which consists of two reverse curves meeting at a common tangent cannot be adopted in the case of

mixed-gauge roads, because the radii of the broad and narrow-gauge reverse curves are so nearly alike, that the necessary clearance between the two rails which start from contact with the main-line rail common to both gauges would never be attained nearer to the springing than the crossing of the middle rail. It would therefore be impossible to use a switch, and practically impossible to use a fast point with a proper length of splay, because it would be too weak towards the point. This may be easily proved. When the main lines are straight,

$$* \frac{r^b}{r^n} = \frac{\sqrt{s+g+d}}{\sqrt{s+g}};$$

therefore

$$\frac{r^b r^n}{r^b - r^n} = \frac{r^b \sqrt{s+g}}{\sqrt{s+g+d} - \sqrt{s+g}}.$$

Therefore the distance of the heel of the switch of the narrow-gauge rail from the springing is equal to

$$\sqrt{\frac{2 r^b r^n (c+t)}{r^b - r^n}} = \sqrt{\frac{2 r^b (c+t) \sqrt{s+g}}{\sqrt{s+g+d} - \sqrt{s+g}}};$$

or is equal to the distance of the heel of the switch of the broad-gauge road from the springing multiplied by

$$\sqrt{\frac{\sqrt{s+g}}{\sqrt{s+g+d} - \sqrt{s+g}}}.$$

For the 7 ft. and 4 ft. 8½ in. gauge roads: the sum of the clearance plus the thickness of the rail being 4½ in.,

$$\sqrt{\frac{s+g}{s+g+d}} = \frac{12}{13} \text{ nearly};$$

therefore

$$\sqrt{\frac{\sqrt{s+g}}{\sqrt{s+g+d} - \sqrt{s+g}}} = 3\tfrac{1}{2}.$$

Since the distance of the heel of the switch of the broad-gauge rail from the springing is also 3½ times the lead of the

* r^b is the radius of the reverse curves of the broad-gauge cross-over, r^n of the narrow gauge.

crossing b_1^3, the distance of the heel of the switch of the narrow-gauge rail is $12\frac{1}{4}$ times the lead of the crossing b_1^3.

Now, the distance of the crossing n_1^2 from the springing is equal to (Fig. 38)

$$\sqrt{2r^n(g-d)} = \frac{\sqrt{2r^b(g-d)\sqrt{s+g}}}{\sqrt[4]{s+g+d}}$$

$$= \frac{2\sqrt{g(g-d)\sqrt{s+g}}}{b_1^3\sqrt[4]{s+g+d}}$$

$$= \frac{13}{b_1^3} \text{ nearly};$$

or the distance of the heel of the switch of the narrow-gauge rail from the springing exceeds that of the crossing n_1^2 by $\frac{3}{4}$ths of the lead of the crossing b_1^3, which can never be sharper than 1 in 5.

The splay of the fast point of the narrow-gauge rail is equal to

$$\sqrt{\frac{2r^n r^b(c+t-c-e)}{r^b-r^n}} = \sqrt{\frac{2r^b(e+t-c-e)\sqrt{s+g}}{\sqrt{s+g+d}-\sqrt{s+g}}};$$

or is equal to $3\frac{1}{2}$ times the splay of the fast point of the broad-gauge rail, which is itself equal to $\frac{9}{10}$ths of the lead of the crossing b_1^3, or the splay can never be less than 16 feet long.

When the main lines are curved, the ratios of the radii of the narrow-gauge road to those of the broad gauge are in one case less, and in the other greater, than when the main line is straight, but so slightly different that the same reasoning applies in both cases.

We must therefore use two reverse curves separated by an interval of straight. The best combination of these is fully explained in the next section on through roads, for which reverse curves meeting at a common tangent cannot be adopted for the reasons just stated, *a fortiori* because the radii are much more nearly equal.

Section V. must therefore be read before Section IV. See also Chapter III., Section V., Cases I. and III., and the numerical evaluations in Chapters V. and VI.

CASE I.—*Main Line Straight: Reverse Curves separated by an interval of Straight* (*Fig.* 38).

α. $s \gtreqless g$.

$$r^b = \frac{2g\,(D - 2l)^2}{\{s + 3g - 4\sqrt{g\,(c+t)}\}^2},$$

$$r^a = r^b - \frac{d\,r^b}{2g}.$$

Interval between springings of reverse curves $\left.\begin{matrix}\\ \end{matrix}\right\} = \frac{(s+3g)\sqrt{r^b}}{\sqrt{2g}},$

$$b_1^2 = b_2^5 = \frac{2\sqrt{2r^b(g-d)}}{2r^b + (g-d)},$$

$$b_1^3 = b_2^4 = \frac{2\sqrt{2r^b g}}{2r^b + g},$$

$$n_1^2 = n_2^5 = \frac{2\sqrt{2r^a(g-d)}}{2r^a + g},$$

$$n_1^3 = n_2^4 = \frac{2\sqrt{2r^a g}}{2r^a + g - d},$$

$$n_2^3 = n_1^4 = \frac{2\sqrt{2r^a d}}{2r^a - g + d}$$

$$S_2 b_1^2 = S'_5 b_2^5 = \sqrt{2r^b(g-d)},$$

$$S_3 b_1^3 = S'_4 b_2^4 = \sqrt{2r^b g},$$

$$S_2 n_1^2 = S'_5 n_2^5 = \sqrt{2r^a(g-d)},$$

$$S_3 n_1^3 = S'_4 n_1^4 = \sqrt{2r^a g},$$

$$S_3 n_2^3 = S'_3 n_2^3 = \sqrt{2r^a d}.$$

β. $s < g$, but $\gtreqless g - \frac{d}{2}$.

$$r^b = \frac{2s\,(D - 2l)^2}{\{3s + g - 4\sqrt{s\,(c+t)}\}^2},$$

$$r^a = r^b - \frac{d\,r^b}{2s}.$$

$$\left.\begin{matrix}\text{Interval between springings}\\ \text{of reverse curves .. ..}\end{matrix}\right\} = \frac{(3s+g)\sqrt{r^b}}{\sqrt{2s}},$$

$$b_1^3 = b_2^5 = \frac{2\sqrt{2r^b(g-d)}}{2r^b+g},$$

$$b_1^3 = b_2^4 = n_1^3 = n_2^4 = \frac{2\sqrt{2r^b s}}{2r^b-g},$$

$$n_1^2 = n_2^5 = \frac{2\sqrt{2r^n(g-d)}}{2r^n+g-d},$$

$$n_2^3 = n_1^4 = \frac{2\sqrt{2r^n d}}{2r^n-g+d},$$

$$S_3 b_1^3 = S'_5 b_2^5 = \sqrt{2r^b(g-d)},$$

$$S_3 b_1^3 = S'_4 b_2^4 = \frac{\sqrt{r^b}(s+g)}{\sqrt{2s}},$$

$$S_2 n_1^2 = S'_5 n_2^5 = \sqrt{2r^n(g-d)},$$

$$S_3 n_1^3 = S'_4 n_2^4 = \frac{\sqrt{r^b}\,(2(s+g)-d)}{2\sqrt{2s}},$$

$$S_3 n_2^3 = S'_4 n_1^4 = \sqrt{2r^n d}.$$

CASE II.—*Main Roads Curved: Reverse Curves separated by an interval of Straight* (*Fig.* 39).

$\alpha.$ $s \gtreqless g.$

There will be two cases: (1st), $R > 2r_1^b$; (2nd), $R < 2r_1^b$.

(1st). $R > 2r_1^b$.

If either of the radii r_1^b, r_2^b be known, the other will be given by one of the equations

$$r_1^b = \frac{R r_2^b g}{R g - (s+g) r_2^b}, \qquad r_2^b = \frac{R r_1^b g}{R g + (s+g) r_1^b},$$

$$O''^2 = \frac{2Rg}{r_2^b(R-r_2^b)}, \qquad O'^2 = \frac{2Rg}{r_1^b(R+r_1^b)},$$

$$O = \sqrt{\frac{2Rg}{r_2^b(R-r_2^b)}} - \sqrt{\frac{2Rg}{r_1^b(R+r_1^b)}},$$

$$r_2^n = r_2^b - \frac{d}{0'''2}, \quad r_1^n = r_1^b - \frac{d}{0'''2},$$

$$b_1^2 = \frac{2\sqrt{2\,R\,r_2^b\,(R - r_2^b)\,(g-d)}}{2\,R\,r_2^b + (R + r_2^b)\,d + (R - r_2^b)\,(g-d)},$$

$$n_1^2 = \frac{2\sqrt{2\,R\,r_2^n\,(R - r_2^n)\,(g-d)}}{2\,R\,r_2^n + (R - r_2^n)\,(g-d)},$$

$$n_2^3 = \frac{2\sqrt{2\,R\,r_2^n\,(R - r_2^n)\,d}}{2\,R\,r_2^n + (R - r_2^n)\,d - (R + r_2^n)\,(g-d)},$$

$$n_1^3 = \frac{2\sqrt{2\,R\,r_2^n\,(R - r_2^n)\,g}}{2\,R\,r_2^n + (R - r_2^n)\,g},$$

$$b_2^4 = \frac{2\sqrt{2\,R\,r_1^b\,(R + r_1^b)\,g}}{2\,R\,r_1^b - 4\,r_1^b\,(s+g) + (R + r_1^b)\,g},$$

$$n_2^4 = \frac{2\sqrt{2\,R\,r_1^n\,(R + r_1^n)\,g}}{2\,R\,r_1^n - 4\,r_1^n\,(s+g) + (R + r_1^n)\,g},$$

$$n_1^4 = \frac{2\sqrt{2\,R\,r_1^n\,(R + r_1^n)\,d}}{2\,R\,r_1^n - 4\,r_1^n\,(s+g) - (R - r_1^n)\,(g-d) + (R + r_1^n)\,d},$$

$$n_2^5 = \frac{2\sqrt{2\,R\,r_1^n\,(R + r_1^n)\,(g-d)}}{2\,R\,r_1^n - 4\,r_1^n\,(s+g) + (R - r_1^n)\,d + (R + r_1^n)\,(g-d)},$$

$$b_2^5 = \frac{2\sqrt{2\,R\,r_1^b\,(R + r_1^b)\,(g-d)}}{2\,R\,r_1^b - 4\,r_1^b\,(s+g) + (R - r_1^b)\,d + (R + r_1^b)\,(g-d)},$$

$$S_2 b_1^2 = \sqrt{\frac{2\,R\,r_2^b\,(g-d)}{R - r_2^b}},$$

$$S_2 n_1^2 = \sqrt{\frac{2\,R\,r_2^n\,(g-d)}{R - r_2^n}},$$

$$S_2 b_1^3 = \sqrt{\frac{2\,R\,r_2^b\,g}{R - r_2^b}},$$

$$S_2 n_2^3 = \sqrt{\frac{2\,R\,r_2^n\,d}{R - r_2^n}},$$

$$S'_5 b_2^5 = \sqrt{\frac{2\,R\,r_1^b\,(g-d)}{R + r_1^b}},$$

$$S'_4 b_2^4 = \sqrt{\frac{2\,R\,r_1^b\,g}{R + r_1^b}},$$

$$S'_5 n_2^5 = \sqrt{\frac{2 R r_1^a (g-d)}{R + r_1^a}},$$

$$S'_4 n_1^4 = \sqrt{\frac{2 R r_1^a d}{R + r_1^a}}.$$

The positions of the crossings n_1^3, n_2^4 are fixed by those of b_1^3, b_2^4, and are distant very nearly $\frac{d}{2 b_1^3}$ and $\frac{d}{2 b_2^4}$ respectively from b_1^3 and b_2^4.

(2nd). $R < 2 r_1^b$.

The circular measures and positions of all the angles will be the same, except those for n_2^5, for which

$$n_2^5 = \frac{2\sqrt{R r_1^b \{2(R + r_1^b) g - 3 r_1^b d\}}}{r_1^b \{2R - 4s - 3g - 2d\}},$$

$$S'_5 n_2^5 = R(n_2^5 - O').$$

β. $s < g$.

Whatever may be the relative values of R and r_2^b, the second rail of the main roads will be cut by the first rails of the curves radii r_2^b, r_2^a, if

$$s \gtrless g - \frac{d}{2}.$$

The same remark applies to the curves radii r_1^b, r_1^a, when

$$R \gtrless 2 r_2^b + \frac{2 r_2^b (s+g)}{g};$$

or, practically, whenever

$$R \gtrless 6 r_1^b,$$

$$r_2^b = \frac{R r_1^b s}{R s + (s+g) r_1^b}, \qquad r_1^b = \frac{R r_2^b s}{R s - (s+g) r_2^b},$$

$$O'' = \frac{2 R s}{r_2^b (R - r_2^b)}, \qquad O'^2 = \frac{2 R s}{r_1^b (R + r_1^b)},$$

$$O = \sqrt{\frac{2 R s}{r_2^b (R - r_2^b)}} - \sqrt{\frac{2 R s}{r_1^b (R + r_1^b)}},$$

$$r_2^a = r_2^b - \frac{d}{O''^2}, \qquad r_1^a = r_1^b - \frac{d}{O'^2}.$$

When

$$R \gtrless 2 r_2^b + \frac{2 r_2^b (s+g)}{g},$$

or

$$R \gtrless 6 r_1^b,$$

the values and positions of $n_1^2\ n_2^5\ n_2^3\ n_1^4\ b_1^2\ b_2^5$ will be the same as those in (*a*), and the rest are

$$b_2^4 = n_2^4 = \frac{2\sqrt{2 R r_2^b (R - r_2^b) s}}{2 R r_2^b + (R - r_2^b) s},$$

$$b_1^3 = n_1^3 = \frac{2\sqrt{2 R r_1^b (R + r_1^b) s}}{2 R r_1^b + (R + r_1^b) s},$$

$$S_3 b_1^3 = R (O'' - b_1^3),$$

$$S'_4 b_2^4 = R (b_2^4 - O'),$$

$$S'_3 b_1^3 = \sqrt{\frac{2 R r_1^b s}{R + r_1^b}},$$

$$S_4 b_2^4 = \sqrt{\frac{2 R r_2^b s}{R - r_2^b}}.$$

When

$$R < 2 r_2^b + \frac{2 r_2^b (s+g)}{g},$$

or

$$R < 6 r_1^b,$$

the only angles affected are b_2^5 and n_2^5,

$$b_2^5 = n_2^5 = \frac{2\sqrt{R r_1^b \{2 (R - r_1^b) s - 3 r_1^b d\}}}{r_1^b (2 R - 4 s - 3 g - 2 d)},$$

$$S_3 b_2^5 = R (b_2^5 - O'),$$

$$b_2^5 n_2^5 = \frac{d}{2 b_2^5} \text{ very nearly.}$$

CLASS II.—*Between two contiguous Siding Roads, where the Centre Line of the space between the two Inner Rails of the Broad-gauge Road does not coincide with that of the space between the two Inner Rails of the Narrow-gauge Road.*

If the distance between the points of the switches be given, the formulæ of Chapter III., Section IV., will give the radii of

the reverse curves of the broad-gauge road; or, if one radius be given, the formulæ will give the other. The radii of the reverse curves of the narrow-gauge road are equal to the corresponding radii of the broad-gauge, increased or diminished by $\frac{d}{2}$, according as the rail common to both gauges is more remote from or nearer to the centre lines of the spaces between the two inner rails than the middle rail. The radii being known, the circular measures and positions of the angles of the crossings of both gauge roads are readily obtained from the same formulæ.

CASE I.—*Main Line Straight* (*Fig.* 40).

As in Case I., Class I., if D be given,

$$r^b = \frac{(D - 2l)^2}{4\left\{\sqrt{s+g} - \sqrt{2(c+t)}\right\}^2},$$

and

$$r^a = r^b \pm \frac{d}{2}.$$

If r^b be given,

$$D = 2l + 2\sqrt{r^b}\left\{\sqrt{s+g} - \sqrt{2(c+t)}\right\}.$$

Let $S_1 S'_1$ and $S_4 S'_4$, both in Case I. and Case II., be the rails which are common to both gauge roads.

a. $s > g$.

Broad Gauge.

$$b_1^2 = \frac{2\sqrt{2r^b(g-d)}}{2r^b+g},$$

$$b_1^3 = b_2^4 = \frac{2\sqrt{2r^b g}}{2r^b+g},$$

$$b_2^5 = \frac{2\sqrt{2r^b d}}{2r^b+g},$$

$$S_2 b_1^2 = \sqrt{2r^b(g-d)},$$

$$S'_2 b_1^2 = 2\sqrt{r^b(s+g)} - \sqrt{2r^b(g-d)},$$

$$S_3 b_1^3 = S'_4 b_2^4$$
$$= \sqrt{2 r^b g},$$
$$S_4 b_2^4 = S'_3 b_1^3$$
$$= 2\sqrt{r^b (s+g)} - \sqrt{2 r^b g},$$
$$S_5 b_2^5 = 2\sqrt{r^b (s+g)} - \sqrt{2 r^b d},$$
$$S'_5 b_2^5 = \sqrt{2 r^b d}.$$

Narrow Gauge.

$$n_1^3 = \frac{2\sqrt{2 r^b d}}{2 r^b - g + 2d},$$
$$n_1^4 = \frac{2\sqrt{2 r^b (g-d)}}{2 r^b + g - 2d},$$
$$S_3 n_1^3 = \sqrt{2 r^b d},$$
$$S'_3 n_1^3 = 2\sqrt{r^b (s+g)} - \sqrt{2 r^b d},$$
$$S_4 n_1^4 = 2\sqrt{r^b (s+g)} - \sqrt{2 r^b (g-d)},$$
$$S'_4 n_1^4 = \sqrt{2 r^b (g-d)}.$$

β. $s \angle g$, but $> g - d$.

Broad Gauge.

$$b_1^2 = \frac{2\sqrt{2 r^b (g-d)}}{2 r^b + g},$$
$$b_1^3 = b_2^4$$
$$= \frac{2\sqrt{2 r^b s}}{2 r^b - g},$$
$$b_2^5 = \frac{2\sqrt{2 r^b d}}{2 r^b + g},$$
$$S_2 b_1^2 = \sqrt{2 r^b (g-d)},$$
$$S'_2 b_1^2 = 2\sqrt{r^b (s+g)} - \sqrt{2 r^b (g-d)},$$
$$S_3 b_1^3 = S'_4 b_2^4$$
$$= 2\sqrt{r^b (s+g)} - \sqrt{2 r^b s},$$

$$S_4 b_2^4 = S'_3 b_1^3$$
$$= \sqrt{2 r^b s},$$
$$S_5 b_2^5 = 2\sqrt{r^b (s+g)} - \sqrt{2 r^b d},$$
$$S'_5 b_2^5 = \sqrt{2 r^b d}.$$

Narrow Gauge.

The circular measures and positions of the crossings are given by the formulæ in (*a*), Case I.

CASE II.—*Main Line Curved* (*Fig.* 41).

If the distance D be given, we have

$$O = \frac{(D - 2l)\sqrt{s+g}}{R\{\sqrt{s+g} - \sqrt{2(c+t)}\}},$$
$$r_1^b = \frac{R^2 O^2}{4(s+g) - R O^2},$$
$$r_2^b = \frac{R^2 O^2}{4(s+g) + R O^2}.$$

If either radius be given, the other is known by one of the equations

$$r_1^b = \frac{R r_2^b}{R - 2 r_2}, \qquad r_2^b = \frac{R r_1^b}{R + 2 r_1^b},$$

and

$$O = \frac{2\sqrt{r_2^b (s+g)}}{\sqrt{R(R - r_2^b)}} = \frac{2\sqrt{r_1^b (s+g)}}{\sqrt{R(R + r_1^b)}},$$
$$r_1^n = r_1^b \pm \frac{d}{2}, \qquad r_2^n = r_2^b \mp \frac{d}{2}.$$

Without trespassing beyond the assigned limits of exactness, we may look upon the centre lines of the space between the two inner rails of the broad-gauge road and of the broad-gauge reverse curves as being coincident with the corresponding centre lines of the narrow-gauge roads, since they are only separated by an interval equal to $\frac{d}{2}$.

a. $s > g$.

Broad Gauge.

$$b_1^2 = \frac{2\sqrt{2\,\mathrm{R}\,r_2^b\,(\mathrm{R}-r_2)\,(g-d)}}{2\,\mathrm{R}\,r_2^b + (\mathrm{R}-r_2^b)\,(g-d)},$$

$$b_1^3 = b_2^4$$

$$= \frac{2\sqrt{2\,\mathrm{R}\,r_2^b\,(\mathrm{R}-r_2^b)\,g}}{2\,\mathrm{R}\,r_2^b + (\mathrm{R}-r_2^b)\,g} = \frac{2\sqrt{2\,\mathrm{R}\,r_1^b\,(\mathrm{R}+r_1^b)\,g}}{2\,\mathrm{R}\,r_1^b + (\mathrm{R}+r_1^b)\,g},$$

$$b_2^5 = \frac{2\sqrt{2\,\mathrm{R}\,r_1^b\,(\mathrm{R}+r_1^b)\,d}}{2\,\mathrm{R}\,r_1^b + (\mathrm{R}+r_1^b)\,d},$$

$$\mathrm{S}_2 b_1^2 = \sqrt{\frac{2\,\mathrm{R}\,r_2^b\,(g-d)}{\mathrm{R}-r_2^b}},$$

$$\mathrm{S}'_2 b_1^2 = \mathrm{R\,O} - \mathrm{S}_2 b_1^2$$

$$= \sqrt{\frac{\mathrm{R}\,r_2^b}{\mathrm{R}-r_2^b}}\left\{2\sqrt{s+g} - \sqrt{2\,(g-d)}\right\},$$

$$\mathrm{S}_2 b_1^3 = \mathrm{S}'_4 b_2^4$$

$$= \sqrt{\frac{2\,\mathrm{R}\,r_2^b\,g}{\mathrm{R}-r_2^b}} = \sqrt{\frac{2\,\mathrm{R}\,r_1^b\,g}{\mathrm{R}+r_1^b}},$$

$$\mathrm{S}'_5 b_2^5 = \sqrt{\frac{2\,\mathrm{R}\,r_1^b\,d}{\mathrm{R}+r_1}},$$

$$\mathrm{S}_5 b_2^5 = \mathrm{R\,O} - \mathrm{S}'_5 b_2^5$$

$$= \sqrt{\frac{\mathrm{R}\,r_1^b}{\mathrm{R}+r_1^b}}\left\{2\sqrt{s+g} - \sqrt{2\,d}\right\}.$$

Narrow Gauge.

$$n_1^3 = \frac{2\sqrt{2\,\mathrm{R}\,r_2^b\,(\mathrm{R}-r_2^b)\,d}}{2\,\mathrm{R}\,r_2^b + (\mathrm{R}-r_2^b)\,d},$$

$$n_1^4 = \frac{2\sqrt{2\,\mathrm{R}\,r_1^b\,(\mathrm{R}+r_1^b)\,(g-d)}}{2\,\mathrm{R}\,r_1^b + (\mathrm{R}+r_1^b)\,(g-d)},$$

$$\mathrm{S}_3 n_1^3 = \sqrt{\frac{2\,\mathrm{R}\,r_2^b\,d}{\mathrm{R}-r_2^b}},$$

$$\mathrm{S}'_3 n_1^3 = \mathrm{R\,O} - \mathrm{S}_1 n_1^3$$

$$= \sqrt{\frac{\mathrm{R}\,r_2^b}{\mathrm{R}-r_2^b}}\left\{2\sqrt{s+g} - \sqrt{2\,d}\right\},$$

$$S'_4 n_1^4 = \sqrt{\frac{2 R r_1^b (g-d)}{R + r_1^b}},$$

$$S_4 n_1^4 = R O - S'_4 n_1^4$$

$$= \sqrt{\frac{R r_1}{R + r_1}} \left\{ 2\sqrt{s+g} - \sqrt{2(g-d)} \right\}.$$

β. $s < g$, but $> g - d$.

Broad Gauge.

$$b_1^2 = \frac{2\sqrt{2 R r_2^b (R - r_2^b)(g-d)}}{2 R r_2^b + (R - r_2)(g-d)},$$

$$b_1^3 = b_2^4$$

$$= \frac{2\sqrt{2 R r_1^b (R + r_1^b) s}}{2 R r_1^b + (R + r_1^b) s} = \frac{2\sqrt{2 R r_2^b (R - r_2^b) s}}{2 R r_2^b + (R - r_2^b) s},$$

$$b_2^5 = \frac{2\sqrt{2 R r_1^b (R + r_1^b) d}}{2 R r_1^b + (R + r_1^b) d},$$

$$S_3 b_1^2 = \sqrt{\frac{2 R r_2^b (g-d)}{R - r_2^b}},$$

$$S'_2 b_1^2 = R O - S_3 b_1^2$$

$$= \sqrt{\frac{R r_2^b}{R - r_2^b}} \left\{ 2\sqrt{s+g} - \sqrt{2(g-d)} \right\},$$

$$S_3 b_1^3 = S'_4 b_2^4$$

$$= \sqrt{\frac{2 R r_2^b s}{R - r_2^b}} = \sqrt{\frac{2 R r_1^b s}{R + r_1^b}},$$

$$S_4 b_1^4 = S'_3 b_1^3$$

$$= R O - S_3 b_1^3$$

$$= \sqrt{\frac{R r_2^b}{R - r_2^b}} \left\{ 2\sqrt{s+g} - \sqrt{2 s} \right\},$$

$$S'_5 b_2^5 = \sqrt{\frac{2 R r_1^b d}{R + r_1^b}},$$

$$S_5 b_2^5 = R O - S'_5 b_2^5$$

$$= \sqrt{\frac{R r_1^b}{R + r_1^b}} \left\{ 2\sqrt{s+g} - \sqrt{2 d} \right\}.$$

Narrow Gauge.

The circular measures and positions of the crossings are given in (*a*), Case II.

Section V.—Through Roads.

As in the case of cross-over roads, there will be two classes of through roads.

Class I. comprises all those which connect two roads separated by any number of intervening roads in which the centre lines of the narrow-gauge roads, including the two roads connected, are in the case of some pairs equidistant from the centre line of the space between the inner rails of the broad-gauge roads, and in others not.

Class II. comprises all those which connect two roads, separated by any number of intervening roads, in which the centre lines of the narrow-gauge roads, including the two roads connected, taken pair by pair, are in no case equidistant from the centre line of the space between the inner rails of the broad-gauge roads.

In Class I. general formulæ applicable to every case would be very complicated, and of no practical use. Special formulæ, therefore, must be worked out for each case; only one case, which is of common occurrence, is here investigated in detail, *viz.*:—

Through road connecting the up or down road with a parallel siding road, separated only by the down or up road; the middle rail of the siding being at the same distance from the centre line of the space between the two inner rails of the intervening road and the siding, taken as a pair, as the middle rail of the intervening road is from the centre of the main line.

Class I.

The communication between the two outside roads must be established by means of two reversing curves, separated by an interval of straight, for the reasons given in Section IV.

The simplest combination of reverse curves, separated by an interval of straight, is that in which the straight parts of both

through roads are parallel to one another, and their centre lines coincident. For the broad-gauge through road the combination adopted is explained in Chapter III., Section V., Cases I. and III. The centre lines of the narrow-gauge reverse curves will meet tangentially the common centre line of the straight part of the through roads, and the centre lines of the narrow-gauge main-line roads at the points where these are cut by the normals, through the springings of the broad-gauge reverse curves.

Let O O′ S_3 S_2 S_1 (Figs. 42 to 47) be the normal through the centres of the reverse curves meeting the rails of the main road in S_1 S_2 S_3. If O P (Figs. 42 and 45), the radius of the broad-gauge reverse curve, which passes through the point where either rail of that reverse curve first cuts a main-road rail, meet the common centre line of the two roads in P, the point P will be the commencement of the straight portion of the broad-gauge through road, and if O′ P′ parallel to O P meet the centre line of the narrow-gauge through road in P′, the point P′ will be the commencement of the straight portion of the narrow-gauge through road. Therefore,

$$\begin{aligned} \text{O P} &= \text{radius of broad-gauge reverse curve} \\ &= r^b; \\ \text{O P}' &= \text{radius of narrow-gauge reverse curve} \\ &= r^n. \end{aligned}$$

Draw O′ Q meeting O P at right angles in Q, then

$$\begin{aligned} \text{O}'\text{P}' &= \text{O P} - \text{O Q} \\ &= \text{O P} - \text{O O}' \cos.\ \text{P O O}', \\ \text{O O}' &= \text{O S}_1 - \text{O}' \text{S}_1 \\ &= r^b + \frac{g}{2} - \left(r^n + \frac{g-d}{2}\right) \\ &= r^b - r^n + \frac{d}{2}. \end{aligned}$$

Therefore, if angle P O O′ be denoted by the letter O,

$$\begin{aligned} r^n &= r^b - \left(r^b - r^n + \frac{d}{2}\right) \cos.\ \text{O} \\ &= r^b - \frac{d \cos.\ \text{O}}{2(1 - \cos.\ \text{O})}. \end{aligned}$$

The preceding investigation applies to each of the three cases:

1st. Main Road Straight.

α. $s > g$.

$$\cos. O = \frac{2r^b - g}{2r^b + g},$$

$$r^a = r^b - \frac{d(2r^b - g)}{4g}$$

$$= \frac{(2g - d)r^b}{2g},$$

neglecting $\frac{d}{4}$.

β. $s \angle g$.

$$\cos. O = \frac{2r^b - g - 2s}{2r^b - g},$$

$$r^a = r^b - \frac{d(2r^b - g - 2s)}{4s}$$

$$= \frac{(2s - d)r^b}{2s},$$

neglecting $\frac{d(g + 2s)}{4s}$.

2nd. Main Roads Curved, Curvature of Reverse Curves in the same direction.

$$O = O'' \qquad r^b = r_2^b,$$

and

$$r_2^a = r_2^b - \frac{d \cos. O''}{2(1 - \cos. O'')}$$

$$= r_2^b - \frac{d\left(1 - 2 \sin.^2 \frac{O''}{2}\right)}{4 \sin.^2 \frac{O''}{2}}$$

$$= r_2^b - \frac{d}{O''^2},$$

putting $\sin. \frac{O''}{2} = \frac{O''}{2}$, and neglecting $\frac{d}{2}$.

3rd. Main Roads Curved, Curvature of Reverse Curves in the opposite direction.

$$O = O' \qquad r^b = r_1^b,$$

$$r_1^a = r_1^b - \frac{d}{O's}.$$

Before the circular measure of the angle n_1^2, where main-road middle rail is cut by the outer rail of the narrow-gauge through road can be obtained, it is necessary to ascertain whether the middle rail is cut by the curved or straight part of the through road.

Now, the interval between the centres of the two reverse curves at the same springing is equal to

$$r^b - r^a + \frac{d}{2},$$

therefore the perpendicular distance between the two radii is equal to

$$\left(r^b - r^a + \frac{d}{2}\right) \text{sin. } O,$$

where O is equal to the angle P O O', P O' O', or P O'' O'_1.

1st. When the main roads are straight, we have

α. $s \gtrless g$.

$$\left(r^b - r^a + \frac{d}{2}\right) \text{sin. } O = \left(\frac{d\,r^b}{2g} + \frac{d}{2}\right) \frac{2\sqrt{2\,r^b g}}{2\,r^b + g}$$

$$= \frac{d\,r^b \sqrt{2\,r^b g}}{2\,r^b g + g^2} \text{ nearly}$$

$$= \frac{d\sqrt{r^b}}{\sqrt{2\,g}}.$$

β. $s < g$.

$$\left(r^b - r^a + \frac{d}{2}\right) \text{sin. } O = \frac{d\sqrt{r^b}}{\sqrt{2\,s}}.$$

2nd. When the roads are curved, curvature of reverse curves in the same direction, we have

$\alpha.$ $s \gtreqless g.$

$$\left(r_2^b - r_2^a + \frac{d}{2}\right) \sin. O'' = \left(\frac{d}{O''^2} + \frac{d}{2}\right) O''$$

$$= \frac{d\sqrt{r_2^b (R - r_2^b)}}{\sqrt{2Rg}}.$$

$\beta.$ $s < g.$

$$\left(r_2^b - r_2^a + \frac{d}{2}\right) \sin. O'' = \frac{d\sqrt{r_2^b (R - r_2^b)}}{\sqrt{2Rs}}.$$

3rd. When the roads are curved, curvature of reverse curves in the opposite direction, we have

$\alpha.$ $s \gtreqless g.$

$$\left(r_1^b - r_1^a + \frac{d}{2}\right) \sin. O' = \left(\frac{d}{O'^2} + \frac{d}{2}\right) O'$$

$$= \frac{d\sqrt{r_1^b \{R + r_1^b - (n+1)(s+g)\}}}{\sqrt{2\{R - (n+1)(s+g)\}g}}.$$

$\beta.$ $s < g.$

$$\left(r_1^b - r_1^a + \frac{d}{2}\right) \sin. O' = \frac{d\sqrt{r_1^b \{R + r_1^b - (n+1)(s+g)\}}}{\sqrt{2\{R - (n+1)(s+g)\}s}}.$$

Let Q (Figs. 42, 43, 44, 45, 46, and 47) be the point where the radius from the centre of the broad-gauge reverse curve, which passes through the end of the interval of straight, is cut by the outer rail of the narrow-gauge reverse curve, and q the point where the rail $S_3 b_1^3$ is cut by a normal to the main line through n_1^2.

The interval between the point Q and n_1^2 is equal to

$$n_1^2 n_1^3 \pm Q n_1^3,$$

$\pm$ accordingly as the point Q falls without or within the space d between the two inner rails $S_2 b_1^2$, $S_3 b_1^3$. The condition that the middle rail should be cut by the reverse curve of the narrow-gauge road, is simply that this interval must not be less than the interval between the broad and narrow-gauge normals, which pass through the adjacent extremities of their respective intervals of straight.

The expression $n_1^2 n_1^3 \pm Q n_1^3$ will have two values in each of the three cases corresponding to the two relative values of s and g.

α. $s \gtreqless g$.

The point Q, in each of the three cases, lies outside the space d between the rails $S_2 b_1^2$ and $S_3 b_2^3$, and

$$Q n_1^3 + n_1^3 n_1^3 = Q b_1^3 \cot. b_1^3 + q n_1^3 \operatorname{cosec}. n_1^3$$

exactly if the main roads be straight, and very nearly if they are curved. Whether the roads are straight or curved,

$$Q b_1^3 \overset{\mp}{=} \frac{d}{2}, \qquad q n_1^3 = d.$$

If the roads are straight,

$$\sin. n_1^3 = \sin. b_1^3 = \frac{2\sqrt{2 r^b g}}{2 r^b + g},$$

$$\cos. n_1^3 = \cos. b_1^3 = \frac{2 r^b - g}{2 r^b + g},$$

and

$$Q b_1^3 \cot. b_1^3 + q n_1^3 \operatorname{cosec}. n_1^3 = \frac{d(2 r^b - g)}{4\sqrt{2 r^b g}} + \frac{d(2 r^b + g)}{2\sqrt{2 r^b g}}$$

$$= \frac{3 d \sqrt{r^b}}{2\sqrt{2 g}}.$$

If the roads are curved, the reverse curves curving in the same direction, very nearly

$$\sin. n_1^3 = \sin. b_1^3 = \frac{2\sqrt{2 R r_2^b (R - r_2^b) g}}{2 R r_2^b + (R - r_2^b) g},$$

$$\cos. n_1^3 = \cos. b_1^3 = \frac{2 R r_2^b - (R - r_2^b) g}{2 R r_2^b + (R - r_2^b) g},$$

and

$$Q b_1^3 \cot. b_1^3 + q n_1^3 \operatorname{cosec}. n_1^3 = \frac{d\{2 R r_2^b - (R - r_2^b) g\}}{4\sqrt{2 R r_2^b (R - r_2^b) g}}$$

$$+ \frac{d\{2 R r_2^b + (R - r_2^b) g\}}{2\sqrt{2 R r_2^b (R - r_2^b) g}}$$

$$= \frac{3 d \sqrt{R r_2^b}}{2\sqrt{2 (R - r_2^b) g}}.$$

Similarly, if the roads are curved, the reverse curves curving in the opposite direction,

$$Q\,b_1^{\,2}\cot.\,b_1^{\,2} + q\,n_1^{\,2}\,\mathrm{cosec.}\,n_1^{\,2} = \frac{3\,d\sqrt{r_1^{\,b}\{R-(n+1)(s+g)\}}}{2\sqrt{2\{R+r_1^{\,b}-(n+1)(s+g)\}}}.$$

And the equations of condition are:

Main Road Straight.

$$\frac{3\,d\sqrt{r^b}}{2\sqrt{2g}} \gtrless \frac{d\sqrt{r^b}}{\sqrt{2g}},$$

or

$$3 > 2;$$

therefore the middle rail is always cut by the reverse-curve rail.

Main Roads Curved, Curvature of Reverse Curves in the same direction.

$$\frac{3\,d\sqrt{R\,r_2^{\,b}}}{2\sqrt{2(R-r_2^{\,b})\,g}} \gtrless \frac{d\sqrt{r_2^{\,b}(R-r_2^{\,b})}}{\sqrt{2Rg}},$$

or

$$3R \gtrless 2(R-r_2^{\,b});$$

therefore the middle rail is always cut by the reverse curve.

Main Roads Curved, Reverse Curve Curving in the opposite direction.

$$\frac{3\,d\sqrt{r_1^{\,b}\{R-(n+1)(s+g)\}}}{2\sqrt{2\{R+r_1^{\,b}-(n+1)(s+g)\}\,g}} = \frac{d\sqrt{r_1^{\,b}\{R+r_1^{\,b}-(n+1)(s+g)\}}}{\sqrt{2\{R-(n+1)(s+g)\}\,g}},$$

or

$$3\{R-(n+1)(s+g)\} \gtrless 2\{R+r_1^{\,b}-(n+1)(s+g)\}$$

$$R-(n+1)(s+g) \gtrless 2r_1^{\,b};$$

therefore, if the radius of the main-line road be less than twice the radius of the reverse curve of the through road, the middle rail will be cut by the straight part of the narrow-gauge road.

β. $s < g$.

The point Q will sometimes lie within and sometimes without

the space d, between the rails $S_2 b_1^2$ and $S_3 b_1^3$; therefore the interval between Q and n_1^2 is

$$n_1^3 n_1^2 \pm Q n_1^3 = q n_1^2 \text{ cosec. } n_1^3 \pm Q c \text{ cot. } b_1^3,$$

c being the point where the radius O Q cuts the rail $S_3 b_1^3$; also very nearly in each case

$$Q c = \pm \left(s \text{ sec. } b_1^3 - g + \frac{d}{2} \right),$$

$\pm$ according as Q lies without or within the space d, between the rails $S_2 b_1^2$ and $S_3 b_1^3$; therefore wherever Q may lie,

$$n_1^2 n_1^3 \pm Q n_1^3 = s \text{ cosec. } b_1^3 - \left(g - \frac{d}{2} \right) \text{cot. } b_1^3 + q n_1^2 \text{ cosec. } n_1^3.$$

When the main line is straight,

$$\text{sin. } n_1^3 = \text{sin. } b_1^3 = \text{sin. } b_2^4 = \frac{2\sqrt{2 r^b s}}{2 r^b - g},$$

$$\text{cos. } n_1^3 = \text{cos. } b_1^3 = \text{cos. } b_2^4 = \frac{2 r^b - 2 s - g}{2 r^b - g},$$

and

$$n_1^3 n_1^3 \pm Q n_1^3 = (s + d) \text{ cosec. } b_1^3 - \left(g - \frac{d}{2} \right) \text{cot. } b_1^3$$

$$= \frac{(s + d)(2 r^b - g)}{2\sqrt{2 r^b s}} - \frac{(2 g - d)(2 r^b - 2 s - g)}{4\sqrt{2 r^b s}}$$

$$= \frac{(2 s - 2 g + 3 d)\sqrt{r^b}}{2\sqrt{2 s}}.$$

When the roads are curved, curvature of reverse curves in the same direction, the exact values of sin. n_1^3, sin. b_1^3, cos. n_1^3, cos. b_1^3, are

$$\text{sin. } n_1^3 = \frac{2\sqrt{2 R r_2^b \{R s - r_2^b (g - d)\}}}{r_2^b (2 R - g)},$$

$$\text{cos. } n_1^3 = \frac{2 R (r_2^b - s) + r_2^b (g - d)}{r_2^b (2 R - g)},$$

$$\text{sin. } b_1^3 = \frac{2\sqrt{2 R r_2^b (R s - r_2^b g)}}{r_2^b (2 R - g)},$$

$$\text{cos. } b_1^3 = \frac{2 R (r_2^b - s) + r_2^b g}{r_2^b (2 R - g)}.$$

Since the value of s lies between g and $g - d$, the deviation from exactness, so far as our present object is concerned, will be inappreciable, if we put

$$\text{sin. } n_1^s = \text{sin. } b_1^s = \frac{2\sqrt{2\,\mathrm{R}\,r_2^b\,(\mathrm{R} - r_2^b)\,s}}{r_2^b\,(2\,\mathrm{R} - g)},$$

$$\text{cos. } n_1^s = \text{cos. } b_1^s = \frac{2\,\mathrm{R}\,(r_2^b - s) + r_2^b\,s}{r_2^b\,(2\,\mathrm{R} - g)}.$$

With these values we get

$$n_1^s\,n_1^s \pm \mathrm{Q}\,n_1^s = \frac{(2\,s - 2\,g + 3\,d)\sqrt{\mathrm{R}\,r_2^b}}{2\sqrt{2\,(\mathrm{R} - r_2^b)\,s}}.$$

Main Line Curved, Curvature of Reverse Curves in the opposite direction.

As in the preceding case, it may be shown that

$$n_1^s\,n_1^s \pm \mathrm{Q}\,n_1^s = \frac{(2\,s - 2\,g + 3\,d)\sqrt{\{\mathrm{R} - (n+1)\,(s+g)\}\,r_1^b}}{2\sqrt{2\,\{\mathrm{R} + r_1^b - (n+1)\,(s+g)\}\,s}}.$$

The equations of condition are:

Main Road Straight.

$$\frac{(2\,s - 2\,g + 3\,d)\sqrt{r^b}}{2\sqrt{2\,s}} \gtrless \frac{d\sqrt{r^b}}{2\sqrt{2\,s}},$$

or

$$s \gtrless g - \frac{d}{2}.$$

Main Roads Curved, Reverse Curves Curving in the same direction.

$$\frac{(2\,s - 2\,g + 3\,d)\sqrt{\mathrm{R}\,r_2^b}}{2\sqrt{2\,(\mathrm{R} - r_2^b)\,s}} \gtrless \frac{d\sqrt{r_2^b\,(\mathrm{R} - r_2^b)}}{\sqrt{2\,\mathrm{R}\,s}},$$

or

$$s \gtrless g - \frac{d}{2} - \frac{d\,r_2^b}{\mathrm{R}}.$$

Main Roads Curved, Reverse Curves Curving in the opposite direction.

$$\frac{(2\,s - 2\,g + 3\,d)\sqrt{\{\mathrm{R} - (n+1)\,(s+g)\}\,r_1^b}}{2\sqrt{2\,\{\mathrm{R} + r_1 - (n+1)\,(s+g)\}\,s}}$$

$$\gtrless \frac{d\sqrt{r_1^b\,\{\mathrm{R} + r_1 - (n+1)\,(s+g)\}}}{\sqrt{2\,\{\mathrm{R} - (n+1)\,(s+g)\}\,s}},$$

or

$$s \gtreqless g - \frac{d}{2} + \frac{d\, r_1 b}{R - (n+1)(s+g)}.$$

In the two first cases the middle rail will always be cut by the reverse curve, if

$$s \gtreqless g - \frac{d}{2}.$$

In the last case it will always be cut by the straight part of the through road, unless

$$R - (n+1)(s+g) > 2\, r_1^b.$$

The relation between r_1^b, r_2^b and R is given by the equation

$$\frac{R}{r_2^b} = \frac{R}{r_1^b} + \frac{(n+1)(s+g)}{s},$$

since we may use for the purpose of this investigation the radius of any of the main roads indiscriminately.

Now r_1^b has a minimum value when r_2^b has: consequently, since in order that the middle rail may be cut by the reverse curve when s just equals g, r_1^b must not be greater than $\frac{1}{2}$ R, the following equation gives the minimum value of R in terms of the minimum value of r_2^b:

$$R = 2\, r_2^b + \frac{(n+1)\, r_2^b\, (s+g)}{s}.$$

If $R = 6\, r_1^b$, the condition that the middle rail should be cut by the reverse curve will be practically identical with

$$s \gtreqless g - \frac{d}{2},$$

and the minimum value of R in terms of the minimum value of r_2^b will be given by the equation

$$R = 6\, r_2^b + \frac{(n+1)\, r_2^b\, (s+g)}{s}.$$

Since in Class I. the centre lines of the two outside rails of the through road at one end meet together at a common tangent, and gradually diverge until at the other end, where they become the two inner rails, they are separated by the interval d, the narrow-gauge rail must commence with a switch or a fast point at a certain distance from the point of the

switch of the broad-gauge rail, which depends upon the relative magnitude of the radii of the two reverse curves. Theoretically it is a matter of no consequence where the point of the switch or the fast point may be situated; practically it is necessary that the centre lines of the rails should be separated by an interval not less than $c + t$ before the first main-line rail is crossed, if a switch be used; since, independently of the difficulty of working the switch in any other position, it must be so situated as not to cross any of the main-line rails, which it will rarely be if the condition above stated is not fulfilled. If a fast point be used, it is immaterial where the separation of the two rails takes place. In any case it is much the best and simplest plan to use a fast point.

The condition that the centre lines of the rails should be separated by an interval not less than $c + t$ before the middle rail is reached, is simply that the distance from the springing of the point where this amount of separation is attained must not be greater than that of the narrow-gauge crossing of the middle rail.

When the middle rail is cut by the rails of both the narrow-gauge and broad-gauge reverse curves, this condition is expressed by the following relations:

Main Roads Straight.

$$\sqrt{\frac{2\,r^n\,r^b\,(c+t)}{r^b - r^n}} < \sqrt{2\,r^n\,(g-d)}.$$

Main Roads Curved.

$$\sqrt{\frac{2\,r_2^n\,r_2^b\,(c+t)}{r_2^b - r_2^n}} < \sqrt{\frac{2\,\mathrm{R}\,r_2^n\,(g-d)}{\mathrm{R} - r_2^n}},$$

$$\sqrt{\frac{2\,r_1^n\,r_1^b\,(c+t)}{r_1^b - r_1^n}} < \sqrt{\frac{2\,\mathrm{R}_1\,r_1^n\,(g-d)}{\mathrm{R}_1 + r_1^n}}.$$

Since the middle rail is always cut by the rails of the reverse curves if

$$s \gtreqless g - \frac{d}{2},$$

except in the case of the reverse curve radius r_1^b, when R_1 is less than $6\,r_1^b$, in which case the crossing will be more remote from

the springing than when it is cut by the curve, therefore the above relations hold good for all cases.

When

$$s \gtreqless g - \frac{d}{2}$$

they assume the following special forms:

CASE I.—*Main Roads Straight, Reverse Curves separated by an interval of Straight.*

$s > g$.

Since

$$r^a = \frac{2g - d}{2g} r^b,$$

the condition becomes

$$\frac{2g(c+t)}{d} < g - d.$$

$s < g$.

Since

$$r^a = \frac{2s - d}{2s} r^b,$$

the condition becomes

$$\frac{2s(c+t)}{d} < g - d.$$

CASE II.—*Main Roads Curved, Reverse Curves separated by an interval of Straight.*

Whether s be $\gtrless g$, we have

$$r_2^a = r_2^b - \frac{d}{O'^2},$$

$$r_1^a = r_1^b - \frac{d}{O'^2},$$

and the conditions for the two reverse curves become

$$\sqrt{\frac{r_2^b(c+t)}{d}} < \sqrt{\frac{R(g-d)}{(R - r_2^b)O'^2 + d}},$$

$$\sqrt{\frac{r_1^b(c+t)}{d}} < \sqrt{\frac{R(g-d)}{(R + r_1^b)O'^2 - d}}.$$

α. $s > g$.

$$O''^2 = \frac{2\,R\,g_4}{r_2^b\,(R - r_2^b)},$$

$$O'^2 = \frac{2\,R\,g}{r_1^b\,(R + r_1^b)},$$

since we may put $R - (n+1)\,(s+g) = R$; and the conditions become

$$\frac{c+t}{d} < \frac{R\,(g-d)}{2\,R\,g + d\,r_2^b},$$

$$\frac{c+t}{d} < \frac{R\,(g-d)}{2\,R\,g - d\,r_1^b}.$$

β. $s < g$.

$$O''^2 = \frac{2\,R\,s}{r_2^b\,(R - r_2^b)},$$

$$O'^2 = \frac{2\,R\,s}{r_1^b\,(R + r_1^b)},$$

and the conditions are

$$\frac{c+t}{d} < \frac{R\,(g-d)}{2\,R\,s + d\,r_2^b},$$

$$\frac{c+t}{d} < \frac{R\,(g-d)}{2\,R\,s - d\,r_1^b}.$$

When the ratios $\frac{r_2^b}{R}$ and $\frac{r_1^b}{R}$ are so small that we may neglect the terms $\frac{d\,r_2^b}{R}$, $\frac{d\,r_1^b}{R}$, the conditions in Case II. become identical with those in Case I.

CASE I.—*Main Roads Straight, Reverse Curves separated by an interval of Straight: one intervening Road.*

See Chapter III., Section V., Case I. For positions of crossings, see Fig. 51.

α. $s \gtrless g$.

$$r^b = \frac{g\,(D - 2\,l)^2}{2\left\{s + 2\,g - 2\sqrt{g\,(c+t)}\right\}^2},$$

$$r'' = \frac{(2\,g - d)\,r^b}{2\,g},$$

$$b_1^2 = b_2^6 = \frac{2\sqrt{2r^b(g-d)}}{2r^b+g},$$

$$b_1^3 = b_2^7 = \frac{2\sqrt{2r^b g}}{2r^b+g},$$

$$n_1^3 = n_2^7 = \frac{2\sqrt{2r^n g}}{2r^n+g},$$

$$n_1^2 = n_2^6 = \frac{2\sqrt{2r^n(g-d)}}{2r^n+g},$$

$$n_2^3 = n_1^7 = \frac{2\sqrt{2r^n d}}{2r^n-g+d}.$$

The circular measures of the angles of the remaining crossings are equal to each other and to b_1^3; also

$$\text{cot. } b_1^2 = \frac{2r^b-g}{2\sqrt{2r^b g}}, \qquad \text{cosec. } b_1^2 = \frac{2r^b+g}{2\sqrt{2r^b g}}.$$

With reference to the positions of the crossings both in this and every subsequent case, see remarks at the end of Section V., Chapter III.

The total distance between the springings of the reverse curves is

$$\frac{2(s+2g)\sqrt{r^b}}{\sqrt{2g}}.$$

$$\begin{aligned} S_2 b_1 &= S'_3 b_2^3 \\ &= \sqrt{2r^b(g-d)}, \\ S_3 b_1 &= S'_7 b_2^7 \\ &= \sqrt{2r^b g}, \\ S_4 b_2^4 &= S'_6 b_1^6 \\ &= S_3 b_1^3 + s \text{ cot. } b_1^3 - g \text{ cosec. } b_1^3 \\ &= \frac{\sqrt{r^b}(s+g)}{\sqrt{2g}}, \end{aligned}$$

$$S_2 n_1^2 = S'_2 n_2^2$$
$$= \sqrt{2 r^n (g - d)},$$
$$S_3 n_1^3 = S'_7 n_2^7$$
$$= S_3 b_1^3 - \frac{d}{2} \operatorname{cosec.} b_1^3$$
$$= \frac{\sqrt{r^b}(4g - d)}{2\sqrt{2g}},$$
$$S_3 n_2^3 = S'_7 n_1^7$$
$$= \sqrt{2 r^n d},$$
$$S_4 n_2^4 = S'_6 n_1^6$$
$$= S_4 b_2^4 + \frac{d}{2} \operatorname{cosec.} b_1^3$$
$$= \frac{\sqrt{r^b}(2s + 2g + d)}{2\sqrt{2g}}.$$

β. $s \angle g$, but $\gtrless g - \frac{d}{2}$.

$$r^b = \frac{s(D - 2l)^2}{2\{2s + g - 2\sqrt{s(c + t)}\}^2},$$
$$r^n = \frac{(2s - d) r^b}{2s},$$
$$b_1^2 = b_2^3$$
$$= \frac{2\sqrt{2 r^b (g - d)}}{2 r^b + g},$$
$$b_1^3 = b_2^7 = n_1^3 = n_2^7 = b_2^4$$
$$= \frac{2\sqrt{2 r^b s}}{2 r^b - g},$$
$$n_1^2 = n_2^3$$
$$= \frac{2\sqrt{2 r^n (g - d)}}{2 r^n + g},$$
$$n_2^3 = n_1^7$$
$$= \frac{2\sqrt{2 r^n d}}{2 r^n - g + d}.$$

The circular measures of the remaining angles of the crossings are equal to each other, and to b_2^4; also

$$\cot. b_2^4 = \frac{2r^b - g - 2s}{2\sqrt{2r^b s}} \qquad \operatorname{cosec}. b_2^4 = \frac{2r^b - g}{2\sqrt{2r^b s}}.$$

The total distance between the springings of the reverse curves is

$$\frac{2(2s+g)\sqrt{r^b}}{\sqrt{2s}}.$$

$$\begin{aligned}
S_2 b_1^2 &= S'_6 b_2^6 \\
&= \sqrt{2r^b(g-d)}, \\
S_4 b_2^4 &= S'_6 b_1^6 \\
&= \sqrt{2r^b s}, \\
S_5 b_1^5 &= S'_7 b_2^7 \\
&= S_4 b_2^4 + g \operatorname{cosec}. b_2^4 - s \cot. b_2^4 \\
&= \frac{\sqrt{r^b}(s+g)}{\sqrt{2s}}, \\
S_2 n_1^2 &= S'_6 n_2^6 \\
&= \sqrt{2r^n(g-d)}, \\
S_3 n_1^3 &= S'_7 n_2^7 \\
&= S_3 b_1^3 - \frac{d}{2} \operatorname{cosec}. b_2^4 \\
&= \frac{\sqrt{r^b}(2s+2g-d)}{2\sqrt{2s}}, \\
S_3 n_2^3 &= S'_7 n_1^7 \\
&= \sqrt{2r^n d}, \\
S_4 n_2^4 &= S'_6 n_1^6 \\
&= S_4 b_2^4 + \frac{d}{2} \operatorname{cosec}. b_2^4 \\
&= \frac{\sqrt{r^b}(4s+d)}{2\sqrt{2s}}.
\end{aligned}$$

CASE II.—*Main Line Curved, Reverse Curves separated by an interval of Straight.*

See Chapter III., Section V., Case III., and Chapter IV., Section I., Cases II. and III.

For positions of crossings, see Fig. 51.

a. $s > g$.

If the radius r_2^b be given,

$$r_1^b = \frac{\{R - 2(s+g)\}\, r_2^b\, g}{R g - 2(s+g)\, r_2^b} = \frac{R\, r_2^b\, g}{R g - 2(s+g)\, r_2^b}.$$

If the radius r_1^b be given,

$$r_2^b = \frac{R\, r_1^b\, g}{R g + 2(s+g)\, r_1^b},$$

also

$$r_2^a = r_2^b - \frac{d}{O''^2},$$

$$r_1^a = r_1^b - \frac{d}{O'^2},$$

$$O''^2 = \frac{2Rg}{r_2^b(R - r_2^b)}, \qquad O'^2 = \frac{2Rg}{r_1^b\{R + r_1^b - 2(s+g)\}},$$

$$O = \sqrt{\frac{2Rg}{r_2^b(R - r_2^b)}} - \sqrt{\frac{2Rg}{r_1^b\{R + r_1^b - 2(s+g)\}}}.$$

Whatever may be the relative values of R and r_2^b, the second and third rail of the main road will be cut by the first rail of the curve radius r_2^b, and the second rail of the main road by the first rail of the curve radius r_2^a; the same remark applies to the curves radii r_1^b, r_1^a, when

$$2\, r_1^b \angle R - 2(s+g);$$

when $2\, r_1^b$ has a greater value than this, the second rail of the main road is cut by the straight part of the first rail next to the curve radius r_1^a.

Where any main-road rail is crossed by the parallel straight rails of the through road, the circular measure of the angles of the crossing of any main-road rail by either of the narrow-gauge rails will be practically equal to that of the crossing of the

same main-road rail by the nearest broad-gauge rail. When therefore

$$2r_1 \angle \mathrm{R} - 2(s+g)$$

we have

$$b_1^2 = \frac{2\sqrt{2\,\mathrm{R}\,r_2^b(\mathrm{R}-r_2^b)(g-d)}}{2\,\mathrm{R}\,r_2^b + (\mathrm{R}+r_2^b)\,d + (\mathrm{R}-r_2^b)(g-d)},$$

$$n_1^2 = \frac{2\sqrt{2\,\mathrm{R}\,r_2^n(\mathrm{R}-r_2^n)(g-d)}}{2\,\mathrm{R}\,r_2^n + (\mathrm{R}-r_2^n)(g-d)},$$

$$n_1^3 = \frac{2\sqrt{2\,\mathrm{R}\,r_2^n(\mathrm{R}-r_2^n)\,g}}{2\,\mathrm{R}\,r_2^n + (\mathrm{R}-r_2^n)\,g},$$

$$b_1^3 = \frac{2\sqrt{2\,\mathrm{R}\,r_2^b(\mathrm{R}-r_2^b)\,g}}{2\,\mathrm{R}\,r_2^b + (\mathrm{R}-r_2^b)\,g},$$

$$n_2^3 = \frac{2\sqrt{2\,\mathrm{R}\,r_2^n(\mathrm{R}-r_2^n)\,d}}{2\,\mathrm{R}\,r_2^n + (\mathrm{R}-r_2^n)\,d - (\mathrm{R}+r_2^n)(g-d)},$$

$$b_1^4 = n_1^4 = \frac{2\sqrt{2\,\mathrm{R}\,r_2^b\{\mathrm{R}\,g - r_2^b(s+g)\}}}{r_2^b(2\,\mathrm{R}-2\,s-g)},$$

$$b_1^5 = n_1^5 = \frac{2\sqrt{2\,\mathrm{R}\,r_2^b\{\mathrm{R}\,g - r_2(s+g+d)\}}}{r_2^b(2\,\mathrm{R}-2\,s-2\,d-g)},$$

$$b_2^4 = n_2^4 = b_1^6 = n_1^6 = \frac{2\sqrt{2\,\mathrm{R}\,r_2^b\{\mathrm{R}\,g - r_2 s\}}}{r_2^b\{2\,\mathrm{R}-2\,s-3\,g\}},$$

$$b_2^5 = n_2^5 = \frac{2\sqrt{2\,\mathrm{R}\,r_2^b\{\mathrm{R}\,g - r_2(s+d)\}}}{r_2^b(2\,\mathrm{R}-2\,s-2\,d-3\,g)},$$

$$b_2^6 = n_2^6 = \frac{2\sqrt{2\,\mathrm{R}\,r_2^b\{\mathrm{R}\,g - r_2^b(s+g)\}}}{r_2^b(2\,\mathrm{R}-2\,s-3\,g)},$$

$$b_2^7 = \frac{2\sqrt{2\{\mathrm{R}-2(s+g)\}\,r_1^b\{\mathrm{R}+r_1^b-2(s+g)\}g}}{2\{\mathrm{R}-2(s+g)\}\,r_1^b + \{\mathrm{R}+r_1^b-2(s+g)\}g}$$

$$= \frac{2\sqrt{2\,\mathrm{R}\,r_1^b(\mathrm{R}+r_1^b)\,g}}{2\,\mathrm{R}\,r_1^b - 4\,r_1^b(s+g) + (\mathrm{R}+r_1^b)\,g},$$

$$n_2^7 = \frac{2\sqrt{2\,\mathrm{R}\,r_1^n(\mathrm{R}+r_1^n)\,g}}{2\,\mathrm{R}\,r_1^n - 4\,r_1^n(s+g) + (\mathrm{R}+r_1^n)\,g}.$$

Similarly,

$$b_2^8 = \frac{2\sqrt{2\,\mathrm{R}\,r_1^b\,(\mathrm{R}+r_1^b)\,(g-d)}}{2\mathrm{R}r_1^b-4r_1^b(s+g)+(\mathrm{R}-r_1^b)d+(\mathrm{R}+r_1^b)(g-d)},$$

$$n_1^7 = \frac{2\sqrt{2\,\mathrm{R}\,r_1^n\,(\mathrm{R}+r_1^n)\,d}}{2\,\mathrm{R}\,r_1^n-4\,r_1^n(s+g)-(\mathrm{R}-r_1^n)\,(g-d)+(\mathrm{R}+r_1^n)\,d},$$

$$n_2^8 = \frac{2\sqrt{2\,\mathrm{R}\,r_1^n\,(\mathrm{R}+r_1^n)\,(g-d)}}{2\,\mathrm{R}\,r_1^n-4\,r_1^n\,(s+g)+(\mathrm{R}-r_1^n)\,d+(\mathrm{R}+r_1^n)\,(g-d)}.$$

The interval between S_1 and S'_1 is equal to R O, and that between S_9 and S'_9 to $\{\mathrm{R}-2\,(s+g)\}$ O. Also,

$$S_3\,b_1^3 = \sqrt{\frac{2\,\mathrm{R}\,r_2^b\,(g-d)}{\mathrm{R}-r_2^b}},$$

$$S_3\,b_1^3 = \sqrt{\frac{2\,\mathrm{R}\,r_2^b\,g}{\mathrm{R}-r_2^b}},$$

$$S_4\,b_2^4 = (\mathrm{R}-s-g)\,(\mathrm{O}''-b_2^4),$$

$$S_2\,n_1^2 = \sqrt{\frac{2\,\mathrm{R}\,r_2^n\,(g-d)}{\mathrm{R}-r_2^n}},$$

$$S_3\,n_2^3 = \sqrt{\frac{2\,\mathrm{R}\,r_2^n\,d}{\mathrm{R}-r_2^n}},$$

$$S'_3\,b_2^3 = \sqrt{\frac{2\,\mathrm{R}\,r_1^b\,(g-d)}{\mathrm{R}+r_1^b-2\,(s+g)}},$$

$$S'_7\,b_2^7 = \sqrt{\frac{2\,\mathrm{R}\,r_1^b\,g}{\mathrm{R}+r_1^b-2\,(s+g)}},$$

$$S'_6\,b_1^6 = (\mathrm{R}-s-g)\,(b_1^6-\mathrm{O}'),$$

$$S'_8\,n_2^8 = \sqrt{\frac{2\,\mathrm{R}\,r_1^n\,(g-d)}{\mathrm{R}+r_1^n-2\,(s+g)}},$$

$$S'_7\,n_2^7 = \sqrt{\frac{2\,\mathrm{R}\,r_1^n\,d}{\mathrm{R}+r_1^n-2\,(s+g)}}.$$

The positions of all the crossings may be readily fixed on the ground by means of the distances given by the preceding formulæ, since the remaining crossings are those belonging to the straight part of the through road.

When

$$2r_1^b > R - 2(s+g),$$

since in this case $n_2^7 = b_2^7$, the rails at these crossings being parallel, the circular measures and positions of all the angles will be the same as those already given, except in the case of n_2^8, for which

$$n_2^8 = \frac{2\sqrt{R r_1^b (2Rg + r_1^b g - 2r_1^b d)}}{r_1^b \{2R - 4(s+g) - d\}},$$

$$S'_8 n_2^8 = \{R - 2(s+g)\}(n_2^8 - O').$$

β. $s \angle g$.

If the radius r_2^b be given,

$$r_1^b = \frac{\{R - 2(s+g)\} r_2^b s}{R s - 2(s+g) r_2^b} = \frac{R r_2^b s}{R s - 2(s+g) r_2^b}.$$

If the radius r_1^b be given,

$$r_2^b = \frac{R r_1^b s}{\{R - 2(s+g)\} s + 2(s+g) r_1^b} = \frac{R r_1^b s}{R s + 2(s+g) r_1^b};$$

also

$$r_2^n = r_2^b - \frac{d}{O''^2},$$

$$r_1^n = r_1^b - \frac{d}{O'^2},$$

$$O''^2 = \frac{2Rs}{r_2^b (R - r_2^b)} \qquad O'^2 = \frac{2Rs}{r_1^b \{R + r_1^b - 2(s+g)},$$

$$O = \sqrt{\frac{2Rs}{(R - r_2^b) r_2^b}} - \sqrt{\frac{2Rs}{r_1^b \{R + r_1^b - 2(s+g)\}}}.$$

The remarks made in (α) with reference to the circular measures of the angles of the crossings of the main-road rails by the straight part of the through road apply also to (β).

Whatever may be the relative values of R and r_2^b, the second rail of the main road will be cut by the first rails of the curves radii r_2^b, r_2^n, if

$$s \gtrless g - \frac{d}{2}.$$

The same remark applies to the curves radii r_1^b, r_1^a, when

$$R \gtrless 2 r_2^b + \frac{2 r_2^b (s+g)}{g},$$

or practically whenever

$$R \gtrless 6 r_1^b.$$

When this last relation holds,

$$b_2^4 = n_2^4 = \frac{2\sqrt{2 R r_2^b (R - r_2^b) s}}{2 R r_2^b + (R - r_2^b) s},$$

$$b_1^3 = n_1^3 = \frac{2\sqrt{2 R r_2^b (R s - r_2^b g)}}{r_2^b (2 R - g)},$$

$$b_1^4 = n_1^4 = \frac{2\sqrt{2 R r_2^b \{R s - r_2^b (s+g)\}}}{r_2^b (2 R - 2 s - g)},$$

$$b_1^5 = n_1^5 = \frac{2\sqrt{2 R r_2^b \{R s - r_2^b (s+g+d)\}}}{r_2^b \{2 R - 2 s - 2 d - g\}},$$

$$b_2^5 = n_2^5 = \frac{2\sqrt{2 R r_2^b \{R s - r_2^b (s+d)\}}}{r_2^b \{2 R - 2 s - 2 d - g\}},$$

$$b_2^6 = n_2^6 = \frac{2\sqrt{2 R r_2^b \{R s - r_2^b (s+g)\}}}{r_2^b (2 R - 2 s - 3 g)},$$

$$b_1^6 = n_1^6 = \frac{2\sqrt{2 R r_1^a (R + r_1^a) s}}{2 R r_1^a + (R + r_1^a) s},$$

$$b_2^7 = n_2^7 = \frac{2\sqrt{2 R r_2^b \{2 R s - r_2^b (s+g)\}}}{r_2^b \{2 R - 4 s - 3 g\}}.$$

As in (a) the interval between S_1 and S'_1 is equal to R O, and that between $S g$ and $S' g$ to $\{R - 2(s+g)\}$ O.

The positions of the angles b_1^3, b_2^4, b_1^6, b_2^7, b_1^2, b_2^8, n_1^2, n_2^3, n_1^7, n_2^8, are given by the formulæ

$$S_3 b_1^3 = R (O'' - b_1^3),$$

$$S_4 b_2^4 = \sqrt{\frac{2 R r_2^b s}{R - r_2^b}},$$

$$S'_6 b_1^6 = \sqrt{\frac{2 R r_1^b s}{R + r_1^b - 2(s+g)}},$$

$$S'_7 b^7 = \{R - 2(s+g)\}(b_2^7 - O'),$$

$$S_2 b_1^2 = \sqrt{\frac{2R r_2^b (g-d)}{R - r_2^b}},$$

$$S'_2 b_2^2 = \sqrt{\frac{2R r_1^b (g-d)}{R + r_1^b - 2(s+g)}},$$

$$S_2 n_1^2 = \sqrt{\frac{2R r_2^n (g-d)}{R - r_2^n}},$$

$$S_2 n_2^2 = \sqrt{\frac{2R r_2^n d}{R - r_2^n}},$$

$$S'_7 n_1^7 = \sqrt{\frac{2R r_1^n d}{R + r_1^n - 2(s+g)}},$$

$$S'_2 n_2^2 = \sqrt{\frac{2R r_1^n (g-d)}{R + r_1^n - 2(s+g)}}.$$

When

$$R \angle 2r_2^b + \frac{2r_2^b(s+g)}{g},$$

or practically when

$$R \angle 6r_1^b,$$

$$n_2^2 = \frac{2\sqrt{R r_1^b \{2Rg + r_1^b g - 2r_1^b d\}}}{r_1^b \{2R - 4(s+g) - d\}},$$

$$S'_2 n_2^2 = \{R - 2(s+g)\}(n_2^2 - O').$$

No example of a double-gauge road either does or is likely to exist in which

$$s \angle g - \frac{d}{2}.$$

The values of the circular measures of the angles and their positions, omitted in (α), are the same as those of the same angles in (β).

CLASS II.—In this class the broad and narrow gauge roads have one rail in common. So far as the values of the radii of the reverse curves, the intervals between their springings, the circular measures and positions of the angles of the crossings of the broad-gauge rails by the broad-gauge rails, are concerned,

the solution of each problem is identical with that for single-gauge roads. Formulæ giving the circular measures and positions of the rest may be easily obtained on the principles already explained for each particular case.

Section VI.—Crossings on the Level.

The circular measure of the angle of the crossing of any broad-gauge rail by a narrow-gauge rail is practically equal to that of the angle of the crossing of the same broad-gauge rail by the nearest broad-gauge rail; and the circular measure of the angle of the crossing of any narrow-gauge rail by a broad-gauge rail is equal to that of the same narrow-gauge rail by the nearest narrow-gauge rail, and therefore these circular measures are given by the formulæ, Section VI., Chapter III., care being taken to give to s and g their proper values.

CHAPTER V.

RECAPITULATION OF FORMULÆ OF CHAPTERS II. AND III., AND REMARKS ON THEIR PRACTICAL USE.

The points c_1^2, c_1^3, &c., are the points of intersection of the centre lines of the rails which cross each other, and consequently not the actual points of the crossings. Since the width of the crossing, where the centre lines of the two rails intersect, is equal to the thickness of the upper flange of one rail, *viz.* t, if t' be the thickness of the point of the crossing, and c the lead of the crossing, the interval between the actual point of the crossing and the intersection of the centre lines of the two rails will be

$$\frac{t - t'}{c}.$$

In Fig. 48 the two dotted lines are the centre lines of the two rails; the point c, where they intersect, is the one whose position is given by the formulæ. If the line of the inner edge of either rail, $b\,d$ or $b'\,d$, be produced till it meets the outer

edge of the other in c' or c'', either of these points will be at the same distance as c from the springing of the curve.

Thus the position of the point c can be readily marked on the crossing by means of a straight-edge or a string.

When the centre lines of both roads are staked out, as in the case of main-line junctions, it is not necessary to fix the positions of the crossings by calculation, but it will very much expedite the work to do so.

In station-yard work—even in the case of a new station—where the roads are supposed to be laid in according to a detail plan, the centre lines of the sidings are rarely staked out. For the subsequent minor alterations after the opening of a railway, plans are rarely provided, and when provided still more rarely worked to—in nine cases out of ten because the leads of the crossings sent on to the ground do not suit the plan. In addition to these cases, catch sidings, cross-over and through roads are continually being put in, for which it would be ridiculous to provide special plans, and for which whatever crossings are at hand are used.

Now, when the centre lines of both roads are not staked out, the springing of the curve is not marked out on the ground, and the position of the crossing therefore is determined by that of the point of the switch; consequently all that is necessary to be known in order to put in the road correctly, is the interval between the heel of the switch and the point of the crossing, which is equal to the difference between the distances of these two points from the springing, and is therefore known from the formulæ.

The radius of a siding road where no fixed plan is adhered to is rarely known by the superintending platelayer who usually uses for the purpose whatever crossing comes first to hand. Hence, in practice, formulæ which give the intervals between the heel of the switch and the point of the crossing in terms of the circular measure of the angle of the crossing will be most generally useful. They are very readily deduced from those given in the preceding chapters.

It must be borne in mind that in these formulæ g is not, strictly speaking, the gauge, or s the interval between the

flanges of the inner rails of a double road, but the interval between the centre line of the rails in each case. Practically, whatever may be the thickness of the rails, in the case of 4 ft. 8½ in. gauge roads we may put $g = 5$ ft. and $s = 6$ ft.; and in the case of 7 ft. gauge roads, $g = 7$ ft. 3 in. and $s = 6$ ft.

The formulæ given in Chapters III. and IV. have been obtained by strictly carrying out the rule for simplifying the results laid down at the commencement, *viz.* the rejection of all terms of the order $g^2 s^2 d^2$, $g d$, $g s$, $s d$. As they stand they still give results much more exact than are required in practice; and it will be shown in this chapter, by the actual working out of a few extreme examples, that they may safely be very much further simplified. Four chains may be taken as the sharpest admissible radius on a 4 ft. 8½ in. gauge road, and five chains on a 7 ft. gauge road.

The value of the circular measure of the angle of a crossing is expressed by a fraction whose numerator is unity. The value of the denominator will be sufficiently exact if it be put equal to the nearest integer if the lead of the crossing is long, or to the nearest half integer if the lead is short. Thus the leads of ordinary crossings will be expressed by the fractions

$$\frac{1}{6},\ \frac{1}{6\frac{1}{2}},\ \frac{1}{7},\ \frac{1}{7\frac{1}{2}},\ \frac{1}{8},\ \frac{1}{9},\ \&c.$$

Hence any simplified formula, which gives the value of the denominator within ·25, will be sufficiently exact for crossings sharper than 1 in 8, and within ·5 for crossings easier than 1 in 8.

Section I.—Single-line Junctions.

Case I.—*Curve leaving Straight Line.*

The exact formula is

$$c_1^2 = \frac{2\sqrt{2rg}}{2r+g}.$$

Putting $r = 66 \times 4$ ft. $= 264$ ft., we get for the 4 ft. 8½ in. gauge,

$$c_1^2 = \frac{1}{5\cdot 2}.$$

Putting $r = 66 \times 5$ ft. $= 330$ ft., for the 7 ft. gauge,

$$c_1^2 = \frac{1}{4 \cdot 9}.$$

If we omit the term g in the denominator,

$$c_1^2 = \frac{2\sqrt{2rg}}{2r} = \sqrt{\frac{2g}{r}}.$$

This formula gives for the 4 ft. $8\frac{1}{2}$ in. gauge,

$$c_1^2 = \frac{1}{5 \cdot 1};$$

for the 7 ft. gauge,

$$c_1^2 = \frac{1}{4 \cdot 8}.$$

Hence the latter formula gives the lead within the required degree of exactness.

The converse formula is

$$r = \frac{2g}{c_1^2}.$$

The interval between the heel of the switch and the point of the crossing is

$$\sqrt{2r}\,(\sqrt{g} - \sqrt{c+t}) = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_1^2}.$$

CASE II.—*Curve leaving Curve, Curvature in the same direction.*

Since the radius of the siding road must be much less than that of the main road, we may take eight chains as an extreme value of the latter. The exact formula is

$$c_1^2 = \frac{2\sqrt{2\,\mathrm{R}\,r\,(\mathrm{R}-r)\,g}}{2\,\mathrm{R}\,r + (\mathrm{R}-r)\,g};$$

and we get for the 4 ft. $8\frac{1}{2}$ in. gauge road,

$$c_1^2 = \frac{1}{7 \cdot 3};$$

for the 7 ft. gauge road,

$$c_1^2 = \frac{1}{8}.$$

If we omit the term $(R - r)\,g$ in the denominator, the formula becomes

$$c_1^2 = \frac{2\sqrt{2Rr(R-r)g}}{2Rr}$$

$$= \sqrt{\frac{2(R-r)g}{Rr}}.$$

This formula gives for the 4 ft. 8½ in. gauge,

$$c_1^2 = \frac{1}{7\cdot 3};$$

for the 7 ft. gauge,

$$c_1^2 = \frac{1}{7\cdot 9}.$$

Hence the latter formula gives the lead within the required degree of exactness.

The converse formula is

$$r = \frac{2Rg}{Rc_1^2 + 2g};$$

and the interval between the heel of the switch and the point of the crossing is equal to

$$\sqrt{\frac{2Rr}{R-r}}\left(\sqrt{g} - \sqrt{c+t}\right) = \frac{2\sqrt{g}\left(\sqrt{g} - \sqrt{c+t}\right)}{c_1^2},$$

since

$$Rr \div R - r = \frac{2R^2 g}{Rc_1^2 + 2g} \div \frac{R^2 c_1^2 + 2Rg - 2Rg}{R_1 c_1^2 + 2g}$$

$$= \frac{2g}{c_1^2}.$$

CASE III.—*Curve leaving Curve, Curvature in the opposite direction.*

We may in this case put $R = r$.

The exact formula is

$$c_1^2 = \frac{2\sqrt{2Rr(R+r)g}}{2Rr + (R+r)g};$$

and we get for the 4 ft. 8½ in. gauge road,

$$c_1^2 = \frac{1}{3\cdot 6};$$

for the 7 ft. gauge,

$$c_1^2 = \frac{1}{3 \cdot 5}.$$

If we omit the term $(R + r)\,g$ in the denominator, we get

$$c_1^2 = \frac{2\sqrt{2Rr(R+r)g}}{2Rr}$$

$$= \sqrt{\frac{2(R+r)g}{Rr}}.$$

This formula gives for the 4 ft. $8\frac{1}{2}$ in. gauge,

$$c_1^2 = \frac{1}{3 \cdot 6};$$

for the 7 ft. gauge road,

$$c_1^2 = \frac{1}{3 \cdot 4}.$$

Hence the latter formula gives the lead within the required limits of exactness.

The converse formula is

$$* r = \frac{2Rg}{Rc_1^2 - 2g},$$

and the interval between the heel of the switch and the point of the crossing is equal

$$\sqrt{\frac{2Rr}{R+r}}\,(\sqrt{g} - \sqrt{c+t}) = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_1^2},$$

since

$$Rr \div R + r = \frac{2R^2 g}{Rc_1^2 - 2g} \div \frac{R^2 c_1^4 - 2Rg + 2Rg}{Rc_1^2 - 2g}$$

$$= \frac{2g}{c_1^2}.$$

* If we make use of the exact formulæ to ascertain the value of r, we obtain quadratic equations in terms of r. Neglecting terms of the order g^2, one root of each of these quadratics will be $r = 0$, which corresponds to a straight line from point of switch to crossing; and the other will be very nearly equal to that given by the simplified formula in Case I. differing from it by the length g, in Cases II. and III. by a small fraction of g. If the terms of the order g^2 be retained, we shall obtain two values for r, one positive and very nearly equal to that given by the simplified formula, and the other negative and very small. The curve corresponding to this last value of r will occupy the position of the dotted line in the figure.

Section II.—Double-line Junctions.

Case I.—*Curve leaving Straight Line.*

In the preceding section the formulæ have been simplified by leaving out the term g in the denominator. In Section II. the corresponding term in the case of c_1^2 is $2g + s$. In no other case has this corresponding term a greater value. If, then, its omission in this case does not affect the result, it cannot do so in the others.

The exact formula is

$$c_1^2 = \frac{2\sqrt{2rg}}{2r + 2g + s}.$$

Putting $r = 4 \times 66$ ft. $= 264$ ft., $g = 5$ ft., $s = 6$ ft., we get for the 4 ft. 8½ in. gauge,

$$c_1^2 = \frac{1}{5 \cdot 3}.$$

Putting $r = 5 \times 66$ ft. $= 330$ ft., $g = 7$ ft., $s = 6$ ft., we get for the 7 ft. gauge,

$$c_1^2 = \frac{1}{5}.$$

If we omit the term $2g + s$ in the denominator, the simplified formula is

$$c_1^2 = \frac{2\sqrt{2rg}}{.2r}$$

$$= \sqrt{\frac{2g}{r}}.$$

For the 4 ft. 8½ in. gauge (see Section I., Case I.),

$$c_1^2 = \frac{1}{5 \cdot 1};$$

for the 7 ft. gauge,

$$c_1^2 = \frac{1}{4 \cdot 8}.$$

Therefore in all cases, within the required degree of exactness,

$$c_1^2 = c_2^4 = \sqrt{\frac{2g}{r}},$$

$$c_1^3 = c_2^4 = \sqrt{\frac{2(s+g)}{r}},$$

$$c_1^4 = \sqrt{\frac{2(2g+s)}{r}},$$

$$c_2^3 = \sqrt{\frac{2s}{r}}.$$

The converse formulæ are

$$r = \frac{2g}{c_1^2} = \frac{2(s+g)}{c_2^{4\,2}} = \frac{2(2g+s)}{c_1^{4\,2}} = \frac{2s}{c_2^{3\,2}};$$

consequently the distance from the heel of the switch to the crossing c_1^2 or c_3^4 is

$$\frac{2\sqrt{g}\,(\sqrt{g}-\sqrt{c+t})}{c_1^2};$$

to the crossing c_1^3 or c_2^4,

$$\frac{2\sqrt{s+g}\,(\sqrt{s+g}-\sqrt{c+t})}{c_1^3};$$

to the crossing c_1^4,

$$\frac{2\sqrt{2g+s}\,(\sqrt{2g+s}-\sqrt{c+t})}{c_1^4};$$

to the crossing c_2^3,

$$\frac{2\sqrt{s}\,(\sqrt{s}-\sqrt{c+t})}{c_2^3}.$$

CASE II.—*Curve leaving Curve, Curvature in the same direction.*

In Section I. the formulæ have been simplified by leaving out the term $(R - r)\,g$ in the denominator. In Section II. the corresponding terms are in the case of c_1^2, $(R + r)\,s + 2\,R\,g$. In no other case have they a greater value. If therefore their omission in this case does not affect the result, it cannot do so in the others.

The exact formula is

$$c_1^2 = \frac{2\sqrt{2\,R\,r\,(R-r)\,g}}{2\,R\,r + (R+r)\,s + 2\,R\,g}.$$

With the same values for R and r as in Section I., Case II., for the 4 ft. 8½ in. gauge,

$$c_1^2 = \frac{1}{7 \cdot 4};$$

for the 7 ft. gauge,

$$c_1^2 = \frac{1}{8 \cdot 3}.$$

If we omit the term $(R + r)\, s + 2\, R\, g$ in the denominator, the simplified formula is

$$c_1^2 = \frac{2\sqrt{2\, R\, r\, (R - r)\, g}}{2\, R\, r}$$

$$= \sqrt{\frac{2\,(R - r)\, g}{R\, r}};$$

for the 4 ft. 8½ in. gauge,

$$c_1^2 = \frac{1}{7 \cdot 3};$$

for the 7 ft. gauge,

$$c_1^2 = \frac{1}{7 \cdot 9}.$$

Hence, within the specified limits of exactness the two formulæ give identical values, and therefore for all possible values of R and r,

$$c_1^2 = c_3^4 = \sqrt{\frac{2\,(R - r)\, g}{R\, r}},$$

$$c_1^3 = c_2^4 = \sqrt{\frac{2\,(R - r)\,(s + g)}{R\, r}},$$

$$c_1^4 = \sqrt{\frac{2\,(R - r)\,(s + 2\, g)}{R\, r}},$$

$$c_2^3 = \sqrt{\frac{2\,(R - r)\, s}{R\, r}}.$$

The converse formulæ are

$$r = \frac{2\, R\, g}{R\, c_1^2 + 2\, g} = \frac{2\, R\,(s + g)}{R\, c_1^3 + 2\,(s + g)}$$

$$= \frac{2\, R\, s}{R\, c_2^3 + 2\, s} = \frac{2\, R\,(s + 2\, g)}{2\, R\, c_1^4 + 2\,(s + 2\, g)}.$$

The distance from the heel of the switch to the crossing in each case is given by the formulæ in Section II., Case I.

CASE III.—*Curve leaving Curve, Curvature in the opposite direction.*

The terms to be left out in Case III. in order to simplify the formulæ have the same maximum value as in Case II. Consequently for all possible values of the radii the simplified formulæ are practically exact. They are

$$c_1^2 = c_2^4 = \sqrt{\frac{2(R+r)g}{Rr}},$$

$$c_1^3 = c_2^4 = \sqrt{\frac{2(R+r)(s+g)}{Rr}},$$

$$c_1^4 = \sqrt{\frac{2(R+r)(s+2g)}{Rr}},$$

$$c_2^3 = \sqrt{\frac{2(R+r)s}{Rr}}.$$

The converse formulæ are

$$r = \frac{2Rg}{Rc_1^2 - 2g} = \frac{2R(s+g)}{Rc_1^3 - 2(s+g)}$$
$$= \frac{2Rs}{Rc_2^2 - 2s} = \frac{2R(s+2g)}{Rc_1^4 - 2(s+2g)}.$$

The distance from the heel of the switch to the crossing in each case is given by the formula at the end of Case I.

Except in the case of c_3^4 the simplification of the formulæ has been effected by diminishing the denominator, and therefore if the value of the denominator in the result lie half-way between two consecutive values, the latter ought to be adopted as the nearest in every case except that of c_3^4, in which the less value ought to be adopted.

A little consideration will show that the simplified formulæ rest on this assumption, *viz.* that the radius of the curve of each rail is practically equal to the radius of the curve of the centre line.

SECTION III.—THREE THROWS.

The formulæ in each case are simplified in the manner explained in Sections I. and III.

The only special features are the relations between c_{r_1}, c_{r_2} and c_m, the positions of the heels of the switches, and the distance of either of these from the crossing c_m.

Whether the main line be straight or curved, if the branch roads curve in opposite directions,

$$c_m = \sqrt{2\left(\frac{1}{r_1}+\frac{1}{r_2}\right)g}$$

$$= \sqrt{c_{r_1}^2 + c_{r_2}^2}.$$

If the branch roads curve in the same direction,

$$c_m = \sqrt{2\left(\frac{1}{r_2}-\frac{1}{r_1}\right)g}$$

$$= \sqrt{c_{r_2}^2 - c_{r_1}^2}.$$

When the branch roads curve in opposite directions, since the two contiguous rails of the branch roads are separated by a main-road rail, the clearance in each case takes place between a branch-road rail and a main-road rail, and therefore the distances of the heels of the switches from the springings of the curves are

* $$\sqrt{2r_1(c+t)} = \frac{2\sqrt{g(c+t)}}{c_{r_1}},$$

$$\sqrt{2r_2(c+t)} = \frac{2\sqrt{g(c+t)}}{c_{r_2}}.$$

When therefore $c_{r_1} = c_{r_2}$ the heels of the switches are at the same distance from the springing.

When the branch roads curve in the same direction the contiguous rails of the branch roads lie on the same side of the main-road rail in contact with them, and the clearance at the

* These formulæ in terms of the radius suppose the main line straight. Since the formulæ expressed in terms of the leads of the crossings are the same in each case, the proof holds good when the main line is curved.

heels of the switches in one case takes place between a main-road rail and a branch rail, and in the other between two branch rails. Therefore the distances between the heels of the switches and the springings of the curves are, $r_1 > r_2$,

$$\sqrt{2 r_1 (c+t)} = \frac{2\sqrt{g(c+t)}}{c_{r_1}},$$

$$\sqrt{\frac{2 r_1 r_2 (c+t)}{r_1 - r_2}} = \frac{2\sqrt{g(c+t)}}{c_m}.$$

Hence the heels of the switches will be at the same distance from the springing, if

$$c_{r_1} = c_m = \sqrt{c_{r_2}^2 - c_{r_1}^2};$$

or if

$$c_{r_2} = \sqrt{2 \cdot c_{r_1}};$$

and consequently

$$c_m = c_{r_1} = \frac{c_{r_2}}{\sqrt{2}}.$$

When these relations do not hold, the interval between the heels of the switches is equal in one case to

$$(\sqrt{r_1} - \sqrt{r_2})\sqrt{2(c+t)} = \frac{2(c_{r_2} - c_{r_1})\sqrt{g(c+t)}}{c_{r_1} c_{r_2}};$$

and in the other case to

$$\left(\sqrt{r_1} - \sqrt{\frac{r_1 r_2}{r_1 - r_2}}\right)\sqrt{2(c+t)} = \frac{2(c_m - c_{r_1})\sqrt{g(c+t)}}{c_{r_1} c_m}.$$

Formulæ are given for every case that can occur in Division I. in terms of the radii of the curves, and in terms of the circular measures of the crossings in Division II. The two simplest cases are those in which the heels of the switches of both branch roads are at the same distance from the springing. This three-throw possesses the following properties, whether the main line be straight or curved.

a. Branch Roads Curving in opposite directions.

(1) The circular measures of the angles of the crossings of the main-line rails are equal to each other, and are equidistant from the springing and the heels of the switches.

(2) The circular measure of the angle of the crossing of the branch-road rails is equal to that of the angle of either main-line crossing multiplied by $\sqrt{2}$; and the distance of this crossing from the heels of the switches is equal to

$$\sqrt{\frac{2\,r_1\,r_2}{r_1 - r_2}}\left(\sqrt{g} - \sqrt{2(c+t)}\right) = \frac{2\sqrt{g}\left(\sqrt{g} - \sqrt{2\,(c+t)}\right)}{c_m}.$$

(3) The point of the crossing of the branch road is exactly in the middle of the main road.

Proofs of (1) and that part of (2) which relates to the circular measure of c_m have already been given. Since the clearance between the two branch rails at the heels of the switches is equal to the sum of the clearances between the main-road rail and either branch rail, plus the thickness of the main-road rail, the interval between the centre lines of the two branch rails is equal to $2\,(c + t)$. Hence the distance from the springing of the heels of the switches, expressed in terms of the radii of the branch roads, or of the circular measure of the angle of their crossing, is equal to

$$\sqrt{\frac{4\,r_1\,r_2\,(c+t)}{r_1 - r_2}} = \frac{2\sqrt{2\,g\,(c+t)}}{c_m}.$$

The following is a proof of rule (3). Since the distance of the crossing c_m from the springing is equal to $\frac{2\,g}{c_m}$, the tangential offsets to the three curves which pass through c_m will be respectively

$$\frac{2\,g^2}{\mathrm{R}\,c_m^2}, \quad \frac{2\,g^2}{r_1\,c_m^2}, \quad \text{and} \quad \frac{2\,g^2}{r_2\,c_m^2}.$$

Since the branch road, radius r_1, curves in the opposite direction to the main line, the difference between the offsets to the curves radii R and r_2, and the sum of the offsets to the

curves radii R and r_1 will be equal to the distances of the point of the crossing from the corresponding main-line rail; if therefore these distances are equal,

$$\frac{1}{r_2} - \frac{1}{R} = \frac{1}{R} + \frac{1}{r_1};$$

whence

$$r_2 = \frac{R\,r_1}{R + 2\,r_1},$$

a relation, which we know holds between R, r_1 and r_2 when $c_{r_1} = c_{r_2}$.

β. Branch Roads curving in the same direction, whether the Main Road curves in the same or the opposite direction to the Branch Roads.

(1) The circular measure c_{r_1} of the angle of the easier crossing is equal to c_m that of the crossing of the branch-road rails, and to c_{r_2} the circular measure of the sharper crossing divided by $\sqrt{2}$.

(2) The distance of the crossing c_m of the branch-road rails from the heels of the switches is equal to that of the easier crossing c_{r_1}. The distance of the sharper crossing c_{r_2} from the heels of the switches is equal to

$$\sqrt{2\,r_2}\,(\sqrt{g} - \sqrt{2\,(c+t)}) = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{2\,(c+t)})}{c_{r_2}},$$

or to

$$\sqrt{\frac{2\,R\,r_2}{R \pm r_2}}\,(\sqrt{g} - \sqrt{2\,(c+t)}) = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{2\,(c+t)})}{c_{r_2}},$$

according as the main line is straight or curved.

Proof of Rule (1) has already been given. As to Rule (2), since both contiguous branch rails lie on the same side of the main-road rail in contact with them, the interval between the centre line of the main-road rail, and that of the nearest branch rail, is equal to the interval between the centre lines of the branch rails and to $(c + t)$. Consequently the interval between

the centre of the main-line rail and the more remote branch rail is equal to 2 $(c + t)$, therefore the distance of the heels of the switches from the springing expressed in terms of c_{r_2} is

$$\frac{2\sqrt{2g(c+t)}}{c_{r_2}},$$

and

$$\frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_{r_1}} = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_m},$$

since $c_{r_1} = c_m$.

SECTION IV.—CROSS-OVER ROADS.

The formulæ are simplified in the manner explained in Sections I. and II.

The only point calling for special remark is the relation between the radii of the reverse curves. When the main line is curved,

$$\frac{r_2}{R - r_2} = \frac{r_1}{R + r_1}.$$

Since r_1, the radius of the reverse curve, which curves in the opposite direction to the main line, is always greater than r_2, the radius of the curve, which curves in the same direction, the latter will have a maximum value when the former is infinite, or one half of the cross-over is straight. Since

$$\frac{r_1}{R + r_1} = \frac{1}{\frac{R}{r_1} + 1}$$
$$= 1$$

where r_1 is infinite, it follows that

$$r_2 = \frac{R}{2}.$$

Conversely, if r_2 have a minimum admissible value, a cross-over road cannot be put between two roads, the radius of whose centre line is less than $2r_2$, which we may fix at 8 chains for a 4 ft. 8½ in. gauge and 10 chains for a 7 ft. gauge.

Since $\frac{1}{c_1^2}$ is equal to the factor, which multiplies the elements

s, g, c and t in the formulæ, whether the main line be straight or curved, we get in both Cases I. and II.:

α. $s > g$.

Interval between the heels of the switches measured along centre line of main line,

$$\frac{2\sqrt{g}\left\{\sqrt{2(s+g)} - 2\sqrt{c+t}\right\}}{c_1^2}.$$

Interval between the points of the crossings measured along the centre line of main line,

$$\frac{2\sqrt{g}\left\{\sqrt{2(s+g)} - 2\sqrt{g}\right\}}{c_1^2}.$$

β. $s < g$.

Interval between the heels of the switches measured along the centre line of the main line,

$$\frac{2\sqrt{s}\left\{\sqrt{2(s+g)} - 2\sqrt{c+t}\right\}}{c_1^2}.$$

Interval between the points of the crossings measured along the centre line of the main line,

$$\frac{2\sqrt{s}\left\{\sqrt{2(s+g)} - 2\sqrt{s}\right\}}{c_1^2}.$$

Lines drawn through the points, marked out on the centre line, at right angles to that line, will cut the rails at the points of the crossings and heels of the switches.

Section V.—Through Roads.

In the recapitulation only the formulæ necessary for calculating the circular measures of the angles, and for fixing the position of the centre line of the through road on the ground, are given.

The simplifications in this section are of the same nature as those explained in the preceding sections. The simplified formulæ may therefore be at once written down from the exact ones given in Chapter IV., and therefore only a few points require especially noticing.

In Case I., within the limits of exactness,

$$\text{cosec. } \theta = \cot. \theta = \sqrt{\frac{r}{2g}}, \text{ or } = \sqrt{\frac{r}{2s}},$$

accordingly as $s \gtrless g$.

The direct formulæ in terms of r, and the converse formulæ in terms of the circular measures of the angles of the crossings, which give the interval between the heels of the switches, in Cases I. and II. are very simple. In Cases III. and IV. the direct formulæ in terms of R, r_1 and r_2 are likewise simple, but the converse formulæ are too complicated to be practically of much use.

In Case III. the relation between R, r_1 and r_2 is

$$\frac{R}{r_2} = \frac{R - (n+1)(s+g)}{r_1} + \frac{(n+1)(s+g)}{g},$$

or

$$\frac{R}{r_2} = \frac{R - (n+1)(s+g)}{r_1} + \frac{(n+1)(s+g)}{s},$$

accordingly as $s \gtrless g$. Now r_2 is always less than r_1; if therefore we put $r_1 = \infty$, and give to r_2 its minimum admissible value, we shall find the relation which must subsist between R and the number of intervening roads. If R be less than the value given by this relation, a through road cannot be put in.

For the 4 ft. 8½ in. gauge, putting $r_2 = 4$,

$$R \gtrless \frac{(n+1)(s+g)\,4}{g} \gtrless \frac{(n+1)\,44}{5};$$

consequently, if there be only one intervening road, R need not be greater than 18 chains. If there be two intervening roads, R must not be less than 26 chains.

For the 7 ft. gauge, putting $r_2 = 5$,

$$R \gtrless \frac{(n+1)(s+g)\,5}{s} \gtrless \frac{(n+1)\,65}{6}.$$

If therefore there be only one intervening road, R need not be greater than 22 chains; if there are two, not less than 33 chains, and so on. In Case IV., the relation between the radii, when $r_1 = \infty$, is

$$R = 2r_2 + \frac{(n+1)(s+g)}{2}.$$

For the 4 ft. 8½ in. gauge, putting $r_2 = 4$, we get

$$R = 8 + \frac{(n+1)\,11}{132};$$

unless therefore there are more than eleven roads, R need not exceed 9 chains.

For the 7 ft. gauge, putting $r_2 = 5$, we get

$$R = 10 + \frac{(n+1)\,13}{132};$$

unless therefore there are more than eleven roads, R need not exceed 11 chains.

In Cases III. and IV. the greatest terms rejected in the denominators of the fractions, which give the circular measures of the angles of the crossings, are of the order

$$\frac{r\,m\,(s+g)}{R};$$

this cannot ever be equal to unity when a chain is the unit.

In Case III. it is impossible to get a simple relation between the angular interval between the springings of the reverse curves and one of the radii. Either radius being known, the angular interval between the springing and the first crossing is given in terms of the radius of the reverse curve and of the main-line outside road, and the relation between the radii of the reverse curves obtained by the same method. This relationship may be obtained in another way. It will be interesting to see how the two agree, and be a check on the accuracy of the methods employed.

The circular measures of the angles of the crossings are given in terms of the radius r_2. If in the formula which gives the value c_2^{2m+1}, we put $m = n + 1$, we shall obtain

$$c_2^{n+3} = \sqrt{\frac{2\,(R + r_2)\,g - 2\,r_2\,(n+1)\,(s+g)}{R\,r_2}};$$

but we have also

$$c_2^{n+3} = \sqrt{\frac{2\,(R + r_1)\,g}{R\,r_1}}.$$

Equating these two values, we get

$$R r_2 g = R r_1 g - r_1 r_2 (n+1)(s+g);$$

whence

$$r_2 = \frac{R r_1 g}{R g + r_1 (n+1)(s+g)},$$

$$r_1 = \frac{R r_2 g}{R g - r_2 (n+1)(s+g)}.$$

Comparing these values with those given in Chapter IV., Section V., Case III., it appears that we may use instead of R or $R - (n+1)(s+g)$, the radius of the centre line of any of the main roads without deviating beyond the limits of exactness fixed upon.

Since every angle in this case is different, a cross-over of this sort possesses no advantages over that of a cross-over consisting of two reverse curves meeting at a common tangent. In the latter case the springings of the reverse curves are nearer to each other, and simple formulæ giving either radius in terms of the angular interval between the springings are easily obtained.

In Case IV., likewise, unless the radius of the main roads be very small, and the number of intervening roads very large, the radius of any of the main roads may be used in the formulæ.

Thus the exact formulæ giving the radius of either reverse curve in terms of the other and the radius of the main-line outside road are

$$r_1 = \frac{\left\{R - \frac{(n+1)(s+g)}{2}\right\} r_2}{R - 2r_2 - \frac{(n+1)(s+g)}{2}},$$

$$r_2 = \frac{\left\{R - \frac{(n+1)(s+g)}{2}\right\} r_1}{R + 2r_1 - \frac{(n+1)(s+g)}{2}}.$$

Now, any deviation from exactness will affect the result most when R has a minimum and n a maximum value: in the case of the 4 ft. 8½ in. gauge it has been just shown that these values are respectively 10 chains and 11 intervening roads. In order

that r_2 may have an admissible value in this case r_1 must be infinite, and we get for the correct value of r_2,

$$r_2 = \frac{R}{2} - 3(s+g) = 4 \text{ chains } 33 \text{ feet.}$$

The extreme cases of divergence from this would be

$$r_2 = \frac{R}{2} = 5 \text{ chains,}$$

$$r_2 = \frac{R}{2} - 6(s+g) = 4 \text{ chains,}$$

results not sufficiently exact. If instead of eleven there had only been three intervening roads, the exact radius would have been 4 chains 55 feet, and the two extreme radii 4 chains 44 feet and 5 chains: neither differing from the exact value by more than 11 feet.

If we substitute successively the extreme pairs of values R = 10 chains, $r_2 = 5$ chains, R = 8 chains, $r_2 = 4$ chains, and the correct values R = 9 chains, $r_2 = 4\frac{1}{2}$ chains, in the formula

$$c_1'' = \sqrt{\frac{(R - r_2)\,10\,(s+g)}{R\,r_2}},$$

which gives the circular measure of one of the crossings of the first rail of the $\frac{n-1^{th}}{2}$ intervening road—an extreme case—we get:

Putting R = 10 chains, $r_2 = 5$ chains,

$$c_1'' = \frac{1}{2 \cdot 45}.$$

Putting R = 8 chains, $r_2 = 5$ chains,

$$c_1'' = \frac{1}{2 \cdot 19}.$$

Putting R = 9 chains, $r_2 = 4\frac{1}{2}$ chains,

$$c_1'' = \frac{1}{2 \cdot 32}.$$

So that even in this extreme case the radius of any of the intervening roads might be used indiscriminately, since accord-

ing to the rule laid down at the commencement of the chapter, we must prefer the integer or half integer next higher than the denominator actually obtained, so that the result in each case gives the lead of the crossing $\frac{1}{2\frac{1}{2}}$.

With the exception of the formulæ giving the values of the radii of the reverse curves, the above statement that the radius of any of the main-line roads may be used indiscriminately refers only to those formulæ which give the values of ratios both in Cases III. and IV. In those formulæ which give the positions of the crossings, in which R appears as a simple factor of a ratio, its exact value must be used for that simple factor.

Section VI.—Crossings on the Level.

These are of such rare occurrence that it is not worth while to attempt any simplification of the formulæ. For all information on this point, the reader is referred to Chapter IV., Section VI.

Section VII.—Siding Roads from one of the Main-line Roads across the other.

There remains one class of roads, which has very frequently to be put in by platelayers without the centre of the road being staked out, *viz.* siding roads from the up or down line to sidings on the down or up side of the main line, which are not directly connected with a road parallel to the main line, and which do not come under Section V., Through Roads.

When this siding road is a continuous curve, the formulæ for double-line junctions give the circular measures and positions of the angles in terms of the radius, and *vice versâ* the radius circular measures and positions of the angles in terms of the circular measure of any one of them.

When the siding road after crossing either the second rail when $s \gtreqless g$, or the third rail when $s < g$, continues straight, we obtain at once the required formulæ from those for through roads, Cases II. and III.

Section VIII.—Recapitulation of Formulæ.

Division I.—*Formulæ expressed in terms of the radii.*

Switches.

Maximum lengths of—

Curve leaving straight line

$$= \sqrt{2r}\left(\sqrt{c+t} - \sqrt{e}\right);$$

Curve leaving curve, curvature in the same direction

$$= \sqrt{\frac{2Rr}{R-r}}\left(\sqrt{c+t} - \sqrt{e}\right);$$

Curve leaving curve, curvature in opposite directions

$$= \sqrt{\frac{2Rr}{R+r}}\left(\sqrt{c+t} - \sqrt{e}\right).$$

Minimum lengths of—

Curve leaving straight line

$$= \sqrt{2r}\left(\sqrt{c+t} - \sqrt{e+e'}\right);$$

Curve leaving curve, curvature in the same direction

$$= \sqrt{\frac{2Rr}{R-r}}\left(\sqrt{c+t} - \sqrt{e+e'}\right);$$

Curve leaving curve, curvature in the same direction

$$= \sqrt{\frac{2Rr}{R+r}}\left(\sqrt{c+t} - \sqrt{e+e'}\right).$$

Ratio of splayed part to whole length—

Bridge rails without top flanges

$$= \frac{\sqrt{t} - \sqrt{e}}{\sqrt{c+t} - \sqrt{e}};$$

Rails with top flanges

$$= \frac{\sqrt{t\left(\sqrt{c+t} - \sqrt{e}\right)} - w\left(\sqrt{c+t} - \sqrt{t}\right) - \sqrt{e}\sqrt{\sqrt{c+t} - \sqrt{e}}}{\left(\sqrt{c+t} - \sqrt{e}\right)^{\frac{3}{2}}}.$$

Distance of the heels from the springing—

Curve leaving straight line

$$= \sqrt{2r(c+t)};$$

Curve leaving curve, curvature in the same direction

$$= \sqrt{\frac{2\,R\,r}{R-r}(c+t)};$$

Curve leaving curve, curvature in opposite directions

$$= \sqrt{\frac{2\,R\,r}{R+r}(c+t)}.$$

Fast Points.

Length of splayed part—

Curve leaving straight line

$$= \sqrt{2r}\left(\sqrt{c+t} - \sqrt{c+e}\right);$$

Curve leaving curve, curvature in the same direction

$$= \sqrt{\frac{2\,R\,r}{R-r}}\left(\sqrt{c+t} - \sqrt{c+e}\right);$$

Curve leaving curve, curvature in opposite directions

$$= \sqrt{\frac{2\,R\,r}{R+r}}\left(\sqrt{c+t} - \sqrt{c+e}\right).$$

Distance of point from the springing—

Curve leaving straight line

$$= \sqrt{2\,r\,(c+e)};$$

Curve leaving curve, curvature in the same direction

$$= \sqrt{\frac{2\,R\,r}{R-r}(c+e)};$$

Curve leaving curve, curvature in opposite directions

$$= \sqrt{\frac{2\,R\,r}{R+r}(c+e)}.$$

Crossings.

Circular measures and position of—

Distance of point of intersection of centre lines of rails of crossing from the actual point

$$= \frac{t-t'}{c}.$$

Single-line Junctions.

CASE I.—*Curve leaving Straight Line.*

$$c_1^2 = \sqrt{\frac{2g}{r}},$$

$$S_2 c_1^2 = \sqrt{2rg}.$$

CASE II.—*Curve leaving Curve, Curvature in the same direction.*

$$c_1^2 = \sqrt{\frac{2(R-r)g}{Rr}},$$

$$S_2 c_1^2 = \sqrt{\frac{2Rrg}{R-r}}.$$

CASE. III.—*Curve leaving Curve, Curvature in opposite directions.*

$$c_1^2 = \sqrt{\frac{2(R+r)g}{Rr}},$$

$$S_2 c_1^2 = \sqrt{\frac{2Rrg}{R+r}}.$$

Double-line Junctions.

CASE I.—*Curve leaving Straight Line.*

$$c_1^2 = c_3^4 = \sqrt{\frac{2g}{r}},$$

$$c_1^3 = c_2^4 = \sqrt{\frac{2(s+g)}{r}},$$

$$c_1^4 = \sqrt{\frac{2(s+2g)}{r}},$$

$$c_2^3 = \sqrt{\frac{2s}{r}},$$

$$S_2 c_1^2 = S_4 c_3^4 = \sqrt{2rg},$$

$$S_3 c_1^3 = S_4 c_2^4 = \sqrt{2r(s+g)},$$

$$S_4 c_1^4 = \sqrt{2r(s+2g)},$$

$$S_3 c_2^3 = \sqrt{2rs}.$$

CASE II.—*Curve leaving Curve, Curvature in the same direction.*

$$c_1^2 = c_3^4 = \sqrt{\frac{2(R-r)g}{Rr}},$$

$$c_1^3 = c_2^4 = \sqrt{\frac{2(R-r)(s+g)}{Rr}},$$

$$c_1^4 = \sqrt{\frac{2(R-r)(s+2g)}{Rr}},$$

$$c_2^3 = \sqrt{\frac{2(R-r)s}{Rr}},$$

$$S_2 c_1^2 = S_4 c_3^4 = \sqrt{\frac{2Rrg}{R-r}},$$

$$S_3 c_1^3 = S_4 c_2^4 = \sqrt{\frac{2Rr(s+g)}{R-r}},$$

$$S_4 c_1^4 = \sqrt{\frac{2Rr(s+2g)}{R-r}},$$

$$S_3 c_2^3 = \sqrt{\frac{2Rrs}{R-r}}.$$

CASE III.—*Curve leaving Curve, Curvature in the opposite direction.*

$$c_1^2 = c_3^4 = \sqrt{\frac{2(R+r)g}{Rr}},$$

$$c_1^3 = c_2^4 = \sqrt{\frac{2(R+r)(s+g)}{Rr}},$$

$$c_1^4 = \sqrt{\frac{2(R+r)(s+2g)}{Rr}},$$

$$c_2^3 = \sqrt{\frac{2(R+r)s}{Rr}},$$

$$S_2 c_1^2 = S_4 c_3^4 = \sqrt{\frac{2Rrg}{R+r}},$$

$$S_3 c_1^3 = S_4 c_2^4 = \sqrt{\frac{2Rr(s+g)}{R+r}},$$

$$S_3 c_1^4 = \sqrt{\frac{2\,R\,r\,(s+2\,g)}{R+r}},$$

$$S_3 c_2^3 = \sqrt{\frac{2\,R\,r\,s}{R+r}}.$$

Three Throws.

CASE I.—*Main Line Straight, Branch Roads Curving in opposite directions.*

$$c_{r_1} = \sqrt{\frac{2\,g}{r_1}},$$

$$c_{r_2} = \sqrt{\frac{2\,g}{r_2}},$$

$$c_m = \sqrt{\frac{2\,(r_1+r_2)\,g}{r_1\,r_2}},$$

$$S_1 c_{r_1} = \sqrt{2\,r_1\,g},$$

$$S_2 c_{r_2} = \sqrt{2\,r_2\,g},$$

$$S_1 c_m = S_2 c_m = \sqrt{\frac{2\,r_1\,r_2\,g}{r_1+r_2}}.$$

CASE II.—*Main Line Straight, Branch Roads Curving in the same direction.*

$$c_{r_1} = \sqrt{\frac{2\,g}{r_1}},$$

$$c_{r_2} = \sqrt{\frac{2\,g}{r_2}},$$

$$c_m = \sqrt{\frac{2\,(r_1-r_2)\,g}{r_1\,r_2}},$$

$$S_2 c_{r_1} = \sqrt{2\,r_1\,g},$$

$$S_2 c_{r_2} = \sqrt{2\,r_2\,g},$$

$$S_1 c_m = S_2 c_m = \sqrt{\frac{2\,r_1\,r_2\,g}{r_1-r_2}}.$$

CASE III.—*Main Line Curved, Branch Roads Curving in opposite directions.*

$$c_{r_1} = \sqrt{\frac{2(R + r_1)g}{R r_1}},$$

$$c_{r_2} = \sqrt{\frac{2(R - r_2)g}{R r_2}},$$

$$c_m = \sqrt{\frac{2(r_1 + r_2)g}{r_1 r_2}},$$

$$S_1 c_{r_1} = \sqrt{\frac{2 R r_1 g}{R + r_1}},$$

$$S_2 c_{r_2} = \sqrt{\frac{2 R r_2 g}{R - r_2}},$$

$$S_1 c_m = S_2 c_m = \sqrt{\frac{2 r_1 r_2 g}{r_1 + r_2}}.$$

CASE IV.—*Main Line Curved, both Branch Roads Curving in the same direction as the Main Line.*

$$c_{r_1} = \sqrt{\frac{2(R - r_1)g}{R r_1}},$$

$$c_{r_2} = \sqrt{\frac{2(R - r_2)g}{R r_2}},$$

$$c_m = \sqrt{\frac{2(r_1 - r_2)g}{r_1 r_2}},$$

$$S_2 c_{r_1} = \sqrt{\frac{2 R r_1 g}{R - r_1}},$$

$$S_2 c_{r_2} = \sqrt{\frac{2 R r_2 g}{R - r_2}},$$

$$S_1 c_m = S_2 c_m = \sqrt{\frac{2 r_1 r_2 g}{r_1 - r_2}}.$$

CASE V.—*Main Line Curved, both Branch Roads Curving in the opposite direction to the Main Line.*

$$c_{r_1} = \sqrt{\frac{2(R + r_1)g}{R r_1}},$$

$$c_{r_2} = \sqrt{\frac{2(R + r_2)g}{R r_2}},$$

$$c_m = \sqrt{\frac{2(r_1 - r_2)g}{r_1 r_2}},$$

$$S_2 c_{r_1} = \sqrt{\frac{2 R r_1 g}{R + r_1}},$$

$$S_2 c_{r_2} = \sqrt{\frac{2 R r_2 g}{R + r_2}},$$

$$S_1 c_m = S_2 c_m = \sqrt{\frac{2 r_1 r_2 g}{r_1 - r_2}}.$$

So long as the interval, $c + t$, between the centre lines of contiguous rails at the heels of the switches remains the same, the positions of the heels of the switches in each case are given by the following formulæ:

CASE I.

Distance of heel of switch of curve, radius r_1, from springing

$$= \sqrt{2 r_1 (c + t)};$$

Interval between heels of switches

$$= (\sqrt{r_1} - \sqrt{r_2})\sqrt{2(c + t)}.$$

Heel, radius r_1, being more remote from springing than heel radius r_2.

CASE II.

Distance of heel of switch of curve, radius r_1, from springing

$$= \sqrt{2 r_1 (c + t)}.$$

The interval between the heels of the switches is equal to

$$\left(\sqrt{r_1} - \sqrt{\frac{r_1 r_2}{r_1 - r_2}}\right)\sqrt{2(c + t)} = \sqrt{2 r_1 (c + t)} - \sqrt{2 r_2 (c' + t)}$$

where $c' + t$ in this and the succeeding cases is equal to the interval between the centre lines of the main-line rail and

the rail of curve radius r_2 opposite the heel of the switch of that rail. Heel, radius r_1, being nearer to the springing of the curve than the heel radius r_2.

CASE III.

Distance of heel of switch of curve, radius r_1, from springing

$$= \sqrt{\frac{2\,R\,r_1}{R+r_1}\,(c+t)};$$

Interval between heels of switches

$$= \left(\sqrt{\frac{R\,r_1}{R+r_1}} - \sqrt{\frac{R\,r_2}{R-r_2}}\right)\sqrt{2\,(c+t)}.$$

CASE IV.

Distance of heel of switch of curve, radius r_1, from springing

$$= \sqrt{\frac{2\,R\,r_1}{R-r_1}\,(c+t)};$$

The interval between the heels of the switches is equal to

$$\left(\sqrt{\frac{R\,r_1}{R-r_1}} - \sqrt{\frac{r_1\,r_2}{r_1-r_2}}\right)\sqrt{2\,(c+t)}$$

$$= \sqrt{\frac{2\,R\,r_1\,(c+t)}{R-r_1}} - \sqrt{\frac{2\,R\,r_2\,(c'+t)}{R-r_2}}.$$

CASE V.

Distance of heel of switch of curve, radius r_1, from springing

$$= \sqrt{\frac{2\,R\,r_1\,(c+t)}{R+r_1}};$$

The interval between the heels of the switches is equal to

$$\left(\sqrt{\frac{R\,r_1}{R+r_1}} - \sqrt{\frac{r_1\,r_2}{r_1-r_2}}\right)\left(\sqrt{2\,(c+t)}\right)$$

$$= \sqrt{\frac{2\,R\,r_1\,(c+t)}{R+r_1}} - \sqrt{\frac{2\,R\,r_2\,(c'+t)}{R+r_2}}.$$

The same remark as to the relative distances of the heels of switches from the springing applies to Cases IV. and V. as to Case II.

When the heels of the switches are kept opposite to one another, by varying the interval between the centre lines of the rails at the heels of the switches, we have the following rules:

CASE I.

$$\frac{\text{Interval between centres of main-line rail and rail-radius } r_1}{\text{Interval between centres of main-line rail and rail-radius } r_2} = \frac{r_2}{r_1}.$$

CASE II.

$$\frac{\text{Interval between centres of main-line rail and rail-radius } r_1}{\text{Interval between centres of main-line rail and rail-radius } r_2} = \frac{r_2}{r_1}.$$

CASE III.

$$\frac{\text{Interval between centres of main-line rail and rail-radius } r_1}{\text{Interval between centres of main-line rail and rail-radius } r_2} = \frac{(R+r_1)r_2}{(R-r_2)r_1}.$$

CASE IV.

$$\frac{\text{Interval between centres of main-line rail and rail-radius } r_1}{\text{Interval between centres of main-line rail and rail-radius } r_2} = \frac{(R-r_1)r_2}{(R-r_2)r_1}.$$

CASE V.

$$\frac{\text{Interval between centres of main-line rail and rail-radius } r_1}{\text{Interval between centres of main-line rail and rail-radius } r_2} = \frac{(R+r_1)r_2}{(R+r_2)r_1}.$$

Cross-over Roads.

CASE I.—*Main Line Straight.*

If the radius R be given,

Distance between springings of the reverse curves $= 2\sqrt{r(s+g)}$.

If the distance D between the points of the switches, and the lengths of the switches be given,

$$\text{Radius of reverse curves} = \frac{(D-2l)^2}{4\left\{\sqrt{s+g}-\sqrt{2(c+t)}\right\}^2}.$$

$\alpha.$ $s \gtreqless g.$

$$c_1^2 = c_2^3 = \sqrt{\frac{2g}{r}},$$

$$S_2 c_1^2 = S'_3 c_2^3 = \sqrt{2rg},$$

$$S_3 c_2^3 = S'_2 c_1^2 = 2\sqrt{r(s+g)} - \sqrt{2rg}.$$

Distance between points of crossings parallel to main line

$$= 2\sqrt{r}\left\{\sqrt{s+g} - \sqrt{2g}\right\}.$$

β. $s < g$.

$$c_1^2 = c_2^2 = \sqrt{\frac{2s}{r}},$$

$$S_2 c_1^2 = S'_2 c_2^2 = 2\sqrt{r(s+g)} - \sqrt{2rs},$$

$$S_2 c_1^2 = S'_2 c_1^2 = \sqrt{2rs}.$$

Distance between points of crossings

$$= 2\sqrt{r}\left\{\sqrt{s+g} - \sqrt{2s}\right\}.$$

CASE II.—*Main Line Curved.*

If either radius be given, one of the two equations following will give the other,

$$r_1 = \frac{R r_2}{R - 2r_2}, \qquad r_2 = \frac{R r_1}{R + 2r_1}.$$

The angular interval between the springings of the reverse curves

$$= \frac{2\sqrt{r_2(s+g)}}{\sqrt{R(R-r_2)}} = \frac{2\sqrt{r_1(s+g)}}{\sqrt{R(R+r_1)}}.$$

If the angular interval O between the springings of the reverse curves be given,

$$\text{Radius } r_2 = \frac{R^2 O^2}{4(s+g) + R O^2},$$

$$\text{Radius } r_1 = \frac{R^2 O^2}{4(s+g) - R O^2}.$$

If the distance D between the points of the switches, and the lengths of the switches be given,

The angular interval between springings of the reverse curves

$$= \frac{(D - 2l)\sqrt{s+g}}{R\left\{\sqrt{s+g} - \sqrt{2(c+l)}\right\}}.$$

α. $s \gtrless g$.

$$c_1^2 = c_2^2 = \sqrt{\frac{2(R-r_2)g}{Rr_2}} = \sqrt{\frac{2(R+r_1)g}{Rr_1}},$$

$$S_2 c_1^2 = S'_3 c_2^3 = \sqrt{\frac{2Rr_2 g}{R - r_2}} = \sqrt{\frac{2Rr_1 g}{R+r_1}},$$

$$S_2 c_2^2 = S'_2 c_1^2 = \sqrt{\frac{R r_2}{R - r_2}} \{2\sqrt{s+g} - \sqrt{2g}\}$$

$$= \sqrt{\frac{R r_1}{R + r_1}} \{2\sqrt{s+g} - \sqrt{2g}\}.$$

Distance between points of the crossings

$$= \frac{2\sqrt{R r_2}}{\sqrt{R - r_2}} \{\sqrt{s+g} - \sqrt{2g}\} = \frac{2\sqrt{R r_1}}{\sqrt{R + r_1}} \{\sqrt{s+g} - \sqrt{2g}\}.$$

If radius $r_1 = \infty$, radius $r_2 = \frac{1}{2} R$, and

$$c_1^2 = c_2^2 = \sqrt{\frac{2g}{R}},$$

$$S_2 c_1^2 = S'_2 c_2^2 = \sqrt{2Rg},$$

$$S_2 c_2^2 = S'_2 c_1^2 = \sqrt{R} \{2\sqrt{s+g} - \sqrt{2g}\}.$$

Distance between the points of the crossings

$$= 2\sqrt{R} \{\sqrt{s+g} - \sqrt{2g}\};$$

also

Angular interval between springings

$$= \frac{2\sqrt{s+g}}{\sqrt{R}}.$$

β. $s < g$.

$$c_1^2 = c_2^2 = \sqrt{\frac{2(R - r_2)s}{R r_2}} = \sqrt{\frac{2(R + r_1)s}{R r_1}},$$

$$S_2 c_1^2 = S'_2 c_2^2 = \sqrt{\frac{R r_2}{R - r_2}} \{2\sqrt{s+g} - \sqrt{2s}\}$$

$$= \sqrt{\frac{R r_1}{R - r_1}} \{2\sqrt{s+g} - \sqrt{2s}\},$$

$$S_2 c_2^2 = S'_2 c_1^2 = \sqrt{\frac{2R r_2 s}{R - r_2}} = \sqrt{\frac{2R r_1 s}{R + r_1}}.$$

Distance between the points of the crossing

$$= \frac{2\sqrt{R r_2}}{\sqrt{R - r_2}} (\sqrt{s+g} - \sqrt{2s}) = \frac{2\sqrt{R r_1}}{\sqrt{R + r_1}} (\sqrt{s+g} - \sqrt{2s}).$$

If the radius $r_1 = \infty$, radius $r_2 = \frac{1}{2}\mathrm{R}$, and

$$c_1^2 = c_2^3 = \sqrt{\frac{2s}{\mathrm{R}}},$$

$$\mathrm{S}_2 c_1^2 = \mathrm{S}'_2 c_2^3 = \sqrt{\mathrm{R}}\left\{2\sqrt{s+g} - \sqrt{2s}\right\},$$

$$\mathrm{S}_2 c_2^3 = \mathrm{S}'_2 c_1^2 = \sqrt{2\mathrm{R}s}.$$

Distance between the points of the crossings

$$= 2\sqrt{\mathrm{R}}\left\{\sqrt{s+g} - \sqrt{2s}\right\};$$

Angular interval between the springings

$$= \frac{2\sqrt{s+g}}{\sqrt{\mathrm{R}}}.$$

Through Roads.

Where the through road consists of two reverse curves separated by an interval of straight, it is only necessary to give the positions of the crossings c_1^2, c_1^{2n+2}, c_2^3, and c_2^{2n+3}, since straight lines drawn on the ground from c_1^2 to c_1^{2n+2}, and c_2^3 to c_2^{2n+3}, will mark the position of the rest.

Where the through road consists of two reverse curves, meeting at a common tangent, the simplest way of marking the positions of the various crossings on the ground is to set out the centre line of the through road. In order to do this correctly, it is necessary to know the relative positions of the springings of the curves and the point, where they meet.

CASE I.—*Main Roads Straight: Reverse Curves separated by an interval of Straight, which extends from extreme crossing to extreme crossing.*

a. $s \gtreqless g$.

If the radius r of the reverse curves be known,

Interval between the normals through the springings of the curves

$$= \frac{\{(n+1)s + (n+3)g\}\sqrt{r}}{\sqrt{2g}}.$$

If the distance D between the points of the switches, and the lengths of the switches be known,

The radius of reverse curves

$$= \frac{2g\,(D - 2l)^2}{\{(n+1)s+(n+3)g-4\sqrt{g(c+t)}\}^2};$$

The circular measure of the angle of each crossing

$$= \sqrt{\frac{2g}{r}}.$$

$$S_2 c_1^2 = S'_{2n+3} c_2^{2n+3} = \sqrt{2rg},$$

$$S_3 c_2^3 = S'_{2n+2} c_1^{2n+2} = \frac{\sqrt{r}\,(s+g)}{\sqrt{2g}}.$$

β. $s < g$.

If the radius r of the reverse curves be known,

Interval between the normals through the springings of the reverse curves

$$= \frac{\{(n+3)s+(n+1)g\}\sqrt{r}}{\sqrt{2s}}.$$

If the distance D between the points of the switches, and the lengths of the switches be known,

Radius of the reverse curves

$$= \frac{2s\,(D - 2l)^2}{\{(n+3)s+(n+1)g-4\sqrt{s(c+t)}\}^2};$$

The circular measure of the angle of each crossing

$$= \sqrt{\frac{2s}{r}}.$$

$$S_2 c_1^2 = S'_{2n+3} c_2^{2n+3} = \frac{\sqrt{r}\,(s+g)}{\sqrt{2s}},$$

$$S_3 c_2^3 = S'_{2n+2} c_1^{2n+2} = \sqrt{2rs}.$$

CASE II.—*Main Roads Straight, Reverse Curves of Through Road meeting at a common tangent.*

If the radius r of the reverse curves be known,

Interval between the normals through the springings of the curves

$$= 2\sqrt{(n+1)(s+g)r}.$$

The point where the two curves meet is exactly half-way between the springings.

If the distance D between the points of the switches, and the lengths of the switches be known,

Radius of the reverse curves

$$= \frac{(D - 2l)^2}{4\left\{\sqrt{(n+1)(s+g)} - \sqrt{2(c+t)}\right\}^2}.$$

$$c_1^2 = c_2^{2n+3} = \sqrt{\frac{2g}{r}}.$$

The values of the circular measures of the remaining angles depend partly on the relative value of s and g.

α. $s > g$.

If n be even, we shall obtain the circular measures of the angles of the crossings of the intervening roads, by substituting for m the successive values

$$1, 2, 3 \ldots \text{\&c.} \ldots \frac{n}{2}$$

in the equations

$$c_1^{2m+1} = c_2^{2(n-m+1)+2} = \sqrt{\frac{2m(s+g)}{r}}, \tag{1}$$

$$c_2^{2m+1} = c_1^{2(n-m+1)+2} = \sqrt{\frac{2ms + 2(m-1)g}{r}}, \tag{2}$$

$$c_1^{2m+2} = c_2^{2(n-m+1)+1} = \sqrt{\frac{2ms + 2(m+1)g}{r}}, \tag{3}$$

$$c_2^{2m+2} = c_1^{2(n-m+1)+1} = \sqrt{\frac{2m(s+g)}{r}}. \tag{4}$$

If n be odd, we shall obtain the circular measures of the angles of the crossings of the $(n-1)$ rails on either side of the middle road, by substituting in the equations (1), (2), (3), and (4) for m the successive values

$$1, 2, 3 \ldots \text{\&c.} \ldots \frac{n-1}{2},$$

and the circular measures of the angles of the crossings of the

middle road by putting $m = \frac{n+1}{2}$ in equations (1) and (2). They are therefore

$$c_1^{n+2} = c_2^{n+3} = \sqrt{\frac{(n+1)(s+g)}{r}},$$

$$c_2^{n+2} = c_1^{n+3} = \sqrt{\frac{(n+1)s+(n-1)g}{r}}.$$

β. $s < g$.

If n be even, we shall obtain the circular measures of the angles of the crossings of the $n-2$ intervening rails on either side of the two middle roads, by substituting for m the successive values

$$1, 2, 3 \ldots \text{\&c.} \ldots \frac{n}{2} - 1$$

in the equations (1), (2), (3), and (4), given in (*a*). The values of the circular measures of the remaining angles are given by the formulæ

$$c_1^{n+1} = c_2^{n+4} = \sqrt{\frac{n(s+g)}{r}},$$

$$c_2^{n+1} = c_1^{n+4} = \sqrt{\frac{ns+(n-2)g}{r}},$$

$$c_1^{n+2} = c_2^{n+3} = \sqrt{\frac{(n+2)s+ng}{r}},$$

$$c_2^{n+2} = c_1^{n+3} = \sqrt{\frac{n(s+g)}{r}}.$$

If n be odd, the formulæ must be used as explained in (*a*).

CASE III.—*Main Roads Curved: Reverse Curves separated by an interval of Straight, which extends from extreme crossing to extreme crossing.*

See remarks on this in Section V. of this chapter.

α. $s \gtrless g$.

The radii of the reverse curves are connected by the following relations:

$$r_1 = \frac{\mathrm{R}\, r_2 g}{\mathrm{R}g - (n+1)(s+g) r_2}, \qquad r_2 = \frac{\mathrm{R}\, r_1 g}{\mathrm{R}g + (n+1)(s+g) r_1}.$$

Circular measure of the angle between the normals which pass through c_1^2 and the adjacent springing

$$= \sqrt{\frac{2Rg}{r_2(R-r_2)}};$$

Circular measure of the angle between the normals which pass through c_2^{2n+3} and the adjacent springing

$$= \sqrt{\frac{2Rg}{r_1(R+r_1)}}.$$

Circular measure of the angle between the normals which pass through the two springings

$$= \sqrt{\frac{2Rg}{r_2(R-r_2)}} - \sqrt{\frac{2Rg}{r_1(R+r_1)}}.$$

$$c_1^2 = \sqrt{\frac{2(R-r_2)g}{Rr_2}},$$

$$c_2^{2n+3} = \sqrt{\frac{2(R+r_1)g}{Rr_1}}.$$

If we substitute for m the successive values

$$1, 2, 3 \ldots \text{\&c.} \ldots n$$

in the equations

$$c_1^{2m+1} = \sqrt{\frac{2Rg - 2r_2m(s+g)}{Rr_2}}, \qquad (1)$$

$$c_2^{2m+1} = \sqrt{\frac{2(R+r_2)g - 2r_2m(s+g)}{Rr_2}}, \qquad (2)$$

$$c_1^{2m+2} = \sqrt{\frac{2(R-r_2)g - 2r_2m(s+g)}{Rr_2}}, \qquad (3)$$

$$c_2^{2m+2} = \sqrt{\frac{2Rg - 2r_2m(s+g)}{Rr_2}}, \qquad (4)$$

we shall obtain the values of the circular measures of the angles of the crossings of the n intervening roads.

$$S_2 c_1 = \sqrt{\frac{2Rr_2g}{R-r_2}},$$

$$S_2 c_2^2 = (R-s-g)\left\{\sqrt{\frac{2Rg}{r_2(R-r_2)}} - \sqrt{\frac{2Rg-2r_2s}{Rr_2}}\right\},$$

$$S'_{2n+2}c_2^{2n+2} = \sqrt{\frac{2Rr_1g}{R+r_1}},$$

$$S'_{2n+2}c_1^{2n+2} = \{R - n(s+g)\}\left\{\sqrt{\frac{2(R-r_2)g - 2r_2n(s+g)}{Rr_2}} - \sqrt{\frac{2Rg}{r_1(R+r_1)}}\right\}.$$

$\beta.$ $s < g.$

The radii of the reverse curves are connected by the following equations:

$$r_1 = \frac{Rr_2s}{Rs - r_2(n+1)(s+g)}, \qquad r_2 = \frac{Rr_1s}{Rs + r_1(n+1)(s+g)}.$$

Circular measure of the angle between the normals which pass through c_2^2 and the adjacent springing

$$= \sqrt{\frac{2Rs}{r_2(R-r_2)}};$$

Circular measure of the angle between the normals which pass through c_1^{2n+2} and the adjacent springing

$$= \sqrt{\frac{2Rs}{r_1(R+r_1)}};$$

Circular measure of the angle between the normals which pass through the two springings

$$= \sqrt{\frac{2Rs}{r_2(R-r_2)}} - \sqrt{\frac{2Rs}{r_1(R+r_1)}}.$$

$$c_1^2 = \sqrt{\frac{2(Rs - r_2g)}{Rr_2}},$$

$$c_2^{2n+2} = \sqrt{\frac{2Rs - 2(n+1)r_2s - 2nr_2g}{Rr_2}}.$$

If we substitute for m the successive values

$$1, 2, 3 \ldots \text{\&c.} \ldots n$$

in the equations

$$c_1^{2m+1} = \sqrt{\frac{2Rs - 2r_2m(s+g)}{Rr_2}}, \qquad (1)$$

$$c_2^{2m+1} = \sqrt{\frac{2Rs - 2r_2 m(s+g) + 2r_2 g}{Rr_2}}, \qquad (2)$$

$$c_1^{2m+2} = \sqrt{\frac{2Rs - 2r_2 m(s+g) - 2r_2 g}{Rr_2}}, \qquad (3)$$

$$c_2^{2m+2} = \sqrt{\frac{2Rs - 2r_2 m(s+g)}{Rr_2}}, \qquad (4)$$

we shall obtain the circular measures of the crossings of the n intervening roads.

$$S_2 c_1^2 = R\left\{\sqrt{\frac{2Rs}{r_2(R-r_2)}} - \sqrt{\frac{2(Rs-r_2 g)}{Rr_2}}\right\},$$

$$S_3 c_2^3 = \sqrt{\frac{2(R-r_2)s}{Rr_2}},$$

$$S'_{2n+3} c_2^{2n+3} = \{R-(n+1)(s+g)\}\left\{\sqrt{\frac{2Rs-2(n+1)r_2 s-2nr_2 g}{Rr_2}} - \sqrt{\frac{2Rs}{r_1(R+r_1)}}\right\},$$

$$S'_{2n+2} c_1^{2n+2} = \sqrt{\frac{2Rr_1 s}{R+r_1}}.$$

CASE IV.—*Main Roads Curved: Reverse Curves meeting at a common point of contrary flexure midway between the two Roads, between which communication is established.*

If the radius of either reverse curve is known, the other will be given by one of the formulæ

$$r_1 = \frac{Rr_2}{R-2r_2}, \qquad r_2 = \frac{Rr_1}{R+2r_1}.$$

The angular interval o between the springings of the reverse curves

$$= \frac{2\sqrt{r_2(n+1)(s+g)}}{\sqrt{R(R-r_2)}} = \frac{\sqrt{r_1(n+1)(s+g)}}{\sqrt{R(R+r_1)}};$$

Angular interval between either springing and the point where the reverse curves meet

$$= \frac{RO}{2r_2} = \frac{RO}{2r_1}.$$

If the distance D between the normals through the points of the switches, measured along any centre line, whose radius is R, and the lengths of the switches be known,

Angular interval O

$$= \frac{(D - 2l)\sqrt{2(c+t)}}{R\{\sqrt{2(n+1)(s+g)} - 2\sqrt{c+t}\}},$$

Radius $r_1 = \dfrac{R^2O^2}{4(n+1)(s+g) - RO^2}$,

Radius $r_2 = \dfrac{R^2O^2}{4(n+1)(s+g) + RO^2}$,

$$c_1^2 = c_2^{2n+2} = \sqrt{\frac{2(R-r_2)g}{Rr_2}} = \sqrt{\frac{2(R+r_1)g}{Rr_1}}.$$

a. $s \gtrless g$.

If n be even, we shall obtain the circular measures of the angles of the crossings of the n intervening roads, by substituting for m the successive values

$$1, 2, 3 \ldots \text{\&c.} \ldots \frac{n}{2}$$

in the equations

$$c_1^{2m+1} = c_2^{2(n-m+1)+2} = \sqrt{\frac{2(R-r_2)m(s+g)}{Rr_2}}, \quad (1)$$

$$c_2^{2m+1} = c_1^{2(n-m+1)+2} = \sqrt{\frac{2(R-r_2)\{m(s+g)-g\}}{Rr_2}}, \quad (2)$$

$$c_1^{2m+2} = c_2^{2(n-m+1)+1} = \sqrt{\frac{2(R-r_2)\{m(s+g)+g\}}{Rr_2}}, \quad (3)$$

$$c_2^{2m+2} = c_1^{2(n-m+1)+1} = \sqrt{\frac{2(R-r_2)m(s+g)}{Rr_2}}; \quad (4)$$

or the successive values

$$\frac{n+2}{2}, \frac{n+4}{2} \ldots \text{\&c.} \ldots n$$

in the equations

$$c_1^{2m+1} = c_2^{2(n-m+1)+2} = \sqrt{\frac{2(R+r_1)(n-m+1)(s+g)}{Rr_1}}, \quad (1^a)$$

$$c_2^{2m+1} = c_1^{2(n-m+1)+2} = \sqrt{\frac{2(R+r_1)\{(n-m+1)(s+g)+g\}}{Rr_1}}, \quad (2^a)$$

$$c_1^{2m+2} = c_2^{2(n-m+1)+1} = \sqrt{\frac{2(R+r_1)\{(n-m+1)(s+g)-g\}}{Rr_1}}, \quad (3^a)$$

$$c_2^{2m+2} = c_1^{2(n-m+1)+1} = \sqrt{\frac{2(R+r_1)(n-m+1)(s+g)}{Rr_1}}. \quad (4^a)$$

If n be odd, we shall obtain the circular measures of the angles of the crossings of the $n-1$ roads on either side of the middle road, by substituting for m the successive values

$$1, 2, 3 \ldots \&c. \ldots \frac{n-1}{2}$$

in equations (1), (2), (3), and (4), or the successive values

$$\frac{n+3}{2}, \frac{n+5}{2} \ldots \&c. \ldots n$$

in equations (1^a), (2^a), (3^a), and (4^a); and the circular measures of the angles of the crossings of the middle road, by putting $m = \frac{n+1}{2}$ in either the two equations (1) and (2) or the pair (3^a) and (4^a), so that

$$c_1^{n+2} = c_2^{n+3} = \sqrt{\frac{(R-r_2)(n+1)(s+g)}{Rr_2}} = \sqrt{\frac{(R+r_1)(n+1)(s+g)}{Rr_1}},$$

$$c_2^{n+2} = c_1^{n+3} = \sqrt{\frac{(R-r_2)\{(n+1)s+(n-1)g\}}{Rr_2}} = \sqrt{\frac{(R+r_1)\{(n+1)s+(n-1)g\}}{Rr_1}}.$$

β. $s < g$.

If n be even, for the circular measures of the angles of the crossings of the $\frac{n-2}{2}$ roads on either side of the two middle roads, we must substitute for m the successive values

$$1, 2, 3 \ldots \&c. \ldots \frac{n-2}{2}$$

in equations (1), (2), (3), and (4), or the successive values

$$\frac{n+4}{2}, \frac{n+6}{2} \ldots \&c. \ldots n$$

in the equations (1^a), (2^a), (3^a), and (4^a).

The values of the circular measures of the remaining angles are given by the formulæ

$$c_1^{n+1} = c_2^{n+1} = \sqrt{\frac{(R-r_2)n(s+g)}{Rr_2}} = \sqrt{\frac{(R+r_1)n(s+g)}{Rr_1}},$$

$$c_2^{n+1} = c_1^{n+1} = \sqrt{\frac{(R-r_2)\{ns+(n-2)g\}}{Rr_2}} = \sqrt{\frac{(R+r_1)\{ns+(n-2)g\}}{Rr_1}},$$

$$c_2^{n+2} = c_1^{n+2} = \sqrt{\frac{(R-r_2)n(s+g)}{Rr_2}} = \sqrt{\frac{(R+r_1)n(s+g)}{Rr_1}},$$

$$c_1^{n+2} = c_2^{n+2} = \sqrt{\frac{(R-r_2)\{(n+2)s+ng\}}{Rr_2}} = \sqrt{\frac{(R+r_1)\{(n+2)s+ng\}}{Rr_1}}.$$

If n be odd, we shall obtain the circular measures of the angles by using the formulæ as explained in (*a*).

The following general rules, which will suffice for the solution of almost every one of the previous problems, when the radii of the curves are known, may be very easily remembered.

RULE I.—*Curve leaving Straight Main Line.*

The circular measure of the crossing of any rail parallel to the main road is equal to the square root of twice the interval between the rail crossed and the rail which the crossing rail of the branch road touches, multiplied by the factor $\sqrt{\frac{1}{r}}$; and the distance of the crossing from the springing is equal to the square root of twice the same interval, multiplied by the factor $\sqrt{r}$.

RULE II.—*Curve leaving Curved Main Line, Curvature in the same direction.*

The circular measure of the angle of the crossing of any rail parallel to the main line is equal to the square root of twice the interval between the rail crossed and the rail which the crossing rail of the branch road touches, multiplied by the factor $\sqrt{\frac{R-r}{Rr}}$;

and the distance of the crossing from the springing is equal to the square root of twice the same interval, multiplied by the factor $\sqrt{\frac{R r}{R - r}}$.

RULE III.—*Curve leaving Curved Main Line, Curvature in the opposite direction.*

The circular measure of the crossing of any rail parallel to the main road is equal to the square root of twice the interval between the rail crossed and the rail which the crossing rail of the branch road touches, multiplied by the factor $\sqrt{\frac{R + r}{R r}}$; and the distance of the crossing from the springing is equal to the square root of twice the same interval, multiplied by the factor $\sqrt{\frac{R r}{R + r}}$.

Sidings from one of the Main-line Roads across the other.

When the siding road is a continuous curve, for the values and positions of c_1^2, c_2^3, c_1^4, c_2^3, c_2^4, see the formulæ for double-line junctions.

When the siding road after crossing the second or third rail is straight, the following formulæ give the circular measures and positions of the angles:

CASE I.—*Main Line Straight.*

a. $s \gtrless g$.

$$c_1^2 = c_1^3 = c_1^4 = c_2^3 = c_2^4 = \sqrt{\frac{2g}{r}},$$

$$S_2 c_1^2 = \sqrt{2rg},$$

$$S_2 c_1^3 = S_4 c_2^4 = \frac{(2g + s)\sqrt{r}}{\sqrt{2g}},$$

$$S_4 c_1^4 = \frac{(3g + s)\sqrt{r}}{\sqrt{2g}},$$

$$S_3 c_2^3 = \frac{(s + g)\sqrt{r}}{\sqrt{2g}}.$$

β. $s < g$.

$$c_1^2 = c_1^3 = c_1^4 = c_2^3 = c_2^4 = \sqrt{\frac{2s}{r}},$$

$$S_2 c_1^2 = \frac{(s+g)\sqrt{r}}{\sqrt{2s}},$$

$$S_3 c_1^3 = S_4 c_2^4 = \frac{(2s+g)\sqrt{r}}{\sqrt{2s}},$$

$$S_4 c_1^4 = \frac{2(s+g)\sqrt{r}}{\sqrt{2s}},$$

$$S_3 c_2^3 = \sqrt{2rs}.$$

CASE II.—*Main Line Curved, Siding Road Curving in the same direction.*

α. $s \gtreqless g$.

$$c_1^2 = \sqrt{\frac{2(R-r_2)g}{R r_2}},$$

$$c_1^3 = c_2^4 = \sqrt{\frac{2Rg - 2r_2(s+g)}{R r_2}},$$

$$c_1^4 = \sqrt{\frac{2Rg - 2r_2(s+2g)}{R r_2}},$$

$$c_2^3 = \sqrt{\frac{2Rg - 2r_2 s}{R r_2}},$$

$$S_2 c_1^2 = \sqrt{\frac{2Rr_2 g}{R - r_2}},$$

$$S_3 c_1^3 = S_4 c_2^4 = (R - s - g)\left\{\sqrt{\frac{2Rg}{r_2(R-r_2)}} - \sqrt{\frac{2Rg-2r_2(s+g)}{Rr_2}}\right\},$$

$$S_2 c_1^4 = (R - s - g)\left\{\sqrt{\frac{2Rg}{r_2(R-r_2)}} - \sqrt{\frac{2Rg-2r_2(s+2g)}{Rr_2}}\right\},$$

$$S_3 c_2^3 = (R - s - g)\left\{\sqrt{\frac{2Rg}{r_2(R-r_2)}} - \sqrt{\frac{2Rg-2r_2 s}{Rr_2}}\right\}.$$

β. $s < g$.

$$c_1^2 = \sqrt{\frac{2Rs - 2r_2 g}{Rr_2}},$$

$$c_1^3 = c_2^4 = \sqrt{\frac{2Rs - 2r_2(s+g)}{Rr_2}},$$

$$c_1^4 = \sqrt{\frac{2Rs - 2r_2(s+2g)}{Rr_2}},$$

$$c_2^3 = \sqrt{\frac{2(R-r_2)s}{Rr_2}},$$

$$S_2 c_1^2 = R\left\{\sqrt{\frac{2Rs}{r_2(R-r_2)}} - \sqrt{\frac{2(Rs-r_2 g)}{Rr_2}}\right\},$$

$$S_3 c_1^3 = S_4 c_2^4 = (R-s-g)\left\{\sqrt{\frac{2Rs}{r_2(R-r_2)}} - \sqrt{\frac{2Rs-2r_2(s+g)}{Rr_2}}\right\},$$

$$S_4 c_1^4 = (R-s-g)\left\{\sqrt{\frac{2Rs}{r_2(R-r_2)}} - \sqrt{\frac{2Rs-2r_2(s+2g)}{Rr_2}}\right\},$$

$$S_3 c_2^3 = \sqrt{\frac{2Rr_2 s}{R-r_2}}.$$

CASE III.—*Main Line Curved, Siding Roads Curving in the opposite direction.*

α. $s \gtrless g$.

$$c_1^2 = \sqrt{\frac{2(R+r_1)g}{Rr_1}},$$

$$c_1^3 = c_2^4 = \sqrt{\frac{2Rg + 2r_1(s+g)}{Rr_1}},$$

$$c_1^4 = \sqrt{\frac{2Rg + 2r_1(s+2g)}{Rr_1}},$$

$$c_2^3 = \sqrt{\frac{2Rg + 2r_1 s}{Rr_1}},$$

$$S_2 c_1^2 = \sqrt{\frac{2Rr_1 g}{R+r_1}},$$

$$S_3 c_1^3 = S_4 c_2^4 = (R+s+g)\left\{\sqrt{\frac{2Rg}{r_1(R+r_1)}} - \sqrt{\frac{2Rg+2r_1(s+g)}{Rr_1}}\right\},$$

$$S_4 c_1^4 = (R+s+g)\left\{\sqrt{\frac{2Rg}{r_1(R+r_1)}} - \sqrt{\frac{2Rg+2r_1(s+2g)}{Rr_1}}\right\},$$

$$S_3 c_2^3 = (R+s+g)\left\{\sqrt{\frac{2Rg}{r_1(R+r_1)}} - \sqrt{\frac{2Rg+2r_1 s}{Rr_1}}\right\}.$$

$\beta.$ $s < g.$

$$c_1^2 = \sqrt{\frac{2Rs+2r_1 g}{Rr_1}},$$

$$c_1^3 = c_2^4 = \sqrt{\frac{2Rs+2r_1(s+g)}{Rr_1}},$$

$$c_1^4 = \sqrt{\frac{2Rs+2r_1(s+2g)}{Rr_1}},$$

$$c_2^3 = \sqrt{\frac{2(R+r_1)s}{Rr_1}},$$

$$S_2 c_1^2 = R\left\{\sqrt{\frac{2Rs}{r_1(R+r_1)}} - \sqrt{\frac{2(Rs+r_1 g)}{Rr_1}}\right\},$$

$$S_3 c_1^3 = S_4 c_2^4 = (R+s+g)\left\{\sqrt{\frac{2Rs}{r_1(R+r_1)}} - \sqrt{\frac{2Rs+2r_1(s+g)}{Rr_1}}\right\},$$

$$S_4 c_1^4 = (R+s+g)\left\{\sqrt{\frac{2Rs}{r_1(R+r_1)}} - \sqrt{\frac{2Rs+2r_1(s+2g)}{Rr_1}}\right\},$$

$$S_3 c_2^3 = \sqrt{\frac{2Rr_1 s}{R+r_1}}.$$

If the siding road crosses more than one main-line road, the circular measures and positions of the angles of the crossings of the second, third, &c., road crossed may be obtained in the same way, by giving m the values 2, 3, &c., in the equations in Chapter III., Section V. If there be only one road crossed, for $R \pm (s+g)$ we may write simply R, since the positions of all the crossings will be equally affected, and to such a slight extent that the divergence from exactness will be practically of no consequence.

Division II.—*Formulæ expressed in terms of the circular measures of the angles of the crossings.*

Switches.

Formulæ which give the lengths of switches and the distances of the heels from the springings of the curve in terms of the circular measures of the angles of the crossings are of no practical use.

Those, however, which give the intervals between the heels of the switches and the points of the crossings in terms of the same circular measures are of the greatest practical use; they are stated for each particular case.

Fast Points.

The same remarks apply to the formulæ giving the lengths and positions of fast points.

Single-line Junctions.

Whether the main line be straight or curved, in each of the cases,

$$H_2 c_1^2 = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_1^2}.$$

Length of splayed part of fast point

$$= \frac{2\sqrt{g}\,(\sqrt{c+t} - \sqrt{c+e})}{c_1^2};$$

Distance of point of fast point from point of the crossing

$$= \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+e})}{c_1^2}.$$

Case I.—*Curve leaving Straight Main Line.*

$$r = \frac{2g}{c_1^2}.$$

Case II.—*Curve leaving Curved Main Line, Curvature in the same direction.*

$$r = \frac{2Rg}{Rc_1^2 + 2g}.$$

CASE III.—*Curve leaving Curved Main Line, Curvature in opposite direction.*

$$r = \frac{2\,\mathrm{R}\,g}{\mathrm{R}\,c_1^2 - 2\,g}.$$

Double-line Junctions.

Whether the main line be straight or curved in each of the cases,

$$c_1^3 = c_2^4 = \sqrt{\frac{s+g}{g}} \cdot c_1^2,$$

$$c_1^4 = \sqrt{\frac{s+2g}{g}} \cdot c_1^2,$$

$$c_2^3 = \sqrt{\frac{s}{g}} \cdot c_1^2,$$

$$c_3^4 = c_1^2,$$

$$\mathrm{H}_2 c_1^2 = \mathrm{H}_4 c_3^4 = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_1^2},$$

$$\mathrm{H}_3 c_1^3 = \mathrm{H}_4 c_2^4 = \frac{2\sqrt{s+g}\,(\sqrt{s+g} - \sqrt{c+t})}{c_1^3} = \frac{2\sqrt{g}\,(\sqrt{s+g} - \sqrt{c+t})}{c_1^2},$$

$$\mathrm{H}_4 c_1^4 = \frac{2\sqrt{s+2g}\,(\sqrt{s+2g} - \sqrt{c+t})}{c_1^4} = \frac{2\sqrt{g}\,(\sqrt{s+2g} - \sqrt{c+t})}{c_1^2},$$

$$\mathrm{H}_2 c_2^3 = \frac{2\sqrt{s}\,(\sqrt{s} - \sqrt{c+t})}{c_2^3} = \frac{2\sqrt{g}\,(\sqrt{s} - \sqrt{c+t})}{c_1^2}.$$

The formulæ for the fast points are the same as those for a single-line junction.

In the formulæ which give the radii in terms of the circular measures of the angles, if r_s, r_2, r_1 denote the radii of the branch roads in each of the three cases, since

$$r_s = \frac{2g}{c_1^2},$$

$$\frac{1}{r_2} = \frac{c_1^2}{2g} + \frac{1}{\mathrm{R}} = \frac{1}{r_s} + \frac{1}{\mathrm{R}},$$

$$\frac{1}{r_1} = \frac{c_1^2}{2g} - \frac{1}{R} = \frac{1}{r_s} - \frac{1}{R},$$

we obtain the following relations, between the radii of branch roads, which cut corresponding rails of main-line roads in each case at the same angle,

$$r_s = \frac{R r_2}{R - r_2} = \frac{R r_1}{R + r_1},$$

$$r_2 = \frac{R r_s}{R + r_s} = \frac{R r_1}{R + 2 r_1},$$

$$r_1 = \frac{R r_s}{R - r_s} = \frac{R r_2}{R - 2 r_2}.$$

Three Throws.

If H_{r_1} H_{r_2} denote the positions of the heels of the switches of the curves, radii r_1 and r_2 respectively, when the interval, $c + t$, between the centre lines of two contiguous rails at either heel remains constant, we have:

If the branch roads curve in opposite directions, whether the main line be straight or curved,

$$c_m = \sqrt{c_{r_1}^2 + c_{r_2}^2},$$

$$H_{r_1} c_{r_1} = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_{r_1}},$$

$$H_{r_2} c_{r_2} = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_{r_2}},$$

$$c_m c_{r_1} = \frac{2\,(c_m - c_{r_1})\,g}{c_m c_{r_1}},$$

$$c_m c_{r_2} = \frac{2\,(c_m - c_{r_2})\,g}{c_m c_{r_2}},$$

$$SH_{r_1} - SH_{r_2} = \frac{2\,(c_{r_2} - c_{r_1})\sqrt{g\,(c+t)}}{c_{r_1} c_{r_2}},$$

the heel of the switch of the easiest crossing, c_{r_1} or c_{r_2}, being farthest away from the springing of the curves.

If the branch roads curve in the same direction, whether the main line be straight or curved,

$$c_m = \sqrt{c_{r_2}^2 - c_{r_1}^2},$$

$$H_{r_1} c_{r_1} = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_{r_1}},$$

$$c_{r_2} c_{r_1} = \frac{2\,(c_{r_2} - c_{r_1})\,g}{c_{r_1} c_{r_2}},$$

$$H_{r_2} c_m = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_m},$$

$$c_{r_2} c_m = \frac{2\,(c_{r_2} - c_m)\,g}{c_m c_{r_2}},$$

$$S H_{r_1} - S H_{r_2} = \frac{2\,(c_m - c_{r_1})\sqrt{g\,(c+t)}}{c_m c_{r_1}},$$

the heel of the switch of the easiest crossing, c_{r_1} or c_m, being farthest away from the springing of the curves.

The exact value of c_m, *viz.* $\sqrt{c_{r_2}^2 \pm c_{r_1}^2}$, must always be used.

When the heels of the switches are kept opposite one another, by varying the amount of clearance we have the following rules:

If the branch roads curve in opposite directions, whether the main line be straight or curved,

$$\frac{\text{Interval between centres of main-line rail and rail with crossing } c_{r_1}}{\text{Interval between centres of main-line rail and rail with crossing } c_{r_2}} = \frac{c_{r_1}^2}{c_{r_2}^2};$$

$$\frac{\text{Interval between centres of main-line rail and rail with crossing } c_{r_1}}{\text{Interval between centre of branch-line rails}} = \frac{c_{r_1}^2}{c_{r_1}^2 + c_{r_2}^2}.$$

If the branch roads curve in the same direction, whether the main line be straight or curved,

$$\frac{\text{Interval between centres of main-line rail and rail with crossing } c_{r_1}}{\text{Interval between centres of main-line rail and rail with crossing } c_{r_2}} = \frac{c_{r_1}^2}{c_{r_2}^2};$$

$$\frac{\text{Interval between centres of main-line rail and rail with crossing } c_{r_1}}{\text{Interval between centres of branch-line rails}} = \frac{c_{r_1}^2}{c_{r_2}^2 - c_{r_1}^2}.$$

Cross-over Roads.

Whether the main line be straight or curved,

$$c_1^2 = c_2^2.$$

α. $s \gtreqless g$.

Interval between the heels of the switches parallel to main line

$$= \frac{2\sqrt{g}\left(\sqrt{2(s+g)} - 2\sqrt{c+t}\right)}{c_1^2};$$

Interval between the points of the crossings parallel to main line

$$= \frac{2\sqrt{g}\left(\sqrt{2(s+g)} - 2\sqrt{g}\right)}{c_1^2}.$$

$$H_2 c_1^2 = H'_2 c_2^2 = \frac{2\sqrt{g}\left(\sqrt{g} - \sqrt{c+t}\right)}{c_1^2},$$

$$H_2 c_2^2 = H'_2 c_1^2 = \frac{2\sqrt{g}\left(\sqrt{2(s+g)} - \sqrt{g} - \sqrt{c+t}\right)}{c_1^2}.$$

β. $s < g$.

Interval between the heels of the switches parallel to main line

$$= \frac{2\sqrt{s}\left(\sqrt{2(s+g)} - 2\sqrt{c+t}\right)}{c_1^2};$$

Interval between the points of the crossings parallel to main line

$$= \frac{2\sqrt{s}\left(\sqrt{2(s+g)} - 2\sqrt{s}\right)}{c_1^2}.$$

$$H_s c_1^2 = H'_s c_2^2 = \frac{2\sqrt{s}\,(\sqrt{2(s+g)} - \sqrt{s} - \sqrt{c+t})}{c_1^2},$$

$$H_s c_2^2 = H'_s c_1^2 = \frac{2\sqrt{s}\,(\sqrt{s} - \sqrt{c+t})}{c_1^2}.$$

Through Roads.

Although we may obtain the radii of the reverse curves, and the circular measures of the rest of the angles and their position from formulæ in terms of the circular measure of any one angle, the general formulæ thus obtained would have no practical value except in Case I., where all the angles are equal. There are, however, two special cases of common occurrence, which deserve investigation, *viz.* where there is only one intervening road, and the reverse curves meet at the centre of the middle road, so that the same formulæ apply to both cases of s greater or less than g.

CASE I.—*Main Roads Straight: Reverse Curves separated by an interval of Straight, the number of intervening Roads being unlimited.*

α. $s \gtreqless g$.

Interval between the heels of the switches

$$= \frac{(n+1)s + (n+3)g - 4\sqrt{g(c+t)}}{c_1^2}.$$

$$H_2 c_1^2 = H'_{2n+s} c_2^{2n+3} = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_1^2},$$

$$H_3 c_2^3 = H'_{2n+2} c_1^{2n+2} = \frac{s+g-2\sqrt{g(c+t)}}{c_1^2}.$$

β. $s < g$.

Interval between the heels of the switches

$$= \frac{(n+3)s + (n+1)g - 4\sqrt{s(c+t)}}{c_2^2}.$$

$$H_2 c_1^2 = H'_{2n+s} c_2^{2n+3} = \frac{s+g-2\sqrt{s(c+t)}}{c_2^2},$$

$$H_3 c_2^3 = H_{2n+2} c_1^{2n+2} = \frac{2\sqrt{s}\,(\sqrt{s} - \sqrt{c+t})}{c_2^2}.$$

CASE II.—*One intervening Road, Reverse Curves meeting in the centre of the middle Road.*

Whether the main line be straight or curved,

Interval between the normals through the heels of the switches

$$= \frac{4\sqrt{g}\,(\sqrt{s+g} - \sqrt{c+t})}{c_1^2}.$$

$$c_2^5 = c_1^2,$$

$$c_1^3 = c_2^4 = \sqrt{\frac{s+g}{g}} \cdot c_1^2,$$

$$c_2^3 = c_1^4 = \sqrt{\frac{s}{g}} \cdot c_1^2,$$

$$H_2 c_1^2 = H'_5 c_2^5 = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_1^2},$$

$$H_3 c_1^3 = H'_4 c_2^4 = \frac{2\sqrt{s+g}\,(\sqrt{s+g} - \sqrt{c+t})}{c_1^3} = \frac{2\sqrt{g}\,(\sqrt{s+g} - \sqrt{c+t})}{c_1^2},$$

$$H_3 c_2^3 = H'_4 c_2^4 = \frac{2\sqrt{s}\,(\sqrt{s} - \sqrt{c+t})}{c_2^3} = \frac{2\sqrt{g}\,(\sqrt{s} - \sqrt{c+t})}{c_1^2}.$$

Sidings from one of the Main-line Roads across the other.

When the siding road is a continuous curve, and the angle c_1^2 is given, for the values of c_1^3, c_1^4, c_2^3 and c_2^4 and their positions, see the formulæ for double-line junctions.

When the angle c_1^4 is given, we have

$$c_1^2 = \sqrt{\frac{g}{s+2g}} \cdot c_1^4,$$

$$c_1^3 = c_2^4 = \sqrt{\frac{s+g}{s+2g}} \cdot c_1^4,$$

$$c_2^3 = \sqrt{\frac{s}{s+2g}} \cdot c_1^4,$$

$$H_2 c_1^2 = \frac{2\sqrt{s+2g}\,(\sqrt{g} - \sqrt{c+t})}{c_1^4},$$

$$H_3 c_1^3 = H_4 c_2^4 = \frac{2\sqrt{s+2g}\,(\sqrt{s+g}-\sqrt{c+t})}{c_1^4},$$

$$H_4 c_1^4 = \frac{2\sqrt{s+2g}\,(\sqrt{s+2g}-\sqrt{c+t})}{c_1^4},$$

$$H_3 c_2^3 = \frac{\sqrt{s+2g}\,(\sqrt{s}-\sqrt{c+t})}{c_1^4}.$$

When the siding road, after crossing the second or third main-line rail, is straight, we have:

CASE I.—*Main Line Straight.*

$\alpha.$ $s \gtreqless g.$

$$c_1^2 = c_1^3 = c_1^4 = c_2^3 = c_2^4,$$

$$H_2 c_1^2 = \frac{2\sqrt{g}\,(\sqrt{g}-\sqrt{c+t})}{c_1^2},$$

$$H_3 c_1^3 = H_4 c_2^4 = \frac{s+2g-2\sqrt{g(c+t)}}{c_1^2},$$

$$H_4 c_1^4 = \frac{s+3g-2\sqrt{g(c+t)}}{c_1^2},$$

$$H_3 c_2^3 = \frac{s+g-2\sqrt{g(c+t)}}{c_1^2}.$$

$\beta.$ $s < g.$

$$c_1^2 = c_1^3 = c_1^4 = c_2^3 = c_2^4,$$

$$H_2 c_1^2 = \frac{s+g-2\sqrt{s(c+t)}}{c_2^3},$$

$$H_3 c_1^3 = H_4 c_2^4 = \frac{2s+g-2\sqrt{s(c+t)}}{c_2^3},$$

$$H_4 c_1^4 = \frac{2(s+g)-2\sqrt{s(c+t)}}{c_2^3},$$

$$H_3 c_2^3 = \frac{2s-2\sqrt{s(c+t)}}{c_2^3}.$$

CASE II.—*Main Line Curved, Siding Roads Curving in the same direction.*

α. $s \gtreqless g$.

When the angle c_1^2 is given,

$$c_1^3 = c_2^4 = \sqrt{c_1^2 - \frac{2s}{R}},$$

$$c_1^4 = \sqrt{c_1^2 - \frac{2(s+g)}{R}},$$

$$c_2^3 = \sqrt{c_1^2 - \frac{2(s-g)}{R}}.$$

When the angle c_1^4 is given,

$$c_1^2 = \sqrt{c_1^4 + \frac{2(s+g)}{R}},$$

$$c_1^3 = c_2^4 = \sqrt{c_1^4 + \frac{2g}{R}},$$

$$c_2^3 = \sqrt{c_1^4 + \frac{4g}{R}},$$

$$H_2 c_1^2 = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_1^2},$$

$$H_3 c_1^3 = H_4 c_2^4 = R(c_1^2 - c_1^3) + \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_1^2},$$

$$H_4 c_1^4 = R(c_1^2 - c_1^4) + \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_1^2},$$

$$H_3 c_2^3 = R(c_1^2 - c_2^3) + \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_1^2}.$$

β. $s < g$.

When the angle c_2^3 is given,

$$c_1^2 = \sqrt{c_2^2 - \frac{2(g-s)}{R}},$$

$$c_1^3 = c_2^4 = \sqrt{c_2^2 - \frac{2g}{R}},$$

$$c_1^4 = \sqrt{c_2^2 - \frac{4g}{R}}.$$

When c_1^4 is given,

$$c_2^2 = \sqrt{c_1^4 + \frac{4g}{R}},$$

$$c_1^2 = \sqrt{c_1^4 + \frac{2(s+g)}{R}},$$

$$c_1^3 = c_2^4 = \sqrt{c_1^4 + \frac{2g}{R}},$$

$$H_2 c_1^2 = R(c_2^2 - c_1^2) + \frac{2\sqrt{s}(\sqrt{s} - \sqrt{c+t})}{c_2^2},$$

$$H_3 c_1^3 = H_4 c_2^4 = R(c_2^2 - c_1^2) + \frac{2\sqrt{s}(\sqrt{s} - \sqrt{c+t})}{c_2^2},$$

$$H_4 c_1^4 = R(c_2^2 - c_1^4) + \frac{2\sqrt{s}(\sqrt{s} - \sqrt{c+t})}{c_2^2},$$

$$H_3 c_2^3 = \frac{2\sqrt{s}(\sqrt{s} - \sqrt{c+t})}{c_2^2}.$$

CASE III.—*Main Roads Curved, Siding Road Curving in the opposite direction.*

a. $s \gtreqless g$.

When the angle c_1^2 is given,

$$c_1^3 = c_2^4 = \sqrt{c_1^2 + \frac{2s}{R}},$$

$$c_1^4 = \sqrt{c_1^2 + \frac{2(s+g)}{R}},$$

$$c_2^3 = \sqrt{c_1^2 + \frac{2(s-g)}{R}}.$$

When the angle c_1^4 is given,

$$c_1^2 = \sqrt{c_1^{4\,2} - \frac{2(s+g)}{R}},$$

$$c_1^3 = c_2^4 = \sqrt{c_1^{4\,2} - \frac{2g}{R}},$$

$$c_2^3 = \sqrt{c_1^{4\,2} - \frac{4g}{R}},$$

$$H_2 c_1^2 = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_1^2},$$

$$H_3 c_1^3 = H_4 c_2^4 = R(c_1^3 - c_1^2) + \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_1^2},$$

$$H_4 c_1^4 = R(c_1^4 - c_1^2) + \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_1^2},$$

$$H_3 c_2^3 = R(c_2^3 - c_1^2) + \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_1^2}.$$

β. $s < g$.

When the angle c_2^3 is given,

$$c_1^2 = \sqrt{c_2^{3\,2} + \frac{2(g-s)}{R}},$$

$$c_1^3 = c_2^4 = \sqrt{c_2^{3\,2} + \frac{2g}{R}},$$

$$c_1^4 = \sqrt{c_2^{3\,2} + \frac{4g}{R}}.$$

When c_1^4 is given,

$$c_2^3 = \sqrt{c_1^{4\,2} - \frac{4g}{R}},$$

$$c_1^2 = \sqrt{c_1^{4\,2} - \frac{2(s+g)}{R}},$$

$$c_1^3 = c_2^4 = \sqrt{c_1^{4\,2} - \frac{2g}{R}},$$

$$H_2 c_1^2 = R(c_2^2 - c_1^2) + \frac{2\sqrt{s}(\sqrt{s} - \sqrt{c+t})}{c_2^2},$$

$$H_3 c_1^3 = H_3 c_2^4 = R(c_2^3 - c_1^3) + \frac{2\sqrt{s}(\sqrt{s} - \sqrt{c+t})}{c_2^3},$$

$$H_4 c_1^4 = R(c_2^4 - c_1^4) + \frac{2\sqrt{s}(\sqrt{s} - \sqrt{c+t})}{c_2^4}.$$

Division III.—*Numerical evaluations of the coefficients in the formulæ for 4 ft. 8½ in. narrow-gauge roads, and 7 ft. broad-gauge roads, and application of the results to simple cases of common occurrence.*

For the narrow-gauge roads we may put $g = 5$ ft., $s = 6$ ft., and for the broad gauge $g = 7$ ft. 3 in., $s = 6$ ft., whatever may be the thickness of the rail.

If we put the clearance $c = 2$ in., and the thickness of the rail $t = 2\frac{1}{2}$ in., the sum of the two will be $4\frac{1}{2}$ in. Since the clearance c may vary from $1\frac{1}{2}$ in. to $2\frac{1}{2}$ in., the numerical evaluations, which are obtained by making $c + t = 4\frac{1}{2}$ in., will hold good for rails whose thickness t varies from 2 in. to 3 in. if at the heels of the switches the sum of the clearance and the thickness of the rail is made equal to $4\frac{1}{2}$ in.

Narrow-gauge Roads.

$$2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t}) = 7{\cdot}26 \text{ ft.} = 7\tfrac{1}{4} \text{ ft.}$$

$$2\sqrt{g}\,(\sqrt{g} - \sqrt{c+e}) = 8{\cdot}02 \text{ ft.} = 8 \text{ ft.}$$

$$2\sqrt{g}\,(\sqrt{c+t} - \sqrt{c+e}) = {\cdot}76 \text{ ft.} = \tfrac{3}{4} \text{ ft.}$$

$$\sqrt{\frac{s+g}{g}} = 1{\cdot}48 = 1\tfrac{1}{2}.$$

$$\sqrt{\frac{s+2g}{g}} = 1{\cdot}79 = 1\tfrac{3}{4}.$$

$$\sqrt{\frac{s}{g}} = 1{\cdot}09 = 1\tfrac{1}{10}.$$

$$2\sqrt{g}\,(\sqrt{s+g}-\sqrt{c+t}) = 12{\cdot}1 \text{ ft.} = 12 \text{ ft.}$$

$$2\sqrt{s+g}\,(\sqrt{s+g}-\sqrt{c+t}) = 17{\cdot}92 \text{ ft.} = 18 \text{ ft.}$$

$$2\sqrt{g}\,(\sqrt{s+2g}-\sqrt{c+t}) = 15{\cdot}14 \text{ ft.} = 15\tfrac{1}{8} \text{ ft.}$$

$$2\sqrt{s+2g}\,(\sqrt{s+2g}-\sqrt{c+t}) = 27{\cdot}1 \text{ ft.} = 27 \text{ ft.}$$

$$2\sqrt{g}\,(\sqrt{s}-\sqrt{c+t}) = 8{\cdot}22 \text{ ft.} = 8\tfrac{1}{4} \text{ ft.}$$

$$2\sqrt{s}\,(\sqrt{s}-\sqrt{c+t}) = 9 \text{ ft.} = 9 \text{ ft.}$$

$$2\sqrt{g}\,(\sqrt{g}-\sqrt{2(c+t)}) = 6{\cdot}12 \text{ ft.} = 6\tfrac{1}{10} \text{ ft.}$$

$$2\sqrt{g}\,(\sqrt{2(s+g)}-2\sqrt{c+t}) = 15{\cdot}5 \text{ ft.} = 15\tfrac{1}{2} \text{ ft.}$$

$$2\sqrt{g}\,(\sqrt{2(s+g)}-2\sqrt{g}) = {\cdot}98 \text{ ft.} = 1 \text{ ft.}$$

$$2\sqrt{s+2g}\,(\sqrt{g}-\sqrt{c+t}) = 13 \text{ ft.} = 13 \text{ ft.}$$

$$2\sqrt{s+2g}\,(\sqrt{s+g}-\sqrt{c+t}) = 21{\cdot}62 \text{ ft.} = 21\tfrac{3}{5} \text{ ft.}$$

$$2\sqrt{s+2g}\,(\sqrt{s}-\sqrt{c+t}) = 14{\cdot}7 \text{ ft.} = 14\tfrac{3}{4} \text{ ft.}$$

$$(n+1)s+(n+3)g-4\sqrt{g(c+t)} = (11n+15{\cdot}52)\text{ ft.} = (11n+15\tfrac{1}{2})\text{ ft.}$$

$$\sqrt{\frac{g}{s+2g}} = {\cdot}56 = \tfrac{4}{7}.$$

$$\sqrt{\frac{s+g}{s+2g}} = {\cdot}83 = \tfrac{4}{5}.$$

$$\sqrt{\frac{s}{s+2g}} = {\cdot}61 = \tfrac{3}{5}.$$

$$s+2g-2\sqrt{g(c+t)} = 13{\cdot}26 \text{ ft.} = 13\tfrac{1}{4} \text{ ft.}$$

$$s+3g-2\sqrt{g(c+t)} = 18{\cdot}26 \text{ ft.} = 18\tfrac{1}{4} \text{ ft.}$$

$$s+g-2\sqrt{g(c+t)} = 8{\cdot}26 \text{ ft.} = 8\tfrac{1}{4} \text{ ft.}$$

Broad-gauge Roads.

$$2\sqrt{g}\,(\sqrt{g}-\sqrt{c+t}) = 11{\cdot}20 \text{ ft.} = 11\tfrac{1}{5} \text{ ft.}$$

$$2\sqrt{g}\,(\sqrt{g}-\sqrt{c+e}) = 12{\cdot}10 \text{ ft.} = 12\tfrac{1}{10} \text{ ft.}$$

$$2\sqrt{g}\,(\sqrt{c+t}-\sqrt{c+e}) = {\cdot}9 \text{ ft.} = \tfrac{9}{10} \text{ ft.}$$

$$\sqrt{\frac{s+g}{g}} = 1{\cdot}37 = 1\tfrac{2}{5}.$$

$$\sqrt{\frac{s+2g}{g}} = 1{\cdot}69 = 1\tfrac{7}{10}.$$

$$\sqrt{\frac{s}{g}} = {\cdot}9 = \tfrac{9}{10}.$$

$$2\sqrt{g}\left(\sqrt{s+g} - \sqrt{c+t}\right) = 16{\cdot}3 \text{ ft.} = 16\tfrac{3}{10} \text{ ft.}$$

$$2\sqrt{s+g}\left(\sqrt{s+g} - \sqrt{c+t}\right) = 22{\cdot}04 \text{ ft.} = 22 \text{ ft.}$$

$$2\sqrt{g}\left(\sqrt{s+2g} - \sqrt{c+t}\right) = 21{\cdot}08 \text{ ft.} = 21\tfrac{1}{10} \text{ ft.}$$

$$2\sqrt{s+2g}\left(\sqrt{s+2g} - \sqrt{c+t}\right) = 35{\cdot}46 \text{ ft.} = 35\tfrac{1}{2} \text{ ft.}$$

$$2\sqrt{g}\left(\sqrt{s} - \sqrt{c+t}\right) = 9{\cdot}88 \text{ ft.} = 9\tfrac{9}{10} \text{ ft.}$$

$$2\sqrt{s}\left(\sqrt{s} - \sqrt{c+t}\right) = 9 \text{ ft.} = 9 \text{ ft.}$$

$$2\sqrt{g}\left(\sqrt{g} - \sqrt{2(c+t)}\right) = 9{\cdot}84 \text{ ft.} = 9\tfrac{4}{5} \text{ ft.}$$

$$2\sqrt{s}\left(\sqrt{2(s+g)} - 2\sqrt{c+t}\right) = 19{\cdot}22 \text{ ft.} = 19\tfrac{1}{4} \text{ ft.}$$

$$2\sqrt{s}\left(\sqrt{2(s+g)} - 2\sqrt{s}\right) = 1{\cdot}20 \text{ ft.} = 1\tfrac{1}{5} \text{ ft.}$$

$$2\sqrt{s+2g}\left(\sqrt{g} - \sqrt{c+t}\right) = 18{\cdot}84 \text{ ft.} = 18\tfrac{4}{5} \text{ ft.}$$

$$2\sqrt{s+2g}\left(\sqrt{g+s} - \sqrt{c+t}\right) = 27{\cdot}42 \text{ ft.} = 27\tfrac{2}{5} \text{ ft.}$$

$$2\sqrt{s+2g}\left(\sqrt{s} - \sqrt{c+t}\right) = 16{\cdot}64 \text{ ft.} = 16\tfrac{3}{5} \text{ ft.}$$

$$(n+3)s + (n+1)g - 4\sqrt{s(c+t)} = (13\tfrac{1}{4}n + 19\tfrac{1}{4}) \text{ ft.}$$

$$\sqrt{\frac{g}{s+2g}} = {\cdot}58 = \tfrac{3}{5}.$$

$$\sqrt{\frac{s+g}{s+2g}} = {\cdot}8 = \tfrac{4}{5}.$$

$$\sqrt{\frac{s}{s+2g}} = {\cdot}55 = \tfrac{11}{20}.$$

$$s + g - 2\sqrt{s(c+t)} = 10\tfrac{1}{4} \text{ ft.}$$

$$2s + g - 2\sqrt{s(c+t)} = 16\tfrac{1}{4} \text{ ft.}$$

$$2(s+g) - 2\sqrt{s(c+t)} = 23\tfrac{1}{2} \text{ ft.}$$

$$2s - 2\sqrt{s(c+t)} = 9 \text{ ft.}$$

Since the numerical values of c_1^2, c_1^3, &c., are seldom the exact values of the circular measures of those angles, but only approximations sufficiently near to the exact value, so far as the mere lead of the crossing is concerned; the positions of the crossings, determined from formulæ in terms of the leads of the crossings, will seldom be exactly the same as those determined from formulæ in terms of the radius. This is a matter of no consequence in the case of single-line junctions. In the case of double-line junctions, the position of each crossing must be determined from formulæ in terms of the circular measure of the same angle. When the positions of the crossings are so determined, suppose in terms of c_1^2 or c_1^4, the branch road will form a continuous true curve, whose radius will be $\frac{2g}{c_1^2}$ or $\frac{2(s+2g)}{c_1^4}$, if the main line be straight, and $\frac{2Rg}{Rc_1^2 \pm 2g}$ or $\frac{2R(s+2g)}{Rc_1^4 \pm (s+2g)}$, if the main line be curved, but which may not exactly agree with the radius, from which c_1^2 or c_1^4 may have been determined. These remarks do not apply when the centre line of both roads has been staked out.

Practical Numerical Rules.

The simple general rules given here will enable the foreman platelayer to order his crossings from the shop for any work which he may have to do without a special plan, if he have no crossings in store; or, if he have part of what he requires in store, will enable him to determine the leads of the rest to suit those he may have, without any actual preliminary setting out on the ground, so far as the special cases selected are concerned.

They will not only enable him to do this, but also to set out the positions of the crossings on the ground, without knowing either the radius of the main line or the siding road.

In order to make the rules as concise as possible, the symbols c_1^2, c_2^3, &c., will still in some cases be used to specify the position of the point of the crossing to which the rule relates. It is therefore absolutely necessary to master the meaning of these symbols before attempting to use some of the rules. A careful study of the following explanation and of the figures will make their meaning clear, and fix it firmly in the memory of any man fit to be a foreman platelayer, however unused he may be to bookwork.

The figures at the bottom of the letter *c* refer to the rail of the siding road, those on the top to the rail of the main line.

In single junctions, that rail of the siding road which cuts the rail of the main road is called the first rail of the siding road.

In double junctions, that rail of the siding road which cuts three rails of the main road is called the first; that which cuts two only, the second; that which cuts one only, the third; and that which cuts none, the fourth.

The main-road rails are numbered to match. Thus, the switch of the first rail of the siding road touches the first rail of the main road, the switch of the second rail of the siding road touches the second rail of the main road, and so on.

In the case of through roads, where the siding road may cross several main-line roads, the rails of the main roads are numbered in order from the first. Even the rails of the last road, between which and the first communication is established, are numbered in the same order. Thus, if there be one intervening road, the rails touched by the first siding rail will be the first and fifth, and those touched by the second siding rail, the second and the sixth.

The platelayer must also fix firmly in his memory the meaning of the symbols H_1, H_2, H_3, H_4, &c., which denote either the actual positions of the heels of the switches of the first, second, third, &c., rails, or the points, where a line drawn through the heels of the switches at right angles to the main line cuts the rails of the main roads. Thus, in the case of a cross-over road,

two of the symbols will be at the actual heels of the switches and the remaining two at the points, where a line drawn through the two first at right angles to the centre line of the railway cuts the other two rails. The symbols H'_1, H'_2, H'_3, H'_4, have the same meaning with reference to the heels of the pair of opposite switches, H'_1 being on the same main-line rail as H_1, and so on.

If there be more than four rails, as in the case of through roads, the symbols for the succeeding rails will be H_5, H_6, &c.

Having fully mastered the meaning of these symbols, an intelligent foreman will be able to use not only the rules given here, but the formulæ in Division II.

Since the values of s g, c, t and e, are expressed in feet, where R the radius of the main line occurs its value must be expressed in feet and not in chains.

The point of the crossing, referred to in the following rules, is not the actual point of the crossing, but the point where the centre lines of the rails, which compose the crossing, intersect, if it consist of two of the ordinary main-road rails meeting at the proper angle. In order to mark its position on the crossing, a straight-edge or string must be held close to the inner edge of the rails; the points, where the line of the inner edge of either rail cuts the outer edge of the other, being marked in this way on the crossing, a line joining these two marks will pass through the point of intersection of the centre lines of the two rails.

If the crossing be a solid box, lines must be marked on it parallel to each face of the V, at a distance from the face equal to half the thickness of the rail. The point where these lines intersect is the point required.

The values of c_1^2, c_2^3, &c., are expressed by a fraction, whose numerator is one and denominator a whole number, if the crossing be easy. If the crossing be sharp, the denominator is either a whole number, or lies half-way between two consecutive whole numbers. Thus their values range according to the following series:

$$\ldots\ldots \frac{1}{6}, \frac{1}{6\frac{1}{2}}, \frac{1}{7}, \frac{1}{7\frac{1}{2}}, \frac{1}{8}, \frac{1}{9}.$$

The numbers 6, 6½, 7, 7½, 8 and 9 are called the denominators of the fractions, and represent the length of a crossing in feet, whose width at the open end is one foot between the centre lines of the rails.

The stamp marks on the crossing are thus written:

1 in 6, 1 in 6½, 1 in 7, 1 in 7½, 1 in 8, 1 in 9.

The denominator is generally called the lead of the crossing, and will be so called in the following rules.

Rules for 4 ft. 8½ in. Narrow Gauge.

Single-line Junctions.

Rule I.—When the main line is straight, the radius of the branch line in feet is equal to ten times the square of the lead of the crossing.

Thus, if the crossing be 1 in 8,

$$\text{Radius of branch line} = 10 \times 8^2 \text{ ft.} = 640 \text{ ft.}$$

Rule II.—When the main line is curved and the branch road curves in the same direction, the radius of the branch road in feet is equal to the product of ten times the radius of the main road in feet, multiplied by the square of the lead of the crossing, divided by the sum of the radius of the main road in feet, plus ten times the square of the lead of the crossing. Thus, if the crossing be 1 in 8 and the radius of the main road 20 chains, we have

$$\text{Radius of branch road} = \frac{10 \times 1320 \times 8^2}{1320 + 10 \times 8^2} \text{ ft.} = 432 \text{ ft.}$$

Rule III.—When the main line is curved and the branch road curves in the opposite direction, the radius of the branch road in feet is equal to the product of ten times the radius of the main road in feet, multiplied by the square of the lead of the crossing, divided by the radius of the main road in feet, minus ten times the square of the lead of the crossing.

Thus, if the lead of the crossing be 1 in 8 and the radius of the main line 20 chains,

$$\text{Radius of branch line} = \frac{10 \times 1320 \times 8^2}{1320 - 10 \times 8^2} \text{ ft.} = 1242 \text{ ft.}$$

Rule IV.—The length of the splayed part of a fast point in feet is equal to three-quarters of the lead of the crossing; and the distance of the point of the fast point from the point of the crossing is equal to eight times the lead of the crossing, whether the main line be straight or curved, so long as the clearance at the point end of the fast point is equal to 2 inches, and the sum of the clearance plus the thickness of the rail is equal to $4\frac{1}{2}$ in. at the other end.

Thus, if the crossing be 1 in 8,

Length of splayed part $= 8 \times \frac{3}{4}$ ft. $= 6$ ft.

Distance of point from point of crossing $= 8 \times 8$ ft. $= 64$ ft.

Rule V.—The distance from the heel of the switch to the point of the crossing in feet, whether the main line be straight or curved, is seven and one-quarter times the lead of the crossing, if the sum of the clearance plus the thickness of the rail at the heel of the switch is equal to $4\frac{1}{2}$ in.

Thus, if the lead of the crossing be 1 in 8,

Distance of point of crossing from heel of switch $= 8 \times 7\frac{1}{4}$ ft. $= 58$ ft.

Double-line Junctions.

The first four of the rules for single-line junctions apply exactly to the case of the double-line junctions, the lead of the crossing c_1^2 of the second main-line rail by the first rail of the siding being the one referred to in those rules.

Whether the main line be straight or curved, so long as the clearance at the heels of the switches is equal to $4\frac{1}{2}$ in., we have:

Rule I.—The lead of the crossing c_1^3 is equal to that of the crossing c_2^4, and to two-thirds of that of the crossing c_1^2; the lead of the crossing c_1^4 is equal to four-sevenths of that of the crossing c_1^2; the lead of the crossing c_2^3 is equal to ten-elevenths of that of c_1^2; and the lead of c_3^4 is equal to that of c_1^2.

Thus, if the crossing c_1^2 be 1 in 8,

Lead of the crossing c_1^3 or $c_2^4 = 8 \times \frac{2}{3} = 5\frac{1}{2}$.

" " $c_1^4 = 8 \times \frac{4}{7} = 4\frac{1}{2}$.

" " $c_2^3 = 8 \times \frac{10}{11} = 7\frac{1}{2}$.

" " $c_3^4 = 8$.

Rule II.—The distance in feet from the line of the heels of the switches of the crossing c_1^2 is equal to that of the crossing c_3^4, and to seven and one-quarter times the lead of the crossing c_1^2; the distance of the crossing c_1^3 is equal to that of the crossing c_2^4, and to twelve times the lead of the crossing c_1^2; the distance of the crossing c_1^4 is equal to fifteen and one-fifth times the lead of the crossing c_1^2; and that of the crossing c_2^3 to eight and one-quarter times the lead of the crossing c_1^2.

Thus, if the crossing c_1^2 be 1 in 8,

Distance of crossing c_1^3 or c_2^4 from heel of switch
$= 7\frac{1}{4} \times 8$ ft. $= 58$ ft.

Distance of crossing c_1^3 or c_2^4 from heel of switch
$= 12 \times 8$ ft. $= 96$ ft.

Distance of crossing c_1^4 from heel of switch
$= 15\frac{1}{5} \times 8$ ft. $= 121$ ft. 6 in.

Distance of crossing c_2^3 from heel of switch
$= 8\frac{1}{4} \times 8$ ft. $= 66$ ft.

Three Throws.

It would be easy to give numerical rules for the formulæ in Division II., but it is not worth while to do so, except in the two special cases already discussed, *viz.* that in which $c_{r_1} = c_{r_2}$, when the branch roads curve in opposite directions, and that in which $c_{r_2} = c_{r_1}\sqrt{2}$, when the branch roads curve in the same direction.

These are the simplest forms of three throws, and therefore ought always to be used, unless a special three throw be necessary. When this is the case, the three throw ought to be set out on the ground by the engineer in charge of the work.

CASE I.—*Branch Roads curving in opposite directions, and crossing the Main-line rails at the same angle.*

Whether the main line be straight or curved:

Rule I.—When the clearance at the heels of the switches between contiguous rails is the same for both roads, the heels of the switches are opposite one another.

Rule II.—The lead of the middle crossing c_m of one branch rail by another is equal to five-sevenths of the lead of either of the other crossings.

Thus, if either main line crossing be 1 in 8,

$$\text{Lead of middle crossing } c_m = 8 \times \tfrac{5}{7} = 6.$$

Rule III.—The point of the crossing c_m is exactly in the middle of the main road.

Rule IV.—The distance from the heels of the switches in feet of the crossing c_{r_1} is equal to that of the crossing c_{r_2}, and to seven and one-quarter times the lead of the crossing; that of the middle crossing c_m is equal to four and two-fifths of the lead of either of the crossings c_{r_1} and c_{r_2}. The sum of the thickness plus the clearance between contiguous rails being $4\frac{1}{2}$ in.

Thus, if the main-line crossings be 1 in 8,

$$\text{Distance of } c_{r_1} \text{ or } c_{r_2} \text{ from heel of switch} = 8 \times 7\tfrac{1}{4} \text{ ft.} = 58 \text{ ft.}$$

$$\text{,, } c_m \text{ or } c_{r_2} \text{ ,, ,,} = 8 \times 4\tfrac{2}{5} \text{ ft.} = 35 \text{ ft.}$$

CASE II.—*Branch Roads curving in the same direction, the lead of the easier Main-line crossing being equal to seven-fifths of that of the sharper crossing.*

Whether the main line be straight or curved:

Rule I.—When the clearance at the heels of the switches between contiguous rails is the same for both roads, the heels of the switches are opposite one another.

Rule II.—The lead of the middle crossing c_m is equal to that of the easier crossing c_{r_1} and to seven-fifths of that of the sharper crossing c_{r_2}.

Thus, if the crossing c_{r_2} be 1 in 8,

$$\text{Lead of crossing } c_m = 8 \times \tfrac{7}{5} = 11.$$

Rule III.—The distance from the heel of the switch of the crossing c_{r_1} is equal to ten and one-fifth times the lead of the crossing c_{r_2}; that of the crossing c_{r_2} to six and one-tenth times

that of c_{r_2}; and the distance of c_m to that of c_{r_1}. The sum of the thickness plus the clearance between contiguous rails being $4\frac{1}{2}$ in.

Thus, if the crossing c_{r_2} be 1 in 8,

Distance of c_{r_1} from heel of switch $= 8 \times 10\frac{1}{8}$ ft. $= 81$ ft. 6 in.

" c_{r_2} " " $= 8 \times 6\frac{1}{10}$ ft. $= 49$ ft.

" c_m " " $= 8 \times 10\frac{1}{8}$ ft. $= 81$ ft. 6 in.

Cross-over Roads.

Whether the main line be straight or curved, if the clearance at the heels of the switches plus the thickness of the rail be equal to $4\frac{1}{2}$ in.:

Rule I.—The lead of one crossing is equal to that of the other and to the interval between the heels of the switches expressed in feet, divided by fifteen and a half feet.

Thus, if the interval be 124 ft.,

$$\text{The lead of each crossing} = \frac{124}{15\frac{1}{2}} = 8.$$

Rule II.—The interval between the heels of the switches is equal to fifteen and one-half times the lead of either crossing.

Thus, if the crossing be 1 in 8,

Interval between the heels of the switches $= 15\frac{1}{2} \times 8$ ft. $= 124$ ft.

Rule III.—The distance in feet of either crossing from the heel of the switch on the same rail as the crossing is equal to seven and one-quarter times the lead of the crossing, and the interval between the points of the crossings is equal to the lead of either crossing.

Thus, if the crossing be 1 in 8,

Distance of either crossing from the heel of the nearest switch $= 8 \times 7\frac{1}{4}$ ft. $= 58$ ft.

Interval between points of crossings $=$ 8 ft.

Through Roads.

When there are n intervening roads:

If the main line is straight, and the clearance at the heels of the switches plus the thickness of the rail is $4\frac{1}{2}$ in.; and the

through road consists of two reverse curves, separated by an interval of straight, which extends from extreme crossing to extreme crossing:

Rule I.—The leads of the crossings are equal, and each is equal to the interval between the heels of the switches in feet, divided by eleven times the number of the intervening roads plus fifteen and one-half feet.

Thus, if there be three intervening roads, and the interval be 388 ft.,

$$\text{Lead of the crossing} = \frac{388}{48\frac{1}{2}} = 8.$$

Rule II.—The interval between the heels of the switches in feet is equal to the lead of the crossing, multiplied by eleven times the number of intervening roads plus fifteen and one-half.

Thus, if the lead of the crossing be 1 in 8, and there are three intervening roads,

$$\text{Interval between heels of switches} = 48\tfrac{1}{2} \times 8 \text{ ft.} = 388 \text{ ft.}$$

Rule III.—The distance in feet from the heel of the switch of the crossing c_1^2 is equal to that of the crossing c_2^{2n+3}, and is equal to seven and one-quarter times the lead of the crossing. The distance of the crossing c_2^3 is equal to that of the crossing c_1^{2n+3}, and is equal to eight and one-quarter times the lead of the crossing. Lines drawn from c_1^2 to c_1^{2n+3}, and from c_2^3 to c_2^{2n+3}, will mark out the positions of the rest of the crossings.

Thus, if the lead of the crossing be 1 in 8,

$$\text{Distance of the crossing } c_1^2 \text{ or } c_2^{2n+3} \text{ from the heel} = 7\tfrac{1}{4} \times 8 \text{ ft.} = 58 \text{ ft.}$$
$$\text{„ „ } c_2^3 \text{ or } c_1^{2n+3} \text{ „ „} = 8\tfrac{1}{4} \times 8 \text{ ft.} = 66 \text{ ft.}$$

When the main line is curved, or the through road consists of two reverse curves meeting at a common tangent, the formulæ in Division I. must be used.

When there is only one intervening road, and the reverse curves meet at a common tangent:

Whether the main line be straight or curved, if the clearance plus the thickness of the rail be $4\frac{1}{2}$ in.:

Rule I.—When the interval in feet between the heels of

the switches is given, the lead of the crossing c_1^2 or c_2^5 is equal to this interval divided by twenty-four.

Thus, if the interval be 192 ft.,

$$\text{Lead of the crossing } c_1^2 \text{ or } c_2^5 = \frac{192}{24} = 8.$$

Rule II.—The interval between the heels of the switches is equal to twenty-four times the lead of the crossing c_1^2 or c_2^5.

Thus, if the crossing c_1^2 or c_2^5 be 1 in 8,

$$\text{Interval between the heels of the switches} = 24 \times 8 \text{ ft.} = 192 \text{ ft.}$$

Rule III.—The lead of the crossing c_1^3 is equal to that of the crossing c_2^4, and is equal to two-thirds of the lead of the crossing c_1^2 or c_2^5; that of the crossing c_1^4 is equal to that of the crossing c_2^3, and is equal to ten-elevenths of the lead of the crossing c_1^2 or c_2^5.

Thus, if the lead of c_1^2 or c_2^5 be 1 in 8,

$$\text{Lead of crossings } c_1^3 \text{ and } c_2^4 = 8 \times \tfrac{2}{3} = 5\tfrac{1}{3}.$$
$$\text{,, ,, } c_1^4 \text{ ,, } c_2^3 = 8 \times \tfrac{10}{11} = 7\tfrac{1}{11}.$$

Rule IV.—The distance in feet from the line of the heels of the nearest switches of the crossing c_1^2 is equal to that of the crossing c_2^5, and to seven and one-quarter times the lead of the crossing c_1^3 or c_2^5; the distance of the crossing c_1^3 is equal to that of the crossing c_2^4, and to twelve times the lead of the crossing c_1^2 or c_2^5; the distance of the crossing c_1^4 is equal to that of the crossing c_2^3, and to eight and one-quarter times the lead of the crossing c_1^2 or c_2^5.

Thus, if the crossing c_1^2 or c_2^5 be 1 in 8,

$$\text{Distance of } c_1^3 \text{ or } c_2^5 \text{ from the heels} = 7\tfrac{1}{4} \times 8 \text{ ft.} = 58 \text{ ft.}$$
$$\text{,, } c_1^3 \text{ ,, } c_2^4 \text{ ,, ,,} = 12 \times 8 \text{ ft.} = 96 \text{ ft.}$$
$$\text{,, } c_2^3 \text{ ,, } c_1^4 \text{ ,, ,,} = 8\tfrac{1}{4} \times 8 \text{ ft.} = 66 \text{ ft.}$$

Siding Roads leading from one Main-line Road across the other: Siding Road a continuous curve till it leaves the Main Line.

Whether the main line be straight or curved, if the sum of the clearance plus the thickness of the rail be $4\frac{1}{2}$ in.:

If the lead of the crossing c_1^2 be given:

Rule I.—See Rule I., Double-line Junctions.

Rule II.—See Rule II., Double-line Junctions.

If the lead of the crossing c_1^4 be given:

Rule III.—The lead of the crossing c_1^2 is equal to seven-fourths of that of c_1^4; the lead of c_1^3 is equal to that of c_2^4, and to five-fourth times that of c_1^4; and the lead of c_2^3 is equal to five-thirds the lead of c_1^4.

Thus, if the crossing c_1^4 be 1 in $4\frac{1}{2}$,

Lead of crossing c_1^2 $= 4\frac{1}{2} \times \frac{7}{4} = 8.$

„ „ c_1^3 or c_2^4 $= 4\frac{1}{2} \times \frac{5}{4} = 5\frac{1}{2}.$

„ „ c_2^3 $= 4\frac{1}{2} \times \frac{5}{3} = 7\frac{1}{2}.$

Rule IV.—The distance from the line of the heel of the switch in feet of the crossing c_1^2 is equal to thirteen times the lead of the crossing c_1^4; the distance of c_1^3 is equal to that of c_2^4, and to twenty-one and three-fifths of the lead of c_1^4; the distance of c_1^4 is equal to twenty-seven times the lead of c_1^4; and the distance of c_2^3 to fourteen and three-quarter times the lead of c_1^4.

Thus, if the crossing c_1^4 be 1 in $4\frac{1}{2}$,

Distance of crossing c_1^2 from heel $= 13 \times 4\frac{1}{2}$ ft. $= 58$ ft. 6 in.

„ „ c_1^3 or c_2^4 „ $= 21\frac{3}{5} \times 4\frac{1}{2}$ ft. $= 97$ ft. 2 in.

„ „ c_1^4 „ $= 27 \times 4\frac{1}{2}$ ft. $= 121$ ft. 6 in.

„ „ c_2^3 „ $= 14\frac{3}{4} \times 4\frac{1}{2}$ ft. $= 66$ ft. 6 in.

When the main line is straight, and the siding road, after crossing the second main-line road, continues straight, if the sum of the clearance plus the thickness of the rail be $4\frac{1}{2}$ in., we have:

Rule I.—The leads of the crossings are all equal to each other.

Rule II.—The distance in feet from the line of the heel of the switches of the crossing c_1^2 is equal to seven and one-quarter times the lead of the crossing; the distance of c_1^3 is equal to that of c_2^4, and to thirteen and one-quarter times the lead; the distance of c_1^4 to eighteen and one-quarter times the lead; and that of c_2^3 to eight and one-quarter times the lead.

Thus, if the crossings be 1 in 8,

Distance of c_1^2 from heel = $7\frac{1}{4} \times 8$ ft. = 58 ft.
" c_1^3 or c_2^4 " = $13\frac{1}{4} \times 8$ ft. = 106 ft.
" c_1^4 " = $18\frac{1}{4} \times 8$ ft. = 146 ft.
" c_2^3 " = $8\frac{1}{4} \times 8$ ft. = 66 ft.

When the main line is curved, the formulæ are too complicated to be reduced to simple written rules.

The simplest way of putting in such a siding road is to use a continuous curve as far as the crossing c_1^4. The required direction of the siding on leaving the main line will give the lead of this crossing.

Rules for 7 ft. Broad Gauge.

Single-line Junctions.

Rule I.—When the main line is straight, the radius of the branch line in feet is equal to fourteen and one-half times the square of the lead of the crossing.

Thus, if the crossing be 1 in 8,

$$\text{Radius of the branch line} = 14\tfrac{1}{2} \times 8^2 = 928 \text{ ft.}$$

Rule II.—When the main line is curved and the branch road curves in the same direction, the radius of the branch road in feet is equal to the product of fourteen and one-half times the radius of the main road in feet, multiplied by the square of the lead of the crossing, divided by the sum of the radius of the main road in feet plus fourteen and one-half times the square of the lead of the crossing.

Thus, if the crossing be 1 in 8, and the radius of the main road 20 chains, we have

$$\text{Radius of branch road} = \frac{14\frac{1}{2} \times 1320 \times 8^2}{1320 + 14\frac{1}{2} \times 8^2} = 545 \text{ ft.}$$

Rule III.—When the main line is curved and the branch road curves in the opposite direction, the radius of the branch road in feet is equal to the product of fourteen and one-half times the radius of the main road in feet, multiplied by the

square of the lead of the crossing, divided by the radius of the main road in feet minus fourteen and one-half times the square of the lead of the crossing.

Thus, if the crossing be 1 in 8, and the radius of the main line 20 chains, we have

$$\text{Radius of branch line} = \frac{14\frac{1}{2} \times 1320 \times 8^2}{1320 - 14\frac{1}{2} \times 8^2} = 3023 \text{ ft.}$$

Rule IV.—The length of the splayed part of a fast point in feet is equal to nine-tenths of the lead of the crossing; and the distance of the point of the fast point from the point of the crossing is equal to twelve and one-tenth times the lead of the crossing, whether the main line be straight or curved, so long as the clearance at the point end of the fast point is equal to 2 in., and the sum of the clearance plus the thickness of the rail is equal to $4\frac{1}{2}$ in. at the other.

Thus, if the crossing be 1 in 8,

$$\text{Length of splayed part} = 8 \times \tfrac{9}{10} \text{ ft.} = 7 \text{ ft. } 3 \text{ in.}$$
$$\text{Distance of point from point of crossing} = 8 \times 12\tfrac{1}{10} \text{ ft.} = 96 \text{ ft. } 10 \text{ in.}$$

Rule V.—The distance from the heel of the switch to the point of the crossing in feet, whether the main line be straight or curved, is eleven and one-fifth times the lead of the crossing, if the sum of the clearance plus the thickness of the rail at the heel of the switch is equal to $4\frac{1}{2}$ in.

Thus, if the crossing be 1 in 8,

$$\text{Distance of point of crossing from heel of switch} = 8 \times 11\tfrac{1}{2} \text{ ft.} = 89 \text{ ft. } 7 \text{ in.}$$

Double-line Junctions.

The first four of the rules for single-line junctions apply exactly to the case of the double-line junctions, the lead of the crossing c_1^2 of the second main-line rail by the first rail of the siding being the one referred to in these rules.

Whether the main line be straight or curved, so long as the clearance at the heels of the switches is equal to $4\frac{1}{2}$ in., we have:

Rule I.—The lead of the crossing c_1^3 is equal to that of the

crossing c_2^4, and to five-sevenths of that of the crossing c_1^2; the lead of the crossing c_1^4 is equal to ten-seventeenths of that of c_1^2; the lead of the crossing c_2^3 is equal to ten-ninths of that of c_1^2; and the lead of c_3^4 is equal to that of c_1^2.

Thus, if the crossing c_1^2 be 1 in 8,

Lead of crossing c_1^3 or $c_2^4 = 8 \times \frac{5}{7} = 6.$
" " $c_1^4 = 8 \times \frac{10}{17} = 5.$
" " $c_2^3 = 8 \times \frac{10}{9} = 9.$
" " $c_3^4 = 8.$

Rule II.—The distance in feet from the line of the heels of the switches of the crossing c_1^2 is equal to that of the crossing c_3^4, and to eleven and one-fifth times the lead of the crossing c_1^2; the distance of the crossing c_1^3 is equal to that of the crossing c_2^4, and to sixteen and three-tenth times the lead of the crossing c_1^2; the distance of the crossing c_1^4 is equal to twenty-one and one-tenth times the lead of the crossing c_1^2; and the distance of the crossing c_2^3 is equal to nine and nine-tenth times the lead of the crossing c_1^2.

Thus, if the crossing c_1^2 be 1 in 8,

Distance of c_1^3 or c_3^4 from heels $= 8 \times 11\frac{1}{5}$ ft. = 89 ft. 7 in.
" c_1^3 or c_2^4 " " $= 8 \times 16\frac{3}{10}$ ft. = 130 ft. 4 in.
" c_1^4 " " $= 8 \times 21\frac{1}{10}$ ft. = 168 ft. 9 in.
" c_2^3 " " $= 8 \times 9\frac{9}{10}$ ft. = 79 ft. 9 in.

Three Throws.

It would be easy to give numerical rules for the formulæ in Division II., but it is not worth while to do so, except in the two special cases already discussed, *viz.* that in which $c_{r_1} = c_{r_2}$, when the branch roads curve in opposite directions, and that in which $c_{r_2} = c_{r_1}\sqrt{2}$, when the branch roads curve in the same direction.

These are the simplest forms of three throws, and therefore ought always to be used, unless a special three throw be necessary. When this is the case, the three throw ought to be set out on the ground by the engineer in charge of the work.

CASE I.—*Branch Roads curving in opposite directions, and crossing the Main-line rails at the same angle.*

Whether the main line be straight or curved:

Rule I.—When the clearance at the heels of the switches between contiguous rails is the same for both roads, the heels of the switches are opposite one another.

Rule II.—The lead of the middle crossing c_m of one branch-road rail by the other is equal to five-sevenths of the lead of either of the other crossings.

Thus, if main-line crossings be each 1 in 8,

$$\text{Lead of middle crossing } c_m = 8 \times \tfrac{5}{7} = 6.$$

Rule III.—The point of the crossing c_m is exactly in the middle of the main road.

Rule IV.—The distance in feet from the heels of the switches of the crossing c_{r_1} is equal to that of the crossing c_{r_2}, and to eleven and one-fifth times the lead of the crossings; that of the middle crossing c_m to seven times the lead of the crossings c_{r_1} or c_{r_2}; the sum of the clearance plus the thickness of the rails being $4\frac{1}{2}$ in.

Thus, if the crossing of the main line be 1 in 8,

$$\text{Distance of } c_{r_1} \text{ or } c_{r_2} \text{ from heel} = 8 \times 11\tfrac{1}{5} \text{ ft.} = 89 \text{ ft. } 7 \text{ in.}$$

$$\text{,, } c_m \text{ ,, ,,} = 8 \times 7 \text{ ft.} = 56 \text{ ft.}$$

CASE II.—*Branch Roads curving in the same direction, the lead of the easier Main-line crossing being equal to seven-fifths that of the sharper crossing.*

Whether the main line be straight or curved:

Rule I.—When the clearance at the heels of the switches between contiguous rails is the same for both roads, the heels of the switches are opposite to one another.

Rule II.—The lead of the middle crossing c_m is equal to that of the easier crossing c_{r_1}, and to seven-fifths of that of the sharper crossing c_{r_2}.

Thus, if the crossing c_{r_2} be 1 in 8,

$$\text{Lead of the crossing } c_m = 8 \times \tfrac{7}{8} = 11.$$

Rule III.—The distance from the heel of the switch of the crossing c_{r_1} is equal to fifteen and three-quarter times the lead of the crossing c_{r_2}; the distance of the crossing c_{r_2} to eleven and one-fifth times that of the crossing c_{r_2}; and the distance of c_m to that of c_{r_1}.

Thus, if the crossing c_{r_2} be 1 in 8,

$$\text{Distance of } c_{r_1} \text{ from heel of switch} = 8 \times 15\tfrac{3}{4} \text{ ft.} = 126 \text{ ft.}$$

$$\text{,, } c_{r_2} \text{ ,, ,,} = 8 \times 11\tfrac{1}{5} \text{ ft.} = 89 \text{ ft. } 6 \text{ in.}$$

$$\text{,, } c_m \text{ ,, ,,} = 8 \times 15\tfrac{3}{4} \text{ ft.} = 126 \text{ ft.}$$

Cross-over Roads.

Whether the main line be straight or curved, if the clearance at the heels of the switches plus the thickness of the rail be equal to $4\frac{1}{2}$ in.:

Rule I.—The lead of one crossing is equal to that of the other, and to the interval between the heels of the switches expressed in feet, divided by nineteen and one-quarter feet.

Thus, if the interval be 154 feet,

$$\text{Lead of crossing} = \frac{154}{19\frac{1}{4}} = 8.$$

Rule II.—The interval between the heels of the switches is equal to nineteen and one-quarter times the lead of either crossing.

Thus, if the crossing be 1 in 8,

$$\text{Interval between the heels of the switches} = 19\tfrac{1}{4} \times 8 \text{ ft.} = 154 \text{ ft.}$$

Rule III.—The distance in feet of either crossing from the heel of the switch on the same rail as the crossing is equal to ten and one-quarter times the lead of the crossing, and the interval between the points of the crossings is equal to one and one-fifth times the lead of the crossing.

Thus, if the crossing be 1 in 8,

Distance of either crossing from the heel of the nearest switch
$= 8 \times 10\frac{1}{4}$ ft. $= 82$ ft.

Interval between points of crossings $= 8 \times 1\frac{1}{8}$ ft. $= 9$ ft. 7 in.

Through Roads.

Where there are n intervening roads:

If the main line is straight and the clearance at the heels of the switches plus the thickness of the rail is $4\frac{1}{2}$ in., and the through road consists of two reverse curves, separated by an interval of straight, which extends from extreme crossing to extreme crossing:

Rule I.—The leads of the crossings are equal, and each is equal to the interval between the heels of the switches in feet, divided by thirteen and one-quarter times the number of intervening roads plus nineteen and one-quarter feet.

Thus, if there be three intervening roads, and the interval 472 ft.,

$$\text{Lead of crossing} = \frac{472}{59} = 8.$$

Rule II.—The interval between the heels of the switches in feet is equal to the lead of the crossing, multiplied by thirteen and one-quarter times the number of intervening roads plus nineteen and one-quarter feet.

Thus, if the crossing be 1 in 8, and there are three intervening roads,

Interval between heels of switches $= 59 \times 8$ ft. $= 472$ ft.

Rule III.—The distance in feet of the line of the heel of the switches from the crossing c_1^2 is equal to that of the crossing c_2^{2n+2}, and to ten and one-quarter times the lead of the crossing; the distance of c_2^3 is equal to that of c_1^{2n+2}, and to nine times the lead of the crossing. Lines drawn from c_1^2 to c_1^{2n+2}, and from c_2^3 to c_2^{2n+2}, will give the positions of the rest of the crossings.

Thus, if the crossing be 1 in 8,

Distance of the crossing c_1^2 or c_2^{2n+2} from the heel $= 10\frac{1}{4} \times 8$ ft. $= 82$ ft.

„ „ c_2^3 or c_1^{2n+2} „ „ $= 9 \times 8$ ft. $= 72$ ft.

When the main line is curved, or the through road consists of a pair of reverse curves meeting at a common tangent, the formulæ in Division I. must be used.

When there is only one intervening road, and the reverse curves meet at a common tangent whether the main line be straight or curved, if the clearance plus the thickness of the rail be $4\frac{1}{2}$ in.:

Rule I.—When the interval in feet between the heels of the switches is given, the lead of the crossing c_1^2 or c_2^5 is equal to this interval, divided by thirty-two and three-fifths feet.

Thus, if the interval be 261 lin. ft.,

$$\text{Lead of the crossing } c_1^2 \text{ or } c_2^5 = \frac{261}{32\frac{3}{5}} = 8.$$

Rule II.—The interval between the heels of the switches is equal to thirty-two and three-fifth times the lead of the crossing c_1^2 or c_2^5.

Thus, if the crossing c_1 or c_2^5 be 1 in 8,

$$\text{Interval between the heels of the switches} = 32\tfrac{3}{5} \times 8 \text{ ft.} = 261 \text{ ft.}$$

Rule III.—The lead of the crossing c_1^3 is equal to that of the crossing c_2^4, and is equal to five-sevenths that of the crossing c_1^2; the lead of the crossing c_2^3 is equal to that of the crossing c_1^4, and to ten-ninths of that of the crossing c_1^2.

Thus, if the crossing c_1^2 or c_2^5 be 1 in 8,

$$\text{Lead of crossings } c_1^3 \text{ and } c_2^4 = 8 \times \tfrac{5}{7} = 6.$$
$$\text{,, ,, } c_2^3 \text{ and } c_1^4 = 8 \times \tfrac{10}{9} = 9.$$

Rule IV.—The distance in feet from the line of the heels of the nearest switches of the crossing c_1^2 is equal to that of the crossing c_2^5, and to eleven and one-fifth times the lead of the crossing; the distance of the crossing c_1^3 is equal to that of the crossing c_2^4, and to sixteen and three-tenth times the lead of the crossing c_1^2 or c_2^5; and the distance of the crossing c_2^3 is equal to that of c_1^4, and to nine and nine-tenths the lead of c_1^2 or c_2^5.

Thus, if the crossings c_1^2 and c_2^5 be 1 in 8,

$$\text{Distance of } c_1^2 \text{ or } c_2^5 \text{ from heels} = 11\tfrac{1}{8} \times 8 \text{ ft.} = 89 \text{ ft. } 7 \text{ in.}$$
$$\text{,, } c_1^3 \text{ or } c_2^4 \text{ ,, ,,} = 16\tfrac{3}{10} \times 8 \text{ ft.} = 130 \text{ ft. } 4 \text{ in.}$$
$$\text{,, } c_2^3 \text{ or } c_1^4 \text{ ,, ,,} = 9\tfrac{9}{10} \times 8 \text{ ft.} = 79 \text{ ft. } 3 \text{ in.}$$

Siding Roads leading from one Main-line Road across the other: Siding Road a continuous curve till it leaves the Main Line.

Whether the main line be straight or curved, if the sum of the clearance plus the thickness of the rail be $4\frac{1}{2}$ in., we have:

If the lead of the crossing c_1^2 be given:

Rule I.—See Rule I., Double-line Junctions.

Rule II.—See Rule II., Double-line Junctions.

If the lead of the crossing c_1^4 be given:

Rule III.—The lead of the crossing c_1^2 is equal to five-thirds that of the crossing c_1^4; the lead of the crossing c_1^3 is equal to that of the crossing c_2^4, and to five-fourths that of the crossing c_1^4; and the lead of the crossing c_2^3 is equal to twenty-elevenths of that of c_1^4.

Thus, if the crossing c_1^4 be 1 in 5,

Lead of crossing c_1^2 $= 5 \times \frac{5}{3} = 8.$
" " c_1^3 or c_2^4 $= 5 \times \frac{5}{4} = 6.$
" " c_2^3 $= 5 \times \frac{20}{11} = 9.$

Rule IV.—The distance from the line of the heels of the switches in feet of the crossing c_1^2 is equal to nineteen and three-tenth times the lead of the crossing c_1^4; the distance of the crossing c_1^3 is equal to that of the crossing c_2^4, and to twenty-seven and two-fifth times the lead of the crossing c_1^4; the distance of the crossing c_1^4 is equal to thirty-five and one-half times the lead of c_1^4; and that of c_2^3 to sixteen and two-fifth times that of c_1^4.

Thus, if the crossing c_1^4 be 1 in 5,

Distance from heel of crossing c_1^2 $= 5 \times 19\frac{3}{4}$ ft. $= 99$ ft.
" " " c_1^3 or c_2^4 $= 5 \times 27\frac{2}{5}$ ft. $= 137$ ft.
" " " c_1^4 $= 5 \times 35\frac{1}{2}$ ft. $= 177$ ft. 6 in.
" " " c_2^3 $= 5 \times 16\frac{2}{5}$ ft. $= 82$ ft.

When the main line is straight, and the siding road after crossing the third main-line road continues straight, if the sum of the clearance plus the thickness of the rail be $4\frac{1}{2}$ in., we have:

Rule I.—The leads of the crossing are all equal to each other.

Rule II.—The distance in feet of the line of the heel of the

switches of the crossing c_1^2 is equal to ten and one-quarter times the lead; the distance of c_1^3 is equal to that of c_2^4, and to sixteen and one-quarter times the lead; the distance of c_1^4 is equal to twenty-three and one-half times the lead; and the distance of c_2^3 to nine times the lead.

Thus, if the crossing be 1 in 8,

Distance from heel of	c_1^2	$= 10\frac{1}{4} \times 8$ ft. $= 82$ ft.
„ „	c_1^3 or c_2^4	$= 16\frac{1}{4} \times 8$ ft. $= 130$ ft.
„ „	c_1^4	$= 23\frac{1}{2} \times 8$ ft. $= 188$ ft.
„ „	c_2^3	$= 9 \times 8$ ft. $= 72$ ft.

When the main line is curved, the formulæ are too complicated to be reduced to simple written rules.

The simplest way of putting in such a siding road is to use a continuous curve as far as the crossing c_1^4. The required direction of the siding on leaving the main line will give the lead of this crossing.

Since the constants by which the leads are multiplied in the formulæ in Division II. only give approximations sufficiently exact for practical use, it is evident that the easier the crossing the greater will the deviation from exactness be. So far as the mere leads go, this is of no consequence. The error in the position of the siding rail, measured at right angles to the rail of the siding, is equal to the error in the position of the crossing measured along the main line, divided by the lead of the crossing. Therefore the error in the position of the siding rail, arising from an inexact value of the constants, will be the same whatever may be the lead of the crossings, since the error in the value of the constant is alternately multiplied and divided by the lead of the crossing.

The deviations from exactness here referred to have nothing to do with the errors introduced by using approximate values of the leads. The nature of these has already been fully explained.

CHAPTER VI.

RECAPITULATION OF FORMULÆ IN CHAPTER IV., AND REMARKS ON THEIR PRACTICAL USE.

THE additional simplifications of the formulæ, and the simplified formulæ themselves, so far as simple junctions and three throws are concerned, are identical with those given in Chapter V., and rest on the same assumption, *viz.* that the radius of the curve of each rail may be taken to be equal to that of the centre line.

So far therefore as simple junctions and three throws are concerned, the truth of this principle might have been assumed, and the formulæ written down at once in their simplified form. Instead therefore of again repeating them, it will suffice here to refer the reader to the three general rules for ascertaining the circular measures and positions of the angles of the crossings given in Chapter V., page 170.

The rules given in that chapter to ascertain the positions of the heels of the switches in three throws apply also to the case of mixed-gauge roads.

The nature of the simplification of the formulæ for cross-over and through roads requires special elucidation.

Cross-over Roads.

The only class of cross-over roads discussed in Section IV., Chapter IV., is that in which the broad and narrow-gauge reverse curves have a common springing. If the broad and narrow-gauge reverse curves are wholly disconnected, the simplest system is that of two reverse curves meeting at a common tangent. This system of single cross-over roads is fully discussed in Section IV., Chapter III. In the case of the cross-over from one narrow-gauge road to another, we must substitute in the formulæ for g and s their proper values $g - d$ and $s + 2d$ respectively. In the case of the broad-gauge road, formulæ for the values and positions of b_1^2 and b_2^5, and in the case of the narrow-gauge road formulæ for the values and position of n_2^3, n_1^3, n_2^4 and n_1^4, must be added to those given in

Chapter III., Section IV. This last is by far preferable to the system explained in Chapter IV., since it is much simpler to put in at first, and much more easily maintained. It cannot however in all cases be adopted, for want of space.

Cross-over roads with a common springing do not admit theoretically of such simple formulæ as single-gauge cross-overs. The deviations from exact values allowable in practice, however, admit of the formulæ being rendered almost as simple in the former case as in the latter.

Case I., Section IV., Chapter IV.

The simplifications of the formulæ present no special features.

Case II., Chapter IV., Section IV.

a. $s \gtrless g$.

The relations between the radii of the broad-gauge reverse curves are

$$r_2^b = \frac{\mathrm{R}\, r_1^b\, g}{\mathrm{R}\, g + (s+g)\, r_1^b}, \qquad r_1^b = \frac{\mathrm{R}\, r_2^b\, g}{\mathrm{R}\, g - (s+g)\, r_2^b}.$$

These relations will be identical with those between the same radii, when the reverse curves meet at a common tangent, when

$$\frac{s+g}{g} = 2.$$

How far the divergence of the value of the ratio $\frac{s+g}{g}$ from this value causes the formulæ for the circular measures of the angles of the broad-gauge crossings in Case II., Chapter IV., to diverge from those in Case II., Chapter III., may be shown as follows.

From the above relations between the radii we obtain the following:

$$\frac{\mathrm{R} + r_1^b}{\mathrm{R}\, r_1^b} = \frac{\mathrm{R}\, g - r_2^b\, s}{\mathrm{R}\, r_2^b\, g};$$

since

$$b_2^4 = \sqrt{\frac{2\,(\mathrm{R}+r_1^b)\,g}{\mathrm{R}\, r_1^b}}, \qquad b_1^3 = \sqrt{\frac{2\,(\mathrm{R}-r_2^b)\,g}{\mathrm{R}\, r_2^b}},$$

$$b_1^3 = b_2^4 \sqrt{\frac{(\mathrm{R}-r_2^b)\,g}{\mathrm{R}\, g - r_2^b\, s}}.$$

Now the fraction

$$\frac{(R - r_2^b)\,g}{R\,g - r_2^b\,s}$$

will differ most from unity when R has a minimum value. The minimum value of R corresponds with the minimum value of r_2^b when r_1^b is infinite, and is therefore equal to

$$\frac{(s+g)\,r_2^b}{g}.$$

When R has this value,

$$b_1^s = b_2^4 \sqrt{\frac{s}{g}}.$$

Since $s \gtrless g$, if we evaluate $\sqrt{\frac{s}{g}}$ on the assumption that $s = 6$ ft., $g = 5$ ft., we get

$$\sqrt{\frac{s}{g}} = \frac{12}{11} \text{ nearly.}$$

Since r_2^b must have its minimum value 4 chains, R cannot be less than $8\frac{1}{2}$ chains, and

$$b_2^4 = \frac{1}{8} \text{ nearly;}$$

$$\therefore \quad b_1^s = \frac{1}{8} \times \frac{12}{11} = \frac{1}{7{\cdot}4} = \frac{1}{7\frac{1}{2}} \text{ nearly.}$$

Therefore, even in this extreme case, either lead would be sufficiently accurate in practice. But r_1^b can never be infinite, because in that case r_1^a would be infinite too, and the narrow-gauge road rail would never meet the rail common to both roads.

Since also

$$r_2^a = r_2^b - \frac{d}{O'^2} = r_2^b - \frac{d\,r_2^b\,(R - r_2^b)}{2\,R\,g},$$

$$r_1^a = r_1^b - \frac{d}{O'^2} = r_1^b - \frac{d\,r_1^b\,(R + r_1^b)}{2\,R\,g},$$

$$\frac{(R - r_2^a)}{R r_2^a} = \frac{R - r_2^b + \frac{d r_2^b (R - r_2^b)}{2 R g}}{R \left\{ r_2^b - \frac{d r_2^b (R - r_2^b)}{2 R g} \right\}},$$

$$= \frac{(R - r_2^b) 2g}{R r_2^b (2g - d)};$$

if we neglect the terms,

$$\frac{d r_2^b}{R};$$

therefore

$$n_1^a = b_1^a \sqrt{\frac{2g}{2g - d}} = b_1^a \sqrt{\frac{2(g - d)}{2g - d}},$$

$$n_1^b = b_1^b \sqrt{\frac{2g}{2g - d}},$$

&c. = &c.

Similarly, we may show that

$$\frac{R + r_1^a}{R r_1^a} = \frac{(R + r_1^b) 2g}{R r_1^b (2g - d)}.$$

β. $s < g$.

The relations between the radii of the broad-gauge reverse curves are

$$r_2^b = \frac{R r_1^b s}{R s + (s + g) r_1^b}, \qquad r_1^b = \frac{R r_2^b s}{R s - (s + g) r_2^b}.$$

These relations will be identical with those between the same radii, when the reverse curves meet at a common tangent, when

$$\frac{s + g}{s} = 2.$$

How far the divergence of the value of this ratio from this value affects the formulæ may be shown by the method adopted in (a).

From the relations between the radii we obtain the following:

$$\frac{R + r_1^b}{R r_1^b} = \frac{R g - r_2^b s}{R r_2^b g};$$

since

$$b_2^4 = \sqrt{\frac{2(R - r_2^b)s}{R r_2^b}}, \qquad b_1^3 = \sqrt{\frac{2(R + r_1^b)s}{R r_1^b}},$$

$$\therefore \quad b_2^4 = b_1^3 \sqrt{\frac{(R - r_2^b)s}{Rs - r_2^b g}}.$$

When r_1^b is infinite, $R = \frac{s+g}{s} r_2^b$, and the value of the fraction $\sqrt{\frac{(R - r_2^b)s}{Rs - r_2^b g}}$ becomes equal to $\frac{g}{s}$; therefore

$$b_1^4 = b_1^3 \sqrt{\frac{g}{s}}.$$

Since $s < g$, if we evaluate $\sqrt{\frac{g}{s}}$ on the assumption that $s = 6$ ft., $g = 7$ ft. 3 in., we get

$$\sqrt{\frac{g}{s}} = \frac{13}{12} \text{ nearly.}$$

Since r_2^b must not exceed its minimum value, 5 chains, R cannot be less than 11 chains, and

$$b_1^3 = \frac{1}{8} \text{ nearly,}$$

$$b_2^4 = \frac{1}{8} \times \frac{13}{12} = \frac{1}{7 \cdot 4} = \frac{1}{7\frac{1}{2}} \text{ nearly;}$$

therefore, for the reasons stated in (*a*), either formula would give a value for b_2^4 or b_1^3 sufficiently exact for practice.

As in (*a*), we may show that

$$\frac{R - r_2^a}{R r_2^a} = \frac{(R - r_2^b)\,2s}{R r_2^b (2s - d)},$$

$$\frac{R + r_1^a}{R r_1^a} = \frac{(R + r_1^a)\,2s}{R r_1^a (2s - d)}.$$

Hence we see—

1st. That the angles b_1^2, b_2^3, are practically equal to the angles b_2^5, b_2^4, respectively in Case II. as well as Case I.

2nd. That when the formulæ are expressed in terms of the circular measures of the angles, the same formulæ will apply to Cases I. and II.

For the determination of the position of the heel of the switch, or of the point of the fast point and the length of the splay of the latter, we require the values of the expressions

$$\frac{r_2^b r_2^a}{r_2^b - r_2^a} \quad \text{and} \quad \frac{r_1^b r_1^a}{r_1^b + r_1^a}.$$

$\alpha.$ $s \gtreqless g.$

$$\frac{r_2^b r_2^a}{r_2^b - r_2^a} = \frac{r_2^b \left\{ r_2^b - \dfrac{d r_2^b (\mathrm{R} - r_2^b)}{2 \mathrm{R} g} \right\}}{\dfrac{d r_2^b (\mathrm{R} - r_2^b)}{2 \mathrm{R} g}}$$

$$= \frac{r_2^b \left\{ 2g - d - \dfrac{d r_2^b}{\mathrm{R}} \right\}}{d - \dfrac{d r_2^b}{\mathrm{R}}} = \frac{(2g - d) r_2^b}{d},$$

$$\frac{r_1^b r_1^a}{r_1^b + r_1^a} = \frac{r_1^b \left\{ r_1^b - \dfrac{d r_1^b (\mathrm{R} + r_1^b)}{2 \mathrm{R} g} \right\}}{r_1^b + r_1^b - \dfrac{d r_1^b (\mathrm{R} + r_1^b)}{2 \mathrm{R} g}}$$

$$= \frac{r_1^b \left\{ 2g - d - \dfrac{d r_1^b}{\mathrm{R}} \right\}}{4g - d - \dfrac{d r_1^b}{\mathrm{R}}} = \frac{(2g - d) r_1^b}{4g - d}.$$

$\beta.$ $s < g.$

$$\frac{r_2^b r_2^a}{r_2^b - r_2^a} = \frac{(2s - d) r_2^b}{d},$$

$$\frac{r_1^b r_1^a}{r_1^b + r_1^a} = \frac{(2s - d) r_1^b}{4s - d}.$$

The expression for the angular interval between the springings of the reverse curves admits of a very simple reduction, if we assume

$$\frac{r_2^b}{\mathrm{R} - r_2^b} = \frac{r_1^b}{\mathrm{R} + r_1^b}.$$

α. $s > g$.

$$\text{Angular interval} = \sqrt{\frac{2Rg}{r_2^b(R - r_2^b)}} - \sqrt{\frac{2Rg}{r_1^b(R + r_1^b)}}$$

$$= \frac{2g}{b_1^3}\left(\frac{1}{r_2^b} - \frac{1}{r_1^b}\right) = \frac{2(s+g)}{R\,b_1^3}.$$

Therefore the linear interval is $\frac{2(s+g)}{b_1^3}$. The linear interval between the springings, when the main line is straight, is $\frac{s+3g}{b_1^3}$, which is less than the other by the interval $\frac{s-g}{b_1^3}$, when the leads of the crossings are the same. Now $\frac{2(s+g)}{R\,b_1^3}$ is greater than the correct angular interval, which is equal to

$$2g\left\{\frac{1}{b_1^3 r_2^b} - \frac{1}{r_1^b b_2^4}\right\},$$

because the lead of the crossing b_2^4 exceeds that of the crossing b_1^3, to which it was assumed to be equal.

α. $s < g \gtrless g - \frac{d}{2}$.

$$\text{Angular interval} = \sqrt{\frac{2Rs}{r_2^b(R - r_2^b)}} - \sqrt{\frac{2Rs}{r_1^b(R + r_1^b)}}$$

$$= \frac{2s}{b_2^4}\left\{\frac{1}{r_2^b} - \frac{1}{r_1^b}\right\} = \frac{2(s+g)}{R\,b_2^4}.$$

Therefore the linear interval is $\frac{2(s+g)}{b_2^4}$. The linear interval between the springings, when the main line is straight, is equal $\frac{3s+g}{b_2^4}$, which is less than the former by the interval $\frac{g-s}{b_2^4}$, when the leads of the crossings are the same. Now $\frac{2(s+g)}{R\,b_2^4}$ is greater than the correct angular interval, which is equal to

$$2g\left\{\frac{1}{b_2^4 r_2^b} - \frac{1}{b_1^3 r_1^b}\right\},$$

because the lead of the crossing b_1^3 exceeds that of the crossing b_1^4, to which it was assumed to be equal.

Now we have shown that the difference between the linear distances given by the formulæ for straight main lines, and those given by the formulæ for curved main lines, when expressed in terms of the circular measure of the broad-gauge crossings, never exceeds the difference between s and g divided by the circular measure of the crossing, and also that the correct values of the linear distances for curved main lines lie between the values given by the approximate formulæ and those given by the formulæ for straight main lines; therefore, without deviating beyond the limits of exactness permissible in practice, we may use the latter in all cases.

Through Roads.

The only class of through roads discussed in Section V., Chapter IV., is that in which the broad and narrow-gauge reverse curves have a common springing. If the broad and narrow-gauge reverse curves are wholly disconnected, the simplest system is that of two reverse curves meeting at a common tangent. This system of single through roads is fully discussed in Chapter III., Section V. In the case of the through road from one narrow-gauge road to another, we must not substitute in the formulæ, which give the radii of the reverse curves, and the intervals between the heels of the switches, for g and s the values $g - d$ and $s + 2d$, since the space between the contiguous narrow-gauge rails of the middle road and the outside road is, in one case, $s + d$, and the other $s + 2d$. But for $2(s + g)$, the interval between the centre lines of the outside broad-gauge roads, we must substitute $2(s + g) + d$, the interval between the centre lines of the outside narrow-gauge roads, in the formulæ which give the radii. In the case of the broad-gauge road, formulæ for the positions and values of b_1^2, b_2^8, b_1^5 and b_2^5, and in the case of narrow-gauge roads, formulæ for the positions and values of n_1^3, n_2^3, n_1^4, n_2^4, n_1^7 and n_2^7, must be added to those given in Section V., Chapter III. For the reasons stated in the case of cross-over roads, this system is preferable to the one explained in Chapter IV.

The nature of the simplifications requires no special comment either in Cases I. or II. of Section V., Chapter IV.

Since the radii r_1^b, r_1^n, r_2^b, r_2^n, are connected by the same relations, as in the case of cross-over roads, with the exception of the equation of relation between the two radii r_1^b, r_2^b, so that

$$\frac{r_1^b}{R + r_1^b} \text{ is not equal to } \frac{r_2^b}{R - r_2^b},$$

the values of the fractions $\frac{R r_2^b}{R - r_2^b}$, $\frac{R r_1^b}{R + r_1^b}$, $\frac{r_2^b r_2^n}{r_2^b - r_2^n}$, and $\frac{r_1^b r_1^n}{r_1^b + r_1^n}$, expressed in terms of r_2^b, r_1^b, are the same as those given for cross-over roads.

It has been shown in Chapter V. that the radii R, $R - s - g$, $R - 2(s + g)$ may be used indiscriminately in the formulæ which give the values of the ratios.

When $s > g$,

$$r_2^b = \frac{R r_1^b g}{R g + 2(s + g) r_1^b};$$

therefore the maximum value of r_2^b in terms of R is $\frac{R g}{2(s + g)}$. For the 4 ft. 8½ in. gauge, since r_2^b cannot be less than 4 chains, R cannot be less than 17½ chains. Similarly, it may be shown that in the case of the 7 ft. gauge, R cannot be less than 22 chains. See Chapter V. For reasons already stated in this chapter (see Cross-over Roads), r_1^b can never be infinite, and therefore R must have always greater values than these.

Division I.—*Formulæ expressed in terms of the radii.*

For positions of heels of switches, fast points, and general rules for ascertaining the circular measures and positions of the angles of the crossings of simple junctions and three throws, and for ascertaining the relative positions of the heels of the switches in the case of three throws, when the clearance remains constant, and the ratio of the interval between the centres of the rails at the heels of the switches, when the latter are opposite one another, see Chapter V., Division I.

Cross-over Roads between contiguous Main-line Roads.

CASE I.—*Main Roads Straight: Double-gauge Cross-over, in which reverse curves, separated by an interval of straight, start from a common springing.*

α. $s \gtrless g$.

If the distance D be given,

$$r^b = \frac{2g(D-2l)^2}{\{s+3g-4\sqrt{g(c+l)}\}^2},$$

$$r^n = \frac{(2g-d)\,r^b}{2g}.$$

Interval between the springings of the reverse curves

$$= \frac{(s+3g)\sqrt{r^b}}{\sqrt{2g}}.$$

All the rails are cut by the reverse-curve rails of the broad-gauge cross-over; also the second and fifth $S_2S'_2$ and $S_5S'_5$ by the reverse-curve rails of the narrow-gauge road, but the third and fourth $S_3S'_3$ and $S_4S'_4$ by the straight part of the narrow-gauge road. This remark applies also to (β), and to both (α) and (β), Case II. Therefore,

$$b_1^2 = b_2^5 = \sqrt{\frac{2(g-d)}{r^b}},$$

$$b_1^3 = b_2^4 = n_1^3 = n_2^4 = \sqrt{\frac{2g}{r^b}},$$

$$n_1^2 = n_2^5 = \sqrt{\frac{2(g-d)}{r^n}} = \sqrt{\frac{4g(g-d)}{r^b(2g-d)}},$$

$$n_2^3 = n_1^4 = \sqrt{\frac{2d}{r^n}} = \sqrt{\frac{4dg}{r^b(2g-d)}},$$

$$S_2b_1^2 = S'_5b_2^5 = \sqrt{2r^b(g-d)},$$

$$S_2b_1^3 = S'_4b_2^4 = \sqrt{2r^bg},$$

$$S_2n_1^2 = S'_5n_2^5 = \sqrt{2r^n(g-d)} = \sqrt{\frac{r^b(g-d)(2g-d)}{g}},$$

$$n_1^3 b_1^3 = n_2^4 b_2^4 = \frac{d}{2 b_1^3} \text{ nearly,}$$

$$S_3 n_2^3 = S'_4 n_1^4 = \sqrt{2 r^n d} = \sqrt{\frac{r^b d (2g - d)}{g}}.$$

The distance from the springing of the heel of the switch of the narrow-gauge rail, which starts from contact with the main-road rail common to both roads, is equal to

$$\sqrt{\frac{2 r^n r^b (c + t)}{r^b - r^n}} = \sqrt{\frac{2 r^b (2g - d)(c + t)}{d}}.$$

Similarly, the distance from the springing of the point of the fast point of the same rail, and the length of the fast point, are respectively equal to

$$\sqrt{\frac{2 r^b (2g - d)(c + e)}{d}},$$

and

$$\sqrt{\frac{2 r^b (2g - d)}{d}} \{\sqrt{c + t} - \sqrt{c + e}\}.$$

$\beta.$ $s < g \gtrless g - \frac{d}{2}$.

If the distance D be given,

$$r^b = \frac{2s(D - 2l)^2}{\{3s + g - 4\sqrt{s(c + t)}\}^2},$$

$$r^n = \frac{(2s - d) r^b}{2s}.$$

Interval between the springings of the reverse curves

$$= \frac{(3s + g)\sqrt{r^b}}{\sqrt{2s}}.$$

$$b_1^2 = b_2^5 = \sqrt{\frac{2(g - d)}{r^b}},$$

$$b_1^3 = b_2^4 = n_2^4 = n_1^3 = \sqrt{\frac{2s}{r^b}},$$

$$n_1^2 = n_2^5 = \sqrt{\frac{2(g - d)}{r^n}} = \sqrt{\frac{4s(g - d)}{r^b(2s - d)}},$$

$$n_2^5 = n_1^4 = \sqrt{\frac{2d}{r^5}} = \sqrt{\frac{4ds}{r^5(2s-d)}},$$

$$S_2 b_1^5 = S'_2 b_2^5 = \sqrt{2r^5(g-d)},$$

$$S_3 b_1^3 = S'_4 b_2^4 = \frac{(s+g)\sqrt{r^5}}{\sqrt{2s}},$$

$$S_2 n_1^3 = S'_2 n_2^5 = \sqrt{2r^5(g-d)} = \sqrt{\frac{(g-d)(2s-d)r^5}{s}},$$

$$n_1^3 b_1^3 = n_2^4 b_2^4 = \frac{d}{2b_1^3} \text{ nearly},$$

$$S_3 n_2^3 = S'_4 n_1^4 = \sqrt{\frac{d(2s-d)r^5}{s}}.$$

The distance from the springing of heel of switch of narrow-gauge rail, which starts from contact with main-road rail common to both roads, is equal to

$$\sqrt{\frac{2r^5 r^5(c+t)}{r^5 - r^5}} = \sqrt{\frac{2r^5(2s-d)(c+t)}{d}}.$$

Similarly, the distance from the springing of the point of the fast point of the same rail, and the length of the fast point, are respectively equal to

$$\sqrt{\frac{2r^5(2s-d)(c+e)}{d}},$$

and

$$\sqrt{\frac{2r^5(2s-d)}{d}}\{\sqrt{c+t} - \sqrt{c+e}\}.$$

CASE I^a.—*Main Line Straight: Single-gauge Cross-over, in which reverse curves meet at a common tangent.*

When the cross-over is from one broad gauge to another, the formulæ in Chapter V. give the values of the radii r_1^5, r_2^5, and the values and positions of every angle except b_1^2 and b_2^5; the c_1^2 and c_2^3 of Chapter V. being identical with b_1^3 and b_2^4 of Chapter VI. We have therefore only to add

$$b_1^2 = b_2^5 = \sqrt{\frac{2(g-d)}{r^b}},$$

$$S_2 b_1^2 = S'_3 b_1^2 = \sqrt{2 r^b (g-d)}.$$

When the cross-over road is from one narrow-gauge road to the other, if we substitute in the formulæ in Chapter V. $g - d$ and $s + 2d$ for g and s respectively, we shall obtain the values of the radii r_1^n, r_2^n, and of the circular measures and positions of the angles n_1^2 and n_2^5; the angles c_1^2 and c_2^3 of Chapter V. being identical with n_1^2 and n_2^5 of Chapter VI. We have therefore only to add

$$n_1^3 = n_2^4 = \sqrt{\frac{2g}{r^n}},$$

$$n_2^3 = n_1^4 = \sqrt{\frac{2d}{r^n}},$$

$$S_3 n_1^3 = S'_4 n_2^4 = \sqrt{2 r^n g},$$

$$S_3 n_2^3 = S'_4 n_1^4 = \sqrt{2 r^n d}.$$

CASE II.—*Main Roads Curved: Double-gauge Cross-over, in which reverse curves are separated by an interval of straight.*

a. $s \gtreqless g$.

If either of the radii r_1^b, r_2^b be known, the other will be given by one of the equations

$$r_1^b = \frac{R r_2^b g}{R g - (s+g) r_2^b}, \qquad r_2^b = \frac{R r_1^b g}{R g + (s+g) r_1^b};$$

also

$$r_1^n = r_1^b - \frac{d r_1^b (R + r_1^b)}{2 R g}, \qquad r_2^n = r_2^b - \frac{d r_2^b (R - r_2^b)}{2 R g}.$$

Angular interval between the springings of the reverse curves

$$= \sqrt{\frac{2Rg}{r_2^b (R - r_2^b)}} - \sqrt{\frac{2Rg}{r_1^b (R + r_1^b)}}.$$

Since within the prescribed limits of exactness it has been shown that

$$\frac{r_2^b}{R - r_2^b} = \frac{r_1^b}{R + r_1^b},$$

therefore

$$b_1^s = b_2^s = \sqrt{\frac{2(R - r_2^b)(g - d)}{R r_2^b}},$$

$$b_1^s = b_2^4 = n_2^4 = n_1^s = \sqrt{\frac{2(R - r_2^b)g}{R r_2^b}},$$

$$n_1^s = n_2^5 = \sqrt{\frac{2(R - r_2^n)(g - d)}{R r_2^n}} = \sqrt{\frac{4(R - r_2^b)g(g - d)}{R r_2^b(2g - d)}},$$

$$n_2^s = n_1^4 = \sqrt{\frac{2(R - r_2^n)d}{R r_2^n}} = \sqrt{\frac{4(R - r_2^b)dg}{R r_2^b(2g - d)}},$$

$$S_2 b_1^s = S'_3 b_2^5 = \sqrt{\frac{2R r_2^b(g - d)}{(R - r_2^b)}},$$

$$S_3 b_1^s = S'_4 b_2^4 = \sqrt{\frac{2R r_2^b g}{(R - r_2^b)}},$$

$$S_2 n_1^s = S'_5 n_2^5 = \sqrt{\frac{2R r_2^n(g - d)}{(R - r_2^n)}} = \sqrt{\frac{R r_2^b(g - d)(2g - d)}{(R - r_2^b)g}},$$

$$n_1^s b_1^s = n_2^4 b_2^4 = \frac{d}{2 b_1^s} \text{ very nearly,}$$

$$S_3 n_2^s = S'_4 n_1^4 = \sqrt{\frac{2R r_2^n d}{R - r_2^n}} = \sqrt{\frac{R r_2^b d(2g - d)}{(R - r_2^b)g}}.$$

The distance from the springing of heel of switch of narrow-gauge rail, which starts from contact with main-road rail common to both roads, is equal to

For reverse curves radius r_2^b, r_2^n, $\sqrt{\frac{2 r_2^b(2g - d)(c + t)}{d}}$,

" " r_1^b, r_1^n, $\sqrt{\frac{2 r_1^b(2g - d)(c + t)}{4g - d}}$.

Similarly, the distance from the springing of the point of the fast point of the same rail, and the length of the fast point, are respectively

For reverse curves radii r_2^b, r_2^a,

$$\sqrt{\frac{2r_2^b(2g-d)(c+e)}{d}} \text{ and } \frac{\sqrt{2r_2^b(2g-d)}\{\sqrt{c+t}-\sqrt{c+e}\}}{\sqrt{d}},$$

For the reverse curves radii r_1^b, r_1^a,

$$\sqrt{\frac{2r_1^b(2g-d)(c+e)}{4g-d}} \text{ and } \frac{\sqrt{2r_1^b(2g-d)}\{\sqrt{c+t}-\sqrt{c+e}\}}{\sqrt{4g-d}}.$$

β. $s < g \gtrless g - \frac{d}{2}$.

If either of the radii r_1^b, r_2^b be known, the other is given by the equation

$$r_1^b = \frac{R r_2^b s}{R s - (s+g) r_2^b} \qquad r_2^b = \frac{R r_1^b s}{R s + (s+g) r_1^b};$$

also

$$r_1^a = r_1^b - \frac{d r_1^b (R + r_1^b)}{2 R s} \qquad r_2^a = r_2^b - \frac{d r_2^b (R - r_2^b)}{2 R s}.$$

Angular interval between the springings of the reverse curves

$$= \sqrt{\frac{2Rs}{r_2^b(R-r_2^b)}} - \sqrt{\frac{2Rs}{r_1^b(R+r_1^b)}}.$$

$$b_1^2 = b_2^5 = \sqrt{\frac{2(R-r_2^b)(g-d)}{R r_2^b}},$$

$$b_1^3 = b_2^4 = n_2^4 = n_1^3 = \sqrt{\frac{2(R-r_2^b)s}{R r_2^b}},$$

$$n_1^2 = n_2^5 = \sqrt{\frac{2(R-r_2^a)(g-d)}{R r_2^a}} = \sqrt{\frac{4(R-r_2^b)s(g-d)}{R r_2^b(2s-d)}},$$

$$n_2^3 = n_1^4 = \sqrt{\frac{2(R-r_2^a)d}{R r_2^a}} = \sqrt{\frac{4(R-r_2^b)sd}{R r_2^b(2s-d)}},$$

$$S_2 b_1^2 = S'_3 b_2^5 = \sqrt{\frac{2R r_2^b(g-d)}{R - r_2^b}},$$

$$S'_3 b_1^3 = S_4 b_2^4 = \sqrt{\frac{2R r_2^b s}{R - r_2^b}},$$

$$S_2 n_1^2 = S'_3 n_2^5 = \sqrt{\frac{2R r_2^a(g-d)}{R - r_2^a}} = \sqrt{\frac{R r_2^b(2s-d)(g-d)}{(R-r_2^b)s}},$$

$$n_1\, b_1^3 = n_2{}' b_2{}' = \frac{d}{2\, b_1^3} \text{ very nearly,}$$

$$S_3 n_2^3 = S'_4 n_1^4 = \sqrt{\frac{2\,R\, r_2''\, d}{(R - r_2'')}} = \sqrt{\frac{R\, r_2^b\, d\,(2\,s - d)}{(R - r_2^b)\, s}}.$$

The distance from the springing of heel of switch of narrow-gauge rail, which starts from contact with main-road rail common to both roads, is equal to

$$\text{For the reverse curves } r_2^b,\ r_2'',\ \sqrt{\frac{2\, r_2^b\,(2\,s - d)\,(c + t)}{d}};$$

$$\text{,, ,, ,, } r_1^b,\ r_1'',\ \sqrt{\frac{2\, r_1^b\,(2\,s - d)\,(c + t)}{4\,s - d}}.$$

Similarly, the distances from the springing of the point of the fast point of the same rail, and the length of the fast point, are respectively,

For the reverse curves radii r_2^b, r_2'',

$$\sqrt{\frac{2 r_2^b (2s - d)(c + e)}{d}} \text{ and } \frac{\sqrt{2 r_2^b (2s - d)}\left\{\sqrt{c + t} - \sqrt{c + e}\right\}}{\sqrt{d}};$$

For the reverse curves radii r_1^b, r_1'',

$$\sqrt{\frac{2 r_1^b (2s - d)(c + e)}{4s - d}} \text{ and } \frac{\sqrt{2 r_1^b (2s - d)}\left\{\sqrt{c + t} - \sqrt{c + e}\right\}}{\sqrt{4s - d}}.$$

CASE II^a.—*Main Line Curved: Single-gauge Cross-over, in which reverse curves meet at a common tangent.*

When the cross-over is from one broad-gauge road to another, the formulæ in Chapter V. give the values of the radii r_1^b, r_2^b, and the values and positions of every angle except b_1^2 and b_2^5; c_1^2 and c_2^3 of Chapter V. being identical with b_1^3 and b_2^4 of Chapter VI. We have therefore only to add

$$b_1^2 = b_2^5 = \sqrt{\frac{2\,(R - r_2^b)\,(g - d)}{R\, r_2^b}},$$

$$S_2 b_1^2 = S'_5 b_2^5 = \sqrt{\frac{2\,R\, r_2^b\,(g - d)}{R - r_2^b}}.$$

When the cross-over road is from one narrow-gauge road to the other, if we substitute in the formulæ in Chapter V., $g - d$ and $s + 2d$ for g and s respectively, we shall obtain the values of the radii r_2^n and r_1^n, and of the circular measures and positions of the angles n_1^2 and n_2^5; the angles c_1^2 and c_2^3 of Chapter V. being identical with n_1^2 and n_2^5 of Chapter VI. We have therefore only to add

$$n_1^3 = n_2^4 = \sqrt{\frac{2(R - r_2^n)g}{R r_2^n}},$$

$$n_2^3 = n_1^4 = \sqrt{\frac{2(R - r_2^n)d}{R r_2^n}},$$

$$S_3 n_1^3 = S'_4 n_2^4 = \sqrt{\frac{2R r_2^n g}{R - r_2^n}},$$

$$S_3 n_2^3 = S'_4 n_1^4 = \sqrt{\frac{2R r_2^n d}{R - r_2^n}}.$$

Through Roads from one Main-line Road across the other.

CASE I.—*Main Line Straight: Double-gauge Through Road, in which reverse curves are separated by an interval of straight: one intervening Road.*

a. $s \gtrless g$.

The rails $S_2S'_2$, $S_3S'_3$, $S_7S'_7$, and $S_8S'_8$, are always cut by the rails of the reverse curves, whatever may be the relation between R, r_1^b and r_2^b.

If the distance D between the points of the switches is known,

$$r^b = \frac{g(D - 2l)^2}{2\left(s + 2g - 2\sqrt{g(c + t)}\right)^2},$$

$$r^n = \frac{(2g - d)r^b}{2g}.$$

Interval between springings of reverse curves

$$= \frac{2(s + 2g)\sqrt{r^b}}{\sqrt{2g}}.$$

$$b_1^2 = b_2^8 = \sqrt{\frac{2(g-d)}{r^5}},$$

$$b_1^3 = b_2^7 = \sqrt{\frac{2g}{r^5}},$$

$$n_1^2 = n_2^8 = \sqrt{\frac{2(g-d)}{r^3}} = \sqrt{\frac{4(g-d)g}{r^5(2g-d)}},$$

$$n_1^3 = n_2^7 = \sqrt{\frac{2g}{r^3}} = \frac{2g}{\sqrt{r^5(2g-d)}},$$

$$n_2^3 = n_1^7 = \sqrt{\frac{2d}{r^3}} = \sqrt{\frac{4gd}{r^5(2g-d)}}.$$

The circular measures of the twelve crossings of the middle road are equal to one another and to b_1^3.

$$S_2 b_1^2 = S'_2 b_2^8 = \sqrt{2r^5(g-d)},$$

$$S_3 b_1^8 = S'_7 b_2^7 = \sqrt{2r^5 g},$$

$$S_4 b_2^4 = S'_6 b_1^6 = \frac{(s+g)\sqrt{r^5}}{\sqrt{2g}},$$

$$S_2 n_1^2 = S_8 n_2^8 = \sqrt{2r^3(g-d)} = \sqrt{\frac{r^4(g-d)(2g-d)}{g}},$$

$$S_3 n_1^8 = S'_7 n_2^7 = \sqrt{2r^3 g} = \sqrt{r^5(2g-d)},$$

$$S_3 n_2^8 = S'_7 n_1^7 = \sqrt{2r^3 d} = \sqrt{\frac{r^5 d(2g-d)}{g}}.$$

Straight lines drawn from b_1^3 to b_1^6, and from b_2^4 to b_2^7, will determine the positions of the crossings of the remaining rails by the broad-gauge rails. Lines parallel to these at a distance $\frac{d}{2}$ ft. from them will determine the positions of the crossings of the remaining rails by the narrow-gauge rails.

β. $s < g \gtreqless g - \frac{d}{2}$.

The rails $S_2 S'_2$ and $S_8 S'_8$ are cut by the rails of the reverse curves, the rest by the straight part of the through road, except in the case of the crossings n_2^3 and n_1^7.

If the distance D be given,

$$r^b = \frac{s(D - 2l)^2}{2\left(2s + g - 2\sqrt{s(c+t)}\right)^2},$$

$$r^a = \frac{(2s - d)\, r^b}{2s}.$$

The interval between the springings of the reverse curves

$$= \frac{2(2s+g)\sqrt{r^b}}{\sqrt{2s}}.$$

$$b_1^2 = b_2^8 = \sqrt{\frac{2(g-d)}{r^b}},$$

$$b_2^4 = b_1^6 = \sqrt{\frac{2s}{r^b}},$$

$$n_1^2 = n_2^7 = \sqrt{\frac{2(g-d)}{r^a}} = \sqrt{\frac{4s(g-d)}{r^b(2s-d)}},$$

$$n_2^8 = n_1^7 = \sqrt{\frac{2d}{r^a}} = \sqrt{\frac{4sd}{r^b(2s-d)}}.$$

The circular measures of the angles b_1^3, b_2^7, n_1^3 and n_2^7, and of the remaining ten crossings of the middle road, are equal to one another and to b_2^4.

$$S_2 b_1^2 = S'_8 b_2^8 = \sqrt{2r^b(g-d)},$$

$$S_3 b_1^3 = S'_7 b_2^7 = \frac{(s+g)\sqrt{r^b}}{\sqrt{2s}},$$

$$S_4 b_2^4 = S'_6 b_1^6 = \sqrt{2r^b s},$$

$$S_2 n_1^2 = S'_8 n_2^8 = \sqrt{2r^a(g-d)} = \sqrt{\frac{r^b(g-d)(2s-d)}{s}},$$

$$S_3 n_2^8 = S'_7 n_1^7 = \sqrt{2r^a d} = \sqrt{\frac{r^b d(2s-d)}{s}}.$$

Straight lines drawn from b_1^3 to b_1^6, and from b_2^4 to b_2^7, will determine the positions of the crossings of the remaining rails by the broad-gauge rails. Lines parallel to these at a distance

$\frac{d}{2}$ from them will determine the positions of the crossings of the remaining rails by the narrow-gauge rails.

The distances of the heel of the switch, of the point of the fast point, and the length of the splay of the last, of the narrow-gauge rail, which starts from contact with the rail common to both gauges, are the same as those given in (α) and (β), Case I., Cross-over Roads.

CASE II.—*Main Line Curved: Double-gauge Through Road, in which reverse curves are separated by an interval of straight.*

a. $s > g$.

The rails $S_3 S'_3$ and $S_7 S'_7$ are always cut by the straight part of the narrow-gauge through road.

The rail $S_2 S'_2$ is always cut by the rail of the reverse curve radius r_2^n.

The rail $S_8 S'_8$ is cut by the rail of the reverse curve radius r_1^n, if

$$2 r_1^b < R - 2(s + g).$$

The rail $S_8 S'_8$ is cut by a rail of the straight interval, if

$$2 r_1^b > R - 2(s + g).$$

Whatever may be the values of the radii,

$$r_1^b = \frac{R r_2^b g}{R g - 2(s+g) r_2^b}, \qquad r_2^b = \frac{R r_1^b g}{R g + 2(s+g) r_1^b},$$

$$r_1^n = r_1^b - \frac{d r_1^b (R + r_1^b)}{2 R g}, \qquad r_2^n = r_2^b - \frac{d r_2^b (R - r_2^b)}{2 R g}.$$

Angular interval between springings of the reverse curves

$$= \sqrt{\frac{2 R g}{r_2^b (R - r_2^b)}} - \sqrt{\frac{2 R g}{r_1^b (R + r_1^b)}}.$$

1st. $2 r_1^b < R - 2(s + g)$.

$$b_1^2 = \sqrt{\frac{2 (R - r_2^b)(g - d)}{R r_2^b}},$$

$$b_1^3 = n_1^3 = \sqrt{\frac{2 (R - r_2^b) g}{R r_2^b}},$$

$$b_1^4 = n_1^4 = b_2^6 = n_2^6 = \sqrt{\frac{2\,\mathrm{R}\,g - 2\,r_2^b\,(s+g)}{\mathrm{R}\,r_2^b}},$$

$$b_1^5 = n_1^5 = \sqrt{\frac{2\,\mathrm{R}\,g - 2\,r_2^b\,(s+g+d)}{\mathrm{R}\,r_2^b}},$$

$$b_1^6 = n_1^6 = \sqrt{\frac{2\,\mathrm{R}\,g - 2\,r_2^b\,(s+2\,g)}{\mathrm{R}\,r_2^b}},$$

$$b_2^4 = n_2^4 = \sqrt{\frac{2\,\mathrm{R}\,g - 2\,r_2^b\,s}{\mathrm{R}\,r_2^b}},$$

$$b_2^5 = n_2^5 = \sqrt{\frac{2\,\mathrm{R}\,g - 2\,r_2^b\,(s+d)}{\mathrm{R}\,r_2^b}},$$

$$b_2^7 = n_2^7 = \sqrt{\frac{2\,(\mathrm{R}+r_1^b)\,g}{\mathrm{R}\,r_1^b}},$$

$$b_2^8 = \sqrt{\frac{2\,(\mathrm{R}+r_1^b)\,(g-d)}{\mathrm{R}\,r_1^b}},$$

$$n_1^2 = \sqrt{\frac{2\,(\mathrm{R}-r_2^a)\,(g-d)}{\mathrm{R}\,r_2^a}} = \sqrt{\frac{4\,(\mathrm{R}-r_2^b)\,g\,(g-d)}{\mathrm{R}\,r_2^b\,(2\,g-d)}},$$

$$n_2^3 = \sqrt{\frac{2\,(\mathrm{R}-r_2^a)\,d}{\mathrm{R}\,r_2^a}} = \sqrt{\frac{4\,(\mathrm{R}-r_2^b)\,g\,d}{\mathrm{R}\,r_2^b\,(2\,g-d)}},$$

$$n_1^7 = \sqrt{\frac{2\,(\mathrm{R}+r_1^a)\,d}{\mathrm{R}\,r_1^a}} = \sqrt{\frac{4\,(\mathrm{R}+r_1^b)\,g\,d}{\mathrm{R}\,r_1^b\,(2\,g-d)}},$$

$$n_2^8 = \sqrt{\frac{2\,(\mathrm{R}+r_1^a)\,(g-d)}{\mathrm{R}\,r_1^a}} = \sqrt{\frac{4\,(\mathrm{R}+r_1^b)\,g\,(g-d)}{\mathrm{R}\,r_1^b\,(2\,g-d)}}.$$

2nd. $2\,r_1^b > \mathrm{R} - 2\,(s+g)$.

$$n_2^8 = \sqrt{\frac{2\,\mathrm{R}\,g + 2\,r_1^b\,g - 3\,r_1^b\,d}{\mathrm{R}\,r_1^b}},$$

$$\mathrm{S}_2\,b_1^2 = \sqrt{\frac{2\,r_2^b\,\mathrm{R}\,(g-d)}{\mathrm{R}-r_2^b}},$$

$$\mathrm{S}_3\,b_1^3 = \sqrt{\frac{2\,\mathrm{R}\,r_2^b\,g}{\mathrm{R}-r_2^b}},$$

$$S'_6 b_1^6 = (R - s - g)\left\{b_1^6 - \sqrt{\frac{2Rg}{r_1^b(R + r_1^b)}}\right\},$$

$$S_6 b_2^4 = (R - s - g)\left\{\sqrt{\frac{2Rg}{r_2^b(R - r_2^b)}} - b_2^4\right\},$$

$$S'_7 b_2^7 = \sqrt{\frac{2Rr_1^b g}{R + r_1^b}},$$

$$S'_8 b_2^8 = \sqrt{\frac{2Rr_1^b(g - d)}{R + r_1^b}},$$

$$S_3 n_1^3 = \sqrt{\frac{2Rr_2^a(g - d)}{R - r_2^a}} = \sqrt{\frac{Rr_2^b(g - d)(2g - d)}{(R - r_2^b)g}},$$

$$S_3 n_2^3 = \sqrt{\frac{2Rr_2^a d}{R - r_2^a}} = \sqrt{\frac{Rr_2^b d(2g - d)}{(R - r_2^b)g}},$$

$$S'_7 n_1^7 = \sqrt{\frac{2Rr_1^a d}{R + r_1^a}} = \sqrt{\frac{Rr_1^b d(2g - d)}{(R + r_1^b)g}},$$

$$S'_8 n_2^8 = \sqrt{\frac{2Rr_1^a(g - d)}{R + r_1^a}} = \sqrt{\frac{Rr_1^b(g - d)(2g - d)}{(R + r_1^b)g}}.$$

This value of $S'_8 n_2^8$ only refers to the case of

$$2r_1^b < R - 2(s + g).$$

Straight lines drawn from b_1^3 to b_1^6, and from b_2^4 to b_2^7, will determine the positions of the crossings of the remaining rails by the broad-gauge rails. Lines parallel to these at a distance $\frac{d}{2}$ ft. from them will determine the positions of the crossings of the remaining rails by the narrow-gauge rails.

β. $s < g \gtrless g - \frac{d}{2}$.

The rails $S_2 S'_2$ are always cut by the rails of the reverse curves radii r_2^b and r_2^a.

The rail $S_8 S'_8$ is cut by the rail of the reverse curve radius r_1^b.

The rail $S_8 S'_8$ is cut by the rail of the reverse curve radius r_1^a, when

$$6r_1^b < R - 2(s + g).$$

The rail $S_8 S'_8$ is cut by a rail of the straight interval, when

$$6 r_1^b > R - 2(s+g).$$

Whatever may be the values of the radii

$$r_1^b = \frac{R r_2^b s}{R s - 2(s+g) r_2^b}, \quad r_2^b = \frac{R r_1^b s}{R s + 2(s+g) r_1^b},$$

$$r_1^a = r_1^b - \frac{d r_1^b (R + r_1^b)}{2 R s}, \quad r_2^a = r_2^b - \frac{d r_2^b (R - r_2^b)}{2 R s}.$$

Angular interval between springings of reverse curves

$$= \sqrt{\frac{2Rs}{r_2^b (R - r_2^b)}} - \sqrt{\frac{2Rs}{r_1^b (R + r_1^b)}}.$$

1st. $6 r_1^b < R - 2(s+g)$.

$$b_1^2 = \sqrt{\frac{2(R - r_2^b)(g-d)}{R r_2^b}},$$

$$b_1^3 = n_1^3 = \sqrt{\frac{2(Rs - r_2^b g)}{R r_2^b}},$$

$$b_1^4 = n_1^4 = b_2^6 = n_2^6 = \sqrt{\frac{2Rs - 2r_2^b(s+g)}{R r_2^b}},$$

$$b_1^5 = n_1^5 = \sqrt{\frac{2Rs - 2r_2^b(s+g+d)}{R r_2^b}},$$

$$b_1^6 = n_1^6 = \sqrt{\frac{2(R + r_1^b)s}{R r_1^b}},$$

$$b_2^4 = n_2^4 = \sqrt{\frac{2(R - r_2^b)s}{R r_2^b}},$$

$$b_2^5 = n_2^5 = \sqrt{\frac{2Rs - 2r_2^b(s+d)}{R r_2^b}},$$

$$b_2^7 = n_2^7 = \sqrt{\frac{2Rs - 2r_2^b(2s+g)}{R r_2^b}},$$

$$b_2^8 = \sqrt{\frac{2(R + r_1^b)(g-d)}{R r_1^b}},$$

$$n_1^{\,a} = \sqrt{\frac{2(R - r_2^{\,a})(g - d)}{R\, r_1^{\,a}}} = \sqrt{\frac{4(R - r_2')\, s\,(g - d)}{R\, r_2^{\,b}(2s - d)}},$$

$$n_1^{\,7} = \sqrt{\frac{2(R + r_1^{\,a})\, d}{R\, r_1^{\,a}}} = \sqrt{\frac{4(R + r_1^{\,b})\, s\, d}{R\, r_1^{\,b}(2s - d)}},$$

$$n_2^{\,3} = \sqrt{\frac{2(R - r_2^{\,a})\, d}{R\, r_2^{\,a}}} = \sqrt{\frac{4(R - r_2^{\,b})\, s\, d}{R\, r_2^{\,b}(2s - d)}},$$

$$n_2^{\,8} = \sqrt{\frac{2(R + r_1^{\,a})(g - d)}{R\, r_1^{\,a}}} = \sqrt{\frac{4(R + r_1^{\,b})\, s\,(g - d)}{R\, r_1^{\,b}(2s - d)}}.$$

2nd. $2r_1' > R - 2(s + g)$.

$$n_4^{\,8} = \sqrt{\frac{2Rs + 2r_1^{\,b} g - 3r_1^{\,b} d}{R\, r_1^{\,b}}},$$

$$S_2 b_1^{\,2} = \sqrt{\frac{2R\, r_2^{\,b}(g - d)}{R - r_2^{\,b}}},$$

$$S_3 b_1^{\,3} = R\left\{\sqrt{\frac{2Rs}{r_2^{\,b}(R - r_2^{\,b})}} - b_1^{\,3}\right\},$$

$$S_4 b_2^{\,4} = \sqrt{\frac{2R\, r_2^{\,b} s}{R - r_2^{\,b}}},$$

$$S'_7 n_1^{\,7} = \sqrt{\frac{2R\, r_1^{\,b} s}{R + r_1^{\,b}}},$$

$$S'_7 b_2^{\,7} = \{R - 2(s + g)\}\left\{b_2^{\,7} - \sqrt{\frac{2Rs}{r_1^{\,b}(R + r_1^{\,b})}}\right\},$$

$$S'_8 b_2^{\,8} = \sqrt{\frac{2R\, r_1^{\,b}(g - d)}{R\, r_1^{\,b}}},$$

$$S_2 n_1^{\,2} = \sqrt{\frac{2R\, r_2^{\,a}(g - d)}{(R - r_2^{\,a})}} = \sqrt{\frac{R\, r_2^{\,b}(2s - d)(g - d)}{(R - r_2^{\,b})\, s}},$$

$$S_3 n_2^{\,3} = \sqrt{\frac{2R\, r_2^{\,a} d}{R - r_2^{\,a}}} = \sqrt{\frac{R\, r_2^{\,b}(2s - d)\, d}{(R - r_2^{\,b})\, s}},$$

$$S'_7 n_1^7 = \sqrt{\frac{2 R r_1^a d}{R + r_1^a}} = \sqrt{\frac{R r_1^b (2 s - d) d}{(R + r_1^b) s}},$$

$$S'_8 n_2^8 = \sqrt{\frac{2 R r_1^a (g - d)}{R + r_1^a}} = \sqrt{\frac{R r_1^b (2 s - d)(g - d)}{(R + r_1^b) s}}.$$

This value of $S'_8 n_2^8$ only refers to the case of

$$6 r_1^b < R - 2(s + g).$$

Straight lines drawn from b_1^3 to b_1^6, and from b_2^4 to b_2^7, will determine the positions of the crossings of the remaining rails by the broad-gauge rails. Lines parallel to these at a distance $\frac{d}{2}$ ft. from them will determine the positions of the crossings of the remaining rails by the narrow-gauge rails.

The distances of the heel of the switch, of the point of the fast point, and the length of the splay of the last, of the narrow-gauge rail, which starts from contact with the rail common to both gauges, are the same as those given in (α) and (β), Case II., Cross-over Roads.

Division II.—*Formulæ expressed in terms of the circular measures of the angles.*

For formulæ relating to fast points and the radii of the curves, see Chapter V.

Single-line Junctions.

Whether the main line be straight or curved, we have:

CASE I.—*Branch Road turning from Rail common to both Roads.*

$$\frac{c_1^2}{c_1^3} = \sqrt{\frac{g - d}{g}},$$

$$c_2^3 = c_1^2 \sqrt{\frac{d}{g - d}} = c_1^3 \sqrt{\frac{d}{g}},$$

$$H_2 c_1^2 = \frac{2\sqrt{g - d}\{\sqrt{g - d} - \sqrt{c + t}\}}{c_1^2} = \frac{2\sqrt{g}\{\sqrt{g - d} - \sqrt{c + t}\}}{c_1^3},$$

$$H_2 c_1^3 = \frac{2\sqrt{g-d}\{\sqrt{g}-\sqrt{c+t}\}}{c_1^2} = \frac{2\sqrt{g}\{\sqrt{g}-\sqrt{c+t}\}}{c_1^2},$$

$$H_2 c_2^3 = \frac{2\sqrt{g-d}\{\sqrt{d}-\sqrt{c+t}\}}{c_1^2} = \frac{2\sqrt{g}\{\sqrt{d}-\sqrt{c+t}\}}{c_1^2}.$$

CASE II.—*Branch Road turning towards the Rail common to both Roads.*

$$\frac{c_2^3}{c_1^3} = \sqrt{\frac{g-d}{g}},$$

$$c_1^3 = c_2^3\sqrt{\frac{d}{g-d}} = c_1^3\sqrt{\frac{d}{g}},$$

$$H_2 c_1^3 = \frac{2\sqrt{g-d}(\sqrt{d}-\sqrt{c+t})}{c_2^3} = \frac{2\sqrt{g}(\sqrt{d}-\sqrt{c+t})}{c_1^3},$$

$$H_2 c_1^3 = \frac{2\sqrt{g-d}(\sqrt{g}-\sqrt{c+t})}{c_2^3} = \frac{2\sqrt{g}(\sqrt{g}-\sqrt{c+t})}{c_1^3},$$

$$H_2 c_2^3 = \frac{2\sqrt{g-d}(\sqrt{g-d}-\sqrt{c+t})}{c_2^3} = \frac{2\sqrt{g}(\sqrt{g-d}-\sqrt{c+t})}{c_1^3}.$$

Double-line Junctions.

Whether the main line be straight or curved, we have,
If the circular measure of the angle c_1^2 be given,

$$c_1^3 = c_4^6 = c_1^2\sqrt{\frac{g}{g-d}},$$

$$c_1^4 = c_3^6 = c_1^2\sqrt{\frac{s+g}{g-d}},$$

$$c_1^5 = c_2^6 = c_1^2\sqrt{\frac{s+g+d}{g-d}},$$

$$c_1^6 = c_1^2\sqrt{\frac{s+2g}{g-d}},$$

$$c_2^3 = c_4^5 = c_1^2\sqrt{\frac{d}{g-d}},$$

$$c_2^4 = c_3^5 = c_1^2\sqrt{\frac{s+d}{g-d}},$$

$$c_2^5 = c_1^2\sqrt{\frac{s+2d}{g-d}},$$

$$c_3^4 = c_1^2\sqrt{\frac{s}{g-d}},$$

$$c_5^6 = c_1^2,$$

$$H_2 c_1^2 = H_6 c_5^6 = \frac{2\sqrt{g-d}\,(\sqrt{g-d} - \sqrt{c+t})}{c_1^2},$$

$$H_3 c_1^3 = H_6 c_4^6 = \frac{2\sqrt{g-d}\,(\sqrt{g} - \sqrt{c+t})}{c_1^2},$$

$$H_4 c_1^4 = H_6 c_3^6 = \frac{2\sqrt{g-d}\,(\sqrt{s+g} - \sqrt{c+t})}{c_1^2},$$

$$H_5 c_1^5 = H_6 c_2^6 = \frac{2\sqrt{g-d}\,(\sqrt{s+g+d} - \sqrt{c+t})}{c_1^2},$$

$$H_6 c_1^6 = \frac{2\sqrt{g-d}\,(\sqrt{s+2g} - \sqrt{c+t})}{c_1^2},$$

$$H_3 c_2^3 = H_5 c_4^5 = \frac{2\sqrt{g-d}\,(\sqrt{d} - \sqrt{c+t})}{c_1^2},$$

$$H_4 c_2^4 = H_5 c_3^5 = \frac{2\sqrt{g-d}\,(\sqrt{s+d} - \sqrt{c+t})}{c_1^2},$$

$$H_5 c_2^5 = \frac{2\sqrt{g-d}\,(\sqrt{s+2d} - \sqrt{c+t})}{c_1^2},$$

$$H_4 c_3^4 = \frac{2\sqrt{g-d}\,(\sqrt{s} - \sqrt{c+t})}{c_1^2}.$$

If the circular measure of the angle c_1^6 be given,

$$c_1^2 = c_3^6 = c_1^6\sqrt{\frac{g-d}{s+2g}},$$

$$c_1^3 = c_4^6 = c_1^6\sqrt{\frac{g}{s+2g}},$$

$$c_1^4 = c_2^6 = c_1^6\sqrt{\frac{s+g}{s+2g}},$$

$$c_1^5 = c_2^6 = c_1^6\sqrt{\frac{s+g+d}{s+2g}},$$

$$c_2^3 = c_4^5 = c_1^6\sqrt{\frac{d}{s+2g}},$$

$$c_2^4 = c_3^5 = c_1^6\sqrt{\frac{s+d}{s+2g}},$$

$$c_2^5 = c_1^6\sqrt{\frac{s+2d}{s+2g}},$$

$$c_3^4 = c_1^6\sqrt{\frac{s}{s+2g}},$$

$$H_2 c_1^2 = H_6 c_3^6 = \frac{2\sqrt{s+2g}\,(\sqrt{g-d}-\sqrt{c+t})}{c_1^6},$$

$$H_3 c_1^3 = H_6 c_4^6 = \frac{2\sqrt{s+2g}\,(\sqrt{g}-\sqrt{c+t})}{c_1^6},$$

$$H_4 c_1^4 = H_6 c_2^6 = \frac{2\sqrt{s+2g}\,(\sqrt{s+g}-\sqrt{c+t})}{c_1^6},$$

$$H_5 c_1^5 = H_6 c_2^6 = \frac{2\sqrt{s+2g}\,(\sqrt{s+g+d}-\sqrt{c+t})}{c_1^6},$$

$$H_6 c_1^6 = \frac{2\sqrt{s+2g}\,(\sqrt{s+2g}-\sqrt{c+t})}{c_1^6},$$

$$H_3 c_2^3 = H_5 c_4^5 = \frac{2\sqrt{s+2g}\,(\sqrt{d}-\sqrt{c+t})}{c_1^6},$$

$$H_4 c_2^4 = H_5 c_2^5 = \frac{2\sqrt{s+2g}\,(\sqrt{s+d} - \sqrt{c+t})}{c_1^6},$$

$$H_5 c_2^5 = \frac{2\sqrt{s+2g}\,(\sqrt{s+2d} - \sqrt{c+t})}{c_1^6},$$

$$H_4 c_2^4 = \frac{2\sqrt{s+2g}\,(\sqrt{s} - \sqrt{c+t})}{c_1^6}.$$

Three Throws.

For the positions of the heels of the switches and the points of the crossings, see Chapter V. In order to apply the formulæ there given, we must look upon the double-gauge three throw as made up of three independent three throws.

1st. Broad-gauge three throws. The crossings belonging to this are

$$c_{r_1}^g,\ c_{r_2}^g,\ c_g^g.$$

2nd. Narrow-gauge three throws. The crossings belonging to this are

$$c_{r_1}^{g-d},\ c_{r_2}^{g-d},\ c_g^{g-d}.$$

3rd. Three throw between the rail, whose centres are separated by the interval d. The crossings belonging to this are

$$c_{r_1}^d,\ c_{r_2}^d,\ c_g^d.$$

The circular measures of these angles are connected by the following relations:

When $c_{r_1}^g$ and $c_{r_2}^g$ are given,

$$c_{r_1}^{g-d} = c_{r_1}^g \sqrt{\frac{g-d}{g}},$$

$$c_{r_2}^{g-d} = c_{r_2}^g \sqrt{\frac{g-d}{g}},$$

$$c_{r_1}^d = c_{r_1}^g \sqrt{\frac{d}{g}},$$

$$c_{r_2}^d = c_{r_1}^g \sqrt{\frac{d}{g}},$$

When $c_{r_1}^{g-d}$ and $c_{r_2}^{g-d}$ are given,

$$c_{r_1}^{g} = c_{r_1}^{g-d}\sqrt{\frac{g}{g-d}},$$

$$c_{r_2}^{g} = c_{r_2}^{g-d}\sqrt{\frac{g}{g-d}},$$

$$c_{r_1}^{d} = c_{r_1}^{g-d}\sqrt{\frac{d}{g-d}},$$

$$c_{r_2}^{d} = c_{r_2}^{g-d}\sqrt{\frac{d}{g-d}}.$$

The above relations hold whether the branch roads turn in the same or in opposite directions. When they turn in opposite directions,

$$c_g^g = \sqrt{c_{r_1}^{g\,2} + c_{r_2}^{g\,2}} = \sqrt{\frac{g\left(c_{r_1}^{g-d\,2} + c_{r_2}^{g-d\,2}\right)}{g-d}},$$

$$c_g^{g-d} = \sqrt{c_{r_1}^{g-d\,2} + c_{r_2}^{g-d\,2}} = \sqrt{\frac{(g-d)\left(c_{r_1}^{g\,2} + c_{r_2}^{g\,2}\right)}{g}},$$

$$c_g^d = \sqrt{c_{r_1}^{d\,2} + c_{r_2}^{d\,2}} = \sqrt{\frac{d\left(c_{r_1}^{g-d\,2} + c_{r_2}^{g-d\,2}\right)}{g-d}}$$

$$= \sqrt{\frac{d\left(c_{r_1}^{g\,2} + c_{r_2}^{g\,2}\right)}{g}}.$$

When they turn in the same direction,

$$c_g^g = \sqrt{c_{r_2}^{g\,2} - c_{r_1}^{g\,2}} = \sqrt{\frac{g\left(c_{r_2}^{g-d\,2} - c_{r_1}^{g-d\,2}\right)}{g-d}},$$

$$c_g^{g-d} = \sqrt{c_{r_2}^{g-d\,2} - c_{r_1}^{g-d\,2}} = \sqrt{\frac{(g-d)\left(c_{r_2}^{g\,2} - c_{r_1}^{g\,2}\right)}{g}},$$

$$c_g^d = \sqrt{c_{r_2}^{d^2} - c_{r_1}^{d^2}} = \sqrt{\frac{d\left(c_{r_2}^{g-d^2} - c_{r_1}^{g-d^2}\right)}{g-d}}$$

$$= \sqrt{\frac{d\left(c_{r_2}^{g^2} - c_{r_1}^{g^2}\right)}{g}}.$$

In applying the formulæ in Chapter V. to ascertain the *positions* of the heels of the switches and points of the crossings, the *exact* values of the circular measures of the crossings, determined in terms of the circular measures of the two, whose values are given, must be used. For g in Chapter V. we must substitute g, $g - d$, and d, in Cases I., II., and III. respectively. Thus the formulæ for the positions of the crossings in the special cases chosen for the written rules in Division III. are:

I.—*Branch Roads curving in opposite directions.*

$$H_1 c_{r_1}^g = H_2 c_{r_2}^g = \frac{2\sqrt{g}\left(\sqrt{g} - \sqrt{c+t}\right)}{c_{r_1}^g},$$

$$H_1 c_g^g = H_2 c_g^g = \frac{2\sqrt{g}\left(\sqrt{g} - \sqrt{2(c+t)}\right)}{c_{r_1}^g\sqrt{2}},$$

$$H_1 c_{r_1}^{g-d} = H_2 c_{r_2}^{g-d} = \frac{2\sqrt{g}\left(\sqrt{g-d} - \sqrt{c+t}\right)}{c_{r_1}^g},$$

$$H_1 c_g^{g-d} = H_2 c_g^{g-d} = \frac{2\sqrt{g}\left(\sqrt{g-d} - \sqrt{2(c+t)}\right)}{c_{r_1}^g\sqrt{2}},$$

$$H_2 c_{r_1}^d = H_3 c_{r_2}^d = \frac{2\sqrt{g}\left(\sqrt{d} - \sqrt{c+t}\right)}{c_{r_1}^g},$$

$$H_2 c_g^d = H_3 c_g^d = \frac{2\sqrt{g}\left(\sqrt{d} - \sqrt{2(c+t)}\right)}{c_{r_1}^g\sqrt{2}}.$$

II.—*Branch Roads curving in the same direction.*

$$H_3 c_{r_1}^{g} = H_2 c_g^{g} = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{c_{r_1}^{g}},$$

$$H_3 c_{r_2}^{g} = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{2(c+t)})}{c_{r_1}^{g}\sqrt{2}},$$

$$H_3 c_{r_1}^{g-d} = H_2 c_g^{g-d} = \frac{2\sqrt{g}\,(\sqrt{g} - d - \sqrt{c+t})}{c_{r_1}^{g}},$$

$$H_3 c_{r_2}^{g-d} = \frac{2\sqrt{g}\,(\sqrt{g} - d - \sqrt{2(c+t)})}{c_{r_1}^{g}\sqrt{2}},$$

$$H_3 c_{r_1}^{d} = H_2 c_g^{d} = \frac{2\sqrt{g}\,(\sqrt{d} - \sqrt{c+t})}{c_{r_1}^{g}},$$

$$H_3 c_{r_2}^{d} = \frac{2\sqrt{g}\,(\sqrt{d} - \sqrt{2(c+t)})}{c_{r_1}^{g}}.$$

The suffixes to the letter H refer to the cases of either both the branch roads, or the road radius r_2, turning from the rail common to both roads. When the reverse is the case, the formulæ will remain the same, but we must interchange the suffixes 3 and 1, in order to make the symbols in the formulæ agree with those in the figures.

When the branch roads curve in opposite directions, if $c_{r_1}^{g} = c_{r_2}^{g}$, then also $c_{r_1}^{g-d} = c_{r_2}^{g-d}$ and $c_{r_1}^{d} = c_{r_2}^{d}$. The crossings $c_{r_1}^{g}$, $c_{r_1}^{g-d}$ and $c_{r_1}^{d}$ are at the same distance from the springing as the crossings $c_{r_2}^{g}$, $c_{r_2}^{g-d}$ and $c_{r_2}^{d}$ respectively, and the heels of the switches are opposite one another. Also

$$c_g^{g} = c_{r_1}^{g}\sqrt{2} = c_{r_1}^{g-d}\sqrt{\frac{2g}{g-d}},$$

$$c_g^{g-d} = c_{r_1}^{g-d}\sqrt{2} = c_{r_1}^{g}\sqrt{\frac{2(g-d)}{g}},$$

$$c_g^{d} = c_{r_1}^{d}\sqrt{2} = c_{r_1}^{g}\sqrt{\frac{2d}{g}} = c_{r_1}^{g-d}\sqrt{\frac{2d}{g-d}}.$$

When the branch roads curve in the same direction, if $c_{r_2}^{g} = c_{r_1}^{g}\sqrt{2}$, then also $c_{r_2}^{g-d} = c_{r_1}^{g-d}\sqrt{2}$, and $c_{r_2}^{d} = c_{r_1}^{d}\sqrt{2}$. The crossings $c_{r_1}^{g}$, $c_{r_1}^{g-d}$ and $c_{r_1}^{d}$ are at the same distance from the springing as the crossings c_g^g, c^{g-d} and c_g^d respectively, and the heels of the switches are opposite to one another. Also

$$c_g^g = c_{r_1}^{g} = c_{r_1}^{g-d}\sqrt{\frac{g}{g-d}},$$

$$c_g^{g-d} = c_{r_1}^{g-d} = c_{r_1}^{g}\sqrt{\frac{g-d}{g}},$$

$$c_g^d = r_1^d = c_{r_1}^{g}\sqrt{\frac{d}{g}} = c_{r_1}^{g-d}\sqrt{\frac{d}{g-d}}.$$

Cross-over Roads: Double Gauge.

The following formulæ are given in terms of b_1^3 only. They are more simple than those in terms of b_1^2.

Whether the main line be straight or curved,

a. $s > g$.

Interval between the heels of the switches

$$= \frac{s + 3g - 4\sqrt{g(c+t)}}{b_1^3}.$$

$$b_1^2 = b_2^5 = b_1^3\sqrt{\frac{g-d}{g}},$$

$$n_1^2 = n_2^5 = b_1^3\sqrt{\frac{2(g-d)}{2g-d}},$$

$$n_2^3 = n_1^4 = b_1^3\sqrt{\frac{2d}{2g-d}},$$

$$n_1^3 = n_2^4 = b_2^4 = b_1^3,$$

$$H_2 b_1^3 = H'_3 b_2^5 = \frac{2\sqrt{g}\,(\sqrt{g-d} - \sqrt{c+t})}{b_1^3},$$

$$H_3 b_1^3 = H'_4 b_2^4 = \frac{2\sqrt{g}\,(\sqrt{g} - \sqrt{c+t})}{b_1^3},$$

$$H_2 n_1^2 = H'_5 n_2^5 = \frac{\sqrt{2(2g-d)}\,(\sqrt{g-d}-\sqrt{c+t})}{b_1^2},$$

$$H_3 n_2^3 = H'_4 n_1^4 = \frac{\sqrt{2(2g-d)}\,(\sqrt{d}-\sqrt{c+t})}{b_1^2},$$

$$n_1^3 b_1^3 = n_2^4 b_2^4 = \frac{d}{2b_1^2}.$$

$\beta.$ $s < g \gtrless g - \frac{d}{2}$.

Interval between the heels of the switches

$$= \frac{3s+g-4\sqrt{s(c+t)}}{b_1^2}.$$

$$b_1^2 = b_2^3 = b_1^3\sqrt{\frac{g-d}{s}},$$

$$n_1^2 = n_2^3 = b_1^3\sqrt{\frac{2(g-d)}{2s-d}},$$

$$n_2^3 = n_1^4 = b_1^3\sqrt{\frac{2d}{2s-d}},$$

$$n_1^3 = n_2^4 = b_2^4 = b_1^3,$$

$$H_2 b_1^3 = H'_5 b_2^5 = \frac{2\sqrt{s}\,(\sqrt{g-d}-\sqrt{c+t})}{b_1^2},$$

$$H_3 b_1^3 = H'_4 b_2^4 = \frac{s+g}{b_1^2},$$

$$H_2 n_1^2 = H'_5 n_2^5 = \frac{\sqrt{2(2s-d)}\,\{\sqrt{g-d}-\sqrt{c+t}\}}{b_1^2},$$

$$H_3 n_2^3 = H'_4 n_1^4 = \frac{\sqrt{2(2s-d)}\,(\sqrt{d}-\sqrt{c+t})}{b_1^2},$$

$$n_1^3 b_1^3 = n_2^4 b_2^4 = \frac{d}{2b_1^2}.$$

Since in the formulæ, which give the position of the heel of the switch, of the point of the fast point, and the splay of the fast point of the narrow-gauge rail $S_1 S'_5$, the radius of the

broad-gauge reverse curve appears as a simple factor, no one set of formulæ can be obtained applicable equally to curved and straight main-line roads; consequently we have the two cases:

Main Roads Straight.

$\alpha.\ \ s > g.$

The interval between the heels of the switches of the rails $S_1 S'_5$ and $S_1 S'_4$ is equal to that between the heels of the switches of the rails $S_2 S'_6$ and $S_3 S'_6$, and to

$$\left(\frac{2\sqrt{g(2g-d)}}{b_1^3\sqrt{d}} - \frac{2\sqrt{g}}{b_1^3}\right)\sqrt{c+t} = \left(\sqrt{\frac{2g-d}{d}} - 1\right)\frac{2\sqrt{g(c+t)}}{b_1^3}.$$

The interval between the heel of the switch of the rail $S_1 S'_4$, and the point of the fast point of the rail $S_1 S'_5$, is equal to that between the heel of the switch of the rail $S_3 S'_6$, and the point of the fast point of the rail $S_2 S'_6$, and to

$$\frac{2\sqrt{g(2g-d)(c+e)}}{b_1^3\sqrt{d}} - \frac{2\sqrt{g(c+t)}}{b_1^3} = \left(\sqrt{\frac{(2g-d)(c+e)}{d}} - \sqrt{c+t}\right)\frac{2\sqrt{g}}{b_1^3}.$$

The length of the splayed part of the fast point of the rail $S_1 S'_5$ is equal to that of the rail $S_2 S'_6$, and to

$$\frac{2\sqrt{g(2g-d)}}{b_1^3\sqrt{d}}\left(\sqrt{c+t} - \sqrt{c+e}\right).$$

$\beta.\ \ s < g \gtrless g - \frac{d}{2}.$

The distance between the heels of the switches of the rails $S_1 S'_5$, $S_1 S'_4$, and $S_2 S'_6$, $S_3 S'_6$, see (a), is equal to

$$\left(\frac{2\sqrt{s(2s-d)}}{b_1^3\sqrt{d}} - \frac{2\sqrt{s}}{b_1^3}\right)\sqrt{c+t} = \left(\sqrt{\frac{2s-d}{d}} - 1\right)\frac{2\sqrt{s(c+t)}}{b_1^3}.$$

The distance between the heels of the switches, and the points of the fast points of the same rails, see (a), is equal to

$$\frac{2\sqrt{s(2s-d)(c+e)}}{b_1^3\sqrt{d}} - \frac{2\sqrt{s(c+t)}}{b_1^3} = \left(\sqrt{\frac{(2s-d)(c+e)}{d}} - \sqrt{c+t}\right)\frac{2\sqrt{s}}{b_1^3}.$$

The length of the splayed part of the fast point, see (*a*), is equal to

$$\frac{2\sqrt{s(2s-d)}}{b_1^{\,2}\sqrt{d}}\left(\sqrt{c+t}-\sqrt{c+e}\right).$$

Main Roads Curved.

a. $s > g$.

The distance between the heels of the switches of the rails $S_1 S'_5$ and $S_1 S'_4$, radius of reverse curve $r_2^{\,b}$, is equal to

$$\sqrt{\frac{2(2g-d)(c+t)r_2^{\,b}}{d}}-\frac{2\sqrt{g(c+t)}}{b_1^{\,2}}.$$

The distance between the heels of the switches of the rails $S_2 S'_6$ and $S_3 S'_6$, radius of reverse curve $r_1^{\,b}$, is equal to

$$\sqrt{\frac{2(2g-d)(c+t)r_1^{\,b}}{4g-d}}-\frac{2\sqrt{g(c+t)}}{b_1^{\,2}}.$$

The interval between the heel of the switch of the rail $S_1 S'_4$, and the point of the fast point of the rail $S_1 S'_5$, radius of reverse curve $r_2^{\,b}$, is equal to

$$\sqrt{\frac{2(2g-d)(c+e)r_2^{\,b}}{d}}-\frac{2\sqrt{g(c+t)}}{b_1^{\,2}}.$$

The interval between the heel of the switch of the rail $S_3 S'_6$, and the point of the fast point of the rail $S_2 S'_6$, radius $r_1^{\,b}$, is equal to

$$\sqrt{\frac{2(2g-d)(c+e)r_1^{\,b}}{4g-d}}-\frac{2\sqrt{g(c+t)}}{b_1^{\,2}}.$$

The length of the splay of the fast point of the rail $S_1 S'_5$, radius $r_2^{\,a}$, is equal to

$$\sqrt{\frac{2g-d}{d}}\left(\sqrt{c+t}-\sqrt{c+e}\right)\frac{2\sqrt{Rg}}{\sqrt{Rb_1^{\,2}+2g}}.$$

The splay of the fast point of the rail $S_2S'_6$, radius r_1^a, is equal to

$$\sqrt{\frac{2g-d}{4g-d}}\,(\sqrt{c+t}-\sqrt{c+e})\,\frac{2\sqrt{\mathrm{R}g}}{\sqrt{\mathrm{R}b_1^3-2g}}.$$

$\beta.$ $s<g\gtreqless g-\frac{d}{2}.$

The distances between the heels of the switches of the rails $S_1S'_4$, $S_1S'_5$, and $S_2S'_6$, $S_3S'_6$, radii r_2^b, r_1^b respectively, are, see (a), equal respectively to

$$\sqrt{\frac{2(2s-d)(c+t)r_2^b}{d}}-\frac{2\sqrt{2s(c+t)}}{b_1^3}$$

and

$$\sqrt{\frac{2(2s-d)(c+t)r_1^b}{4s-d}}-\frac{2\sqrt{2s(c+t)}}{b_1^3}.$$

The distances between the heels of the switches and the points of the fast points of the rails $S_1S'_4$, $S_1S'_5$, and $S_2S'_6$, $S_3S'_6$, radii r_2^b and r_1^b respectively, are, see (a), equal respectively to

$$\sqrt{\frac{2(2s-d)(c+e)r_2^b}{d}}-\frac{2\sqrt{g(c+t)}}{b_1^3}$$

and

$$\sqrt{\frac{2(2s-d)(c+e)r_1^b}{4s-d}}-\frac{2\sqrt{g(c+t)}}{b_1^3}.$$

The lengths of the splay of the fast points of the rails $S_1S'_4$ and $S_2S'_6$, radii r_2^a, r_1^a respectively, are, see (a), respectively equal to

$$\sqrt{\frac{2s-d}{d}}\,(\sqrt{c+t}-\sqrt{c+e})\,\frac{2\sqrt{\mathrm{R}s}}{\sqrt{\mathrm{R}b_1^3+2s}}$$

and

$$\sqrt{\frac{2s-d}{4s-d}}\,(\sqrt{c+t}-\sqrt{c+e})\,\frac{2\sqrt{\mathrm{R}s}}{\sqrt{\mathrm{R}b_1^3-2s}}.$$

Cross-over Roads: Single Gauge.

Narrow Gauge.

Interval between the heels of the switches

$$= \frac{2\sqrt{g-d}\left\{\sqrt{2(s+g+d)}-2\sqrt{c+t}\right\}}{n_1^2}.$$

$$n_1^3 = n_2^4 = n_1^2\sqrt{\frac{g}{g-d}},$$

$$n_2^3 = n_1^4 = n_1^2\sqrt{\frac{d}{g-d}},$$

$$n_2^5 = n_1^2,$$

$$H_2 n_1^2 = H'_5 n_2^5 = \frac{2\sqrt{g-d}\left(\sqrt{g-d}-\sqrt{c+t}\right)}{n_1^2},$$

$$H_3 n_1^3 = H'_4 n_2^4 = \frac{2\sqrt{g-d}\left(\sqrt{g}-\sqrt{c+t}\right)}{n_1^2},$$

$$H_3 n_2^3 = H'_4 n_1^4 = \frac{2\sqrt{g-d}\left(\sqrt{d}-\sqrt{c+t}\right)}{n_1^2}.$$

Broad Gauge.

The intervals between the heels of the switches and the positions of the broad-gauge crossings are given in (α) and (β), Division II., Chapter V. For the crossings of the narrow-gauge rail,

α. $s \gtrless g$.

$$b_1^2 = b_2^5 = b_1^3\sqrt{\frac{g-d}{g}},$$

$$H_2 b_1^2 = H'_5 b_2^5 = \frac{2\sqrt{g}\left(\sqrt{g-d}-\sqrt{c+t}\right)}{b_1^2}.$$

β. $s < g$.

$$b_1^2 = b_2^5 = b_1^3\sqrt{\frac{g-d}{s}},$$

$$H_2 b_1^2 = H'_5 b_2^5 = \frac{2\sqrt{s}\left(\sqrt{g-d}-\sqrt{c+t}\right)}{b_1^2}.$$

Through Roads.

The formulæ for the through roads expressed in terms of the circular measures of the leads of the crossings, except when

the main line is straight, are no simpler than those expressed in terms of the radii, and therefore useless for the object in view, *viz.* the attainment of simple formulæ for the use of the foreman platelayer. All double-gauge through roads ought to be set out by the engineer in charge.

Division III.—*Numerical evaluations of the coefficients in the formulæ for 4 ft. 8½ in. narrow-gauge roads, and 7 ft. broad-gauge roads, and application of the results to simple cases of common occurrence.*

The mixed-gauge road chosen for illustration is the combination of the two roads selected in Chapter V., *viz.* the 7 ft. and 4 ft. 8½ in. gauge. Therefore $g = 7$ ft. 3 in., $g - d = 5$ ft., and $d = 2$ ft. 3 in. The constants have the values assigned to them in Chapter V.

$$\sqrt{\frac{g-d}{g}} = \cdot 83 = \tfrac{4}{5}.$$

$$\sqrt{\frac{d}{g}} = \cdot 56 = \tfrac{3}{5}.$$

$$\sqrt{g}\left(\sqrt{g-d} - \sqrt{c+t}\right) = 4 \cdot 37 = 4\tfrac{3}{8} \text{ ft.}$$

$$\sqrt{g}\left(\sqrt{g} - \sqrt{c+t}\right) = 5 \cdot 6 = 5\tfrac{3}{5} \text{ ft.}$$

$$\sqrt{g}\left(\sqrt{d} - \sqrt{c+t}\right) = 2 \cdot 39 = 2\tfrac{3}{8} \text{ ft.}$$

$$\sqrt{\frac{s+g}{g-d}} = 1 \cdot 63 = 1\tfrac{3}{10} \text{ ft.}$$

$$\sqrt{\frac{s+g+d}{g-d}} = 1 \cdot 76 = 1\tfrac{3}{4}.$$

$$\sqrt{\frac{s+2g}{g-d}} = 2 \cdot 02 = 2.$$

$$\sqrt{\frac{s+d}{g-d}} = 1 \cdot 28 = 1\tfrac{1}{4}.$$

$$\sqrt{\frac{s+2d}{g-d}} = 1 \cdot 45 = 1\tfrac{1}{2}.$$

$$\sqrt{\frac{s}{g-d}} = 1{\cdot}1 = 1\tfrac{1}{10}.$$

$$\sqrt{\frac{d}{g-d}} = {\cdot}67 = \tfrac{7}{10}.$$

$$\sqrt{g-d}\,(\sqrt{g-d} - \sqrt{c+t}) = 3{\cdot}63 = 3\tfrac{3}{5}\text{ ft.}$$

$$\sqrt{g-d}\,(\sqrt{g} - \sqrt{c+t}) = 4{\cdot}65 = 4\tfrac{7}{10}\text{ ft.}$$

$$\sqrt{g-d}\,(\sqrt{s+g} - \sqrt{c+t}) = 6{\cdot}77 = 6\tfrac{3}{4}\text{ ft.}$$

$$\sqrt{g-d}\,(\sqrt{s+g+d} - \sqrt{c+t}) = 7{\cdot}43 = 7\tfrac{2}{5}\text{ ft.}$$

$$\sqrt{g-d}\,(\sqrt{s+2g} - \sqrt{c+t}) = 8{\cdot}75 = 8\tfrac{3}{4}\text{ ft.}$$

$$\sqrt{g-d}\,(\sqrt{d} - \sqrt{c+t}) = 1{\cdot}98 = 2\text{ ft.}$$

$$\sqrt{g-d}\,(\sqrt{s+d} - \sqrt{c+t}) = 5{\cdot}05 = 5\text{ ft.}$$

$$\sqrt{g-d}\,(\sqrt{s+2d} - \sqrt{c+t}) = 5{\cdot}87 = 5\tfrac{9}{10}\text{ ft.}$$

$$\sqrt{g-d}\,(\sqrt{s} - \sqrt{c+t}) = 4{\cdot}11 = 4\tfrac{1}{10}\text{ ft.}$$

$$\sqrt{\frac{g}{s+2g}} = {\cdot}6 = \tfrac{3}{5}.$$

$$\sqrt{\frac{s+g}{s+2g}} = {\cdot}8 = \tfrac{4}{5}.$$

$$\sqrt{\frac{s+g+d}{s+2g}} = {\cdot}87 = \tfrac{9}{10}.$$

$$\sqrt{\frac{d}{s+2g}} = {\cdot}33 = \tfrac{3}{10}.$$

$$\sqrt{\frac{s+d}{s+2g}} = {\cdot}63 = \tfrac{3}{5}.$$

$$\sqrt{\frac{s+2d}{s+2g}} = {\cdot}72 = \tfrac{7}{10}.$$

$$\sqrt{\frac{s}{s+2g}} = {\cdot}54 = \tfrac{8}{15}.$$

$$\sqrt{s+2g}\,(\sqrt{g-d}-\sqrt{c+t}) = 7{\cdot}35 = 7\tfrac{2}{5}\text{ ft.}$$

$$\sqrt{s+2g}\,(\sqrt{g}-\sqrt{c+t}) = 9{\cdot}40 = 2\tfrac{2}{5}\text{ ft.}$$

$$\sqrt{s+2g}\,(\sqrt{s+g}-\sqrt{c+t}) = 13{\cdot}70 = 13\tfrac{7}{10}\text{ ft.}$$

$$\sqrt{s+2g}\,(\sqrt{s+g+d}-\sqrt{c+t}) = 15{\cdot}05 = 15\text{ ft.}$$

$$\sqrt{s+2g}\,(\sqrt{s+2g}-\sqrt{c+t}) = 17{\cdot}73 = 17\tfrac{3}{4}\text{ ft.}$$

$$\sqrt{s+2g}\,(\sqrt{d}-\sqrt{c+t}) = 4{\cdot}02 = 4\text{ ft.}$$

$$\sqrt{s+2g}\,(\sqrt{s+d}-\sqrt{c+t}) = 10{\cdot}23 = 10\tfrac{1}{4}\text{ ft.}$$

$$\sqrt{s+2g}\,(\sqrt{s+2d}-\sqrt{c+t}) = 11{\cdot}90 = 11\tfrac{9}{10}\text{ ft.}$$

$$\sqrt{s+2g}\,(\sqrt{s}-\sqrt{c+t}) = 8{\cdot}32 = 8\tfrac{3}{10}\text{ ft.}$$

$$3s+g-4\sqrt{s(c+t)} = 19{\cdot}25 = 19\tfrac{1}{4}\text{ ft.}$$

$$\sqrt{\frac{g-d}{s}} = {\cdot}91 = \tfrac{9}{10}.$$

$$\sqrt{\frac{2(g-d)}{2s-d}} = 1{\cdot}01 = 1.$$

$$\sqrt{\frac{2d}{2s-d}} = {\cdot}68 = \tfrac{7}{10}.$$

$$\sqrt{s}\,(\sqrt{g-d}-\sqrt{c+t}) = 3{\cdot}98 = 4\text{ ft.}$$

$$\sqrt{2s-d}\,(\sqrt{g-d}-\sqrt{c+t}) = 5{\cdot}03 = 5\text{ ft.}$$

$$\sqrt{2s-d}\,(\sqrt{d}-\sqrt{c+t}) = 2{\cdot}77 = 2\tfrac{4}{5}\text{ ft.}$$

$$\sqrt{\frac{s(2s-d)}{d}} = 5{\cdot}10 = 5\tfrac{1}{10}.$$

$$\sqrt{\frac{s(2s-d)}{4s-d}} = 1{\cdot}64 = 1\tfrac{3}{5}.$$

$$\sqrt{s(c+t)} = 1{\cdot}50.$$

$$\sqrt{s(c+e)} = 1{\cdot}09.$$

$$\sqrt{c+t} = {\cdot}613.$$

$$\sqrt{c+e} = {\cdot}445.$$

$$\sqrt{g}\left(\sqrt{g} - \sqrt{2(c+t)}\right) = 4{\cdot}92 \text{ ft.}$$

$$\sqrt{g}\left(\sqrt{g-d} - \sqrt{2(c+t)}\right) = 3{\cdot}69 \text{ ft.}$$

$$\sqrt{g}\left(\sqrt{d} - \sqrt{2(c+t)}\right) = 1{\cdot}69 \text{ ft.}$$

$$\sqrt{g-d}\left\{\sqrt{2(s+g+d)} - 2\sqrt{c+t}\right\} = 9{\cdot}71 \text{ ft.}$$

$$2\sqrt{s}\left\{\sqrt{2(s+g)} - 2\sqrt{c+t}\right\} = 19{\cdot}22 \text{ ft.}$$

Numerical Rules.

The foreman platelayer, who has carefully studied the explanations given in Division III., Chapter V., will readily understand the meaning of the symbols used for double-gauge roads, since they are formed on the same principle. Thus in single junctions the first rail of the branch line cuts two rails, the second, one. In double junctions the first rail of the branch line cuts five rails; the second, four; the third, three; the fourth, two; and the fifth, one. The main-road rails are numbered after the branch-road rails which they meet at the point of the switch.

In the case of the three throws, the letter g is used as a suffix for the symbols of the middle crossings, because a broad-gauge rail is always one of the crossing rails. If the letter m had been used, the meaning might have been more clear at first sight, on account of the previous use of the same letter in a similar symbol. Thus we should have had, instead of c_g^s, c_g^{s-d}, c_g^d, which represent the crossings of a broad-gauge rail by a broad-gauge rail, a narrow-gauge rail, and a d ft. gauge rail respectively, the symbols c_m^s, c_m^{s-d}, c_m^d.

In order to shorten the written rules, instead of writing the lead, position, &c., of the crossings c_1^2, c_1^3, &c., the simpler expression lead, position, &c., of c_1, of c_2^3, &c., will be used, and the numerical values expressed, not in words, but numbers. The following symbols, instead of words, will also be used:

Multiplied by	$\times$
Divided by	$\div$
Equal to	$=$
Plus	$+$
Minus	$-$

For the rules for finding the radii of the curves, the lengths and positions of the fast points, see Chapter V., Division III. If the circular measure of c_1^2 be given, the 4 ft. $8\frac{1}{2}$ in. gauge formulæ must be used; if c_1^3, the 7 ft. gauge formulæ.

Single-line Junctions.

Branch Road turning from Rail common to both Roads.

RULE I.

Lead of c_1^2 = lead of $c_1^3 \times 1\frac{1}{4}$.
„ c_2^3 = „ $c_1^3 \times 1\frac{2}{3}$.

RULE II.

Distance of c_1^2 from heel = lead of $c_1^3 \times 8\frac{3}{4}$ ft.
„ c_1^3 „ = „ $c_1^3 \times 11\frac{1}{4}$ ft.
„ c_2^3 „ = „ $c_1^3 \times 4\frac{4}{5}$ ft.

Branch Road turning towards Rail common to both Roads.

RULE I.

Lead of c_1^2 = lead of $c_1^3 \times 1\frac{2}{3}$.
„ c_2^3 = „ $c_1^3 \times 1\frac{1}{4}$.

RULE II.

Distance of c_1^2 from heel = lead of $c_1^3 \times 4\frac{4}{5}$ ft.
„ c_1^3 „ = „ $c_1^3 \times 11\frac{1}{4}$ ft.
„ c_2^3 „ = „ $c_1^3 \times 8\frac{3}{4}$ ft.

Double-line Junctions.

When the lead of the crossing c_1^2 is given:

RULE I.

Lead of c_1^3 = lead of c_4^6 = lead of $c_1^2 \times \frac{4}{5}$.
„ c_1^4 = „ c_3^6 = „ $c_1^2 \times \frac{10}{13}$.
„ c_1^5 = „ c_2^6 = „ $c_1^2 \times \frac{4}{7}$.
„ c_1^6 = „ $c_1^2 \times \frac{1}{2}$.
„ c_2^3 = „ c_4^5 = „ $c_1^2 \times 1\frac{3}{7}$.
„ c_2^4 = „ c_3^5 = „ $c_1^2 \times \frac{4}{5}$.
„ c_2^5 = „ $c_1^2 \times \frac{2}{3}$.
„ c_3^4 = „ $c_1^2 \times \frac{10}{11}$.
„ c_5^6 = „ c_1^2.

RULE II.

Distance of c_1^2 from heel = distance of c_5^6 = lead of $c_1^2 \times 7\frac{1}{4}$ ft.
„ c_1^3 „ = „ c_4^6 = „ $c_1^2 \times 9\frac{3}{10}$ ft.
„ c_1^4 „ = „ c_3^6 = „ $c_1^2 \times 13\frac{1}{2}$ ft.
„ c_1^5 „ = „ c_2^6 = „ $c_1^2 \times 14\frac{4}{5}$ ft.
„ c_1^6 „ = „ $c_1^2 \times 17\frac{1}{2}$ ft.
„ c_2^3 „ = „ c_4^5 = „ $c_1^2 \times 4$ ft.
„ c_2^4 „ = „ c_3^5 = „ $c_1^2 \times 10\frac{1}{10}$ ft.
„ c_2^5 „ = „ $c_1^2 \times 11\frac{3}{4}$ ft.
„ c_3^4 „ = „ $c_1^2 \times 8\frac{1}{4}$ ft.

When the lead of the crossing c_1^6 is given:

RULE I.

Lead of c_1^2 = lead of c_5^6 = lead of $c_1^6 \times 2$.
„ c_1^3 = „ c_4^6 = „ $c_1^6 \times 1\frac{2}{3}$.
„ c_1^4 = „ c_3^6 = „ $c_1^6 \times 1\frac{1}{4}$.
„ c_1^5 = „ c_2^6 = „ $c_1^6 \times 1\frac{1}{5}$.
„ c_2^3 = „ c_4^5 = „ $c_1^6 \times 3\frac{1}{3}$.
„ c_2^4 = „ c_3^5 = „ $c_1^6 \times 1\frac{2}{3}$.
„ c_2^5 = „ $c_1^6 \times 1\frac{3}{7}$.
„ c_3^4 = „ $c_1^6 \times 1\frac{7}{8}$.

RULE II.

Distance of c_1^2 from heel = distance c_5^6 = lead of $c_1^6 \times 14\frac{7}{10}$ ft.
„ c_1^3 „ = „ c_4^6 = „ $c_1^6 \times 18\frac{4}{5}$ ft.
„ c_1^4 „ = „ c_3^6 = „ $c_1^6 \times 27\frac{2}{5}$ ft.
„ c_1^5 „ = „ c_2^6 = „ $c_1^6 \times 30\frac{1}{10}$ ft.
„ c_1^6 „ = „ $c_1^6 \times 35\frac{1}{2}$ ft.
„ c_2^3 „ = „ c_4^5 = „ $c_1^6 \times 8$ ft.
„ c_2^4 „ = „ c_3^5 = „ $c_1^6 \times 20\frac{1}{2}$ ft.
„ c_2^5 „ = „ $c_1^6 \times 23\frac{4}{5}$ ft.
„ c_3^4 „ = „ $c_1^6 \times 16\frac{3}{5}$ ft.

The two cases of the siding road from one main-line road across another, discussed in Chapter V., are included in the formulæ for double-line junctions, when the siding road is curved. When the siding road is straight, after leaving the second or third rail, the formulæ are too complicated to admit of much simplification in terms of the leads of the crossings.

It may be useful to note the connection between the lead of

the crossing c_1^6 and that of the crossing of the sixth rail of the main line by the centre lines of the branch road.

Lead of c_1^6 = lead of crossing of sixth rail by centre line of double-line branch road .. .. } $\times \frac{5}{7}$.

Lead of c_1^6 = lead of crossing of sixth rail by centre line of broad-gauge single-line branch road, which starts from S_1 S_2 } $\times \frac{9}{10}$.

The above relations hold true also between the lead of c_1^4 and those of the same crossings, in the case of broad-gauge single-gauge roads.

The first relation holds true also between the lead of c_1^4 and that of the centre line of the double-line branch road, in the case of narrow-gauge single-gauge roads.

The second becomes

Lead of c_1^4 = lead of crossing of fourth rail by centre line of single-line branch road, which starts from S_1 S_2 .. .. } $\times \frac{10}{11}$.

Three Throws.

When the branch roads turn in opposite directions, and the lead of the crossing $c_{r\,1}^{g}$ is equal to that of the crossing $c_{r\,2}^{g}$:

RULE I.

Lead of $c_{r\,1}^{g-d}$ = lead of $c_{r\,2}^{g-d}$ = lead of $c_{r\,1}^{g} \times 1\frac{1}{2}$.

„ $c_{r\,1}^{d}$ = „ $c_{r\,2}^{d}$ = „ $c_{r\,1}^{g} \times 1\frac{2}{3}$.

„ c_{g}^{g} = „ $c_{r\,1}^{g} \times \frac{5}{7}$.

„ c_{g}^{g-d} = „ $c_{r\,1}^{g} \times \frac{9}{10}$.

„ c_{g}^{d} = „ $c_{r\,1}^{g} \times \frac{11}{12}$.

RULE II.

Distance of $c_{r\,1}^{g}$ from heel = distance of $c_{r\,2}^{g}$ = lead of $c_{r\,1}^{g} \times 11\frac{1}{4}$ ft.

„ $c_{r\,1}^{g-d}$ „ = „ $c_{r\,2}^{g-d}$ = „ $c_{r\,1}^{g} \times 8\frac{3}{4}$ ft.

„ $c_{r\,1}^{d}$ „ = „ $c_{r\,2}^{d}$ = „ $c_{r\,1}^{g} \times 4\frac{4}{5}$ ft.

„ c_{g}^{g} „ = „ $c_{r\,1}^{g} \times 6\frac{9}{10}$ ft.

„ c_{g}^{g-d} „ = „ $c_{r\,1}^{g} \times 5\frac{1}{4}$ ft.

„ c_{g}^{d} „ = „ $c_{r\,1}^{g} \times 2\frac{2}{5}$ ft.

When the branch roads turn in the same direction, and the lead of the crossing $c_{r_2}^{g}$ is equal to that of the crossing $c_{r_1}^{g}$ multiplied by $\frac{5}{7}$:

RULE I.

Lead of $c_{r_2}^{g-d}$ = lead of $c_{r_1}^{g-d} \times \frac{5}{7}$ = lead of $c_{r_1}^{g} \times \frac{9}{10}$.
„ $c_{r_2}^{d}$ = „ $c_{r_1}^{d} \times \frac{5}{7}$ = „ $c_{r_1}^{g} \times \frac{11}{12}$.
„ c_{g}^{g} = „ $c_{r_1}^{g}$.
„ c_{g}^{g-d} = „ $c_{r_1}^{g-d}$ = „ $c_{r_1}^{g} \times 1\frac{1}{4}$.
„ c_{g}^{d} = „ $c_{r_1}^{d}$ = „ $c_{r_1}^{g} \times 1\frac{2}{3}$.

RULE II.

Distance of $c_{r_1}^{g}$ from heel = distance of c_{g}^{g} = lead of $c_{r_1}^{g} \times 11\frac{1}{4}$ ft.
„ $c_{r_2}^{g}$ „ = „ $c_{r_1}^{g} \times 6\frac{9}{10}$ ft.
„ $c_{r_1}^{g-d}$ „ = „ c_{g}^{g-d} = „ $c_{r_1}^{g-d} \times 8\frac{3}{4}$ ft.
„ $c_{r_2}^{g-d}$ „ = „ $c_{r_1}^{g} \times 5\frac{1}{4}$ ft.
„ $c_{r_1}^{d}$ „ = „ c_{g}^{d} = „ $c_{r_1}^{g} \times 4\frac{4}{5}$ ft.
„ $c_{r_2}^{d}$ „ = „ = „ $c_{r_1}^{g} \times 2\frac{2}{5}$ ft.

Cross-over Roads: Single-gauge Cross-over.

Narrow Gauge.

If the lead of the crossing n_1^2 be given,

Interval in feet between heels of switches = lead of $n_1^2 \times 19\frac{2}{5}$ ft.

If the interval between the heels of the switches be given,

Lead of n_1^2 = interval in feet $\div 19\frac{2}{5}$ ft.
Lead of n_1^3 = lead of n_2^4 = lead of $n_1^2 \times \frac{4}{5}$.
„ n_2^3 = „ n_1^4 = „ $n_1^2 \times 1\frac{3}{7}$.
„ n_2^5 = „ n_1^2.

Distance of n_1^2 from heel = distance of n_2^5 = lead of $n_1^2 \times 7\frac{1}{4}$ ft.
„ n_1^3 „ = „ n_2^4 = „ $n_1^2 \times 9\frac{3}{10}$ ft.
„ n_2^3 „ = „ n_1^4 = „ $n_1^2 \times 4$ ft.

Broad Gauge.

If the lead of the crossing b_1^3 be given,

Interval in feet between the heels of the switches = lead of $b_1^3 \times 19\frac{1}{4}$ ft.

If the interval between the heels of the switches be given,

Lead of b_1^3 = interval in feet $\div 19\frac{1}{4}$ ft.
Lead of b_1^2 = lead of b_2^5 = lead of $b_1^3 \times 1\frac{1}{4}$ ft.
„ b_2^4 = „ b_1^3.

Distance of b_1^2 from heel = distance of b_2^5 = lead of $b_1^3 \times 8\frac{3}{4}$ ft.
„ b_1^3 „ = „ b_2^4 = „ $b_1^3 \times 11\frac{1}{8}$ ft.

Double-gauge Cross-over.

If the lead of the crossing b_1^3 be given,

Interval in feet between the heels of the switches = lead of $b_1^3 \times 19\frac{1}{4}$ ft.

If the interval between the heels of the switches be given,

Lead of b_1^3 = interval in feet $\div 19\frac{1}{4}$ ft.
Lead of b_1^2 = lead of b_2^5 = lead of $b_1^3 \times 1\frac{1}{10}$.
„ n_1^2 = „ n_2^5 = „ b_1^3.
„ n_2^3 = „ n_1^4 = „ $b_1^3 \times 1\frac{3}{7}$.
„ $n_1^3 = n_2^4 = b_2^4 = b_1^3$.

Distance of b_1^2 from heel = distance of b_2^5 = lead of $b_1^3 \times 8$ ft.
„ b_1^3 „ = „ b_2^4 = „ $b_1^3 \times 13\frac{1}{4}$ ft.
„ n_1^2 „ = „ n_2^5 = „ $b_1^3 \times 7\frac{1}{10}$ ft.
„ n_2^3 „ = „ n_1^4 = „ $b_1^3 \times 3\frac{9}{10}$ ft.

Distance of n_1^3 from b_1^3 = distance of n_2^4 from b_2^4 = lead of $b_1^3 \times 1\frac{1}{8}$ ft.

The interval between the heels of the switches, of the cross-over rails, which start from contact with the common broad-gauge rail, and the length of the splay of the fast point, are given by the following formulæ:

Main Line Straight.

Interval between heels of switches = lead of $b_1^3 \times 3\frac{1}{4}$ ft.
Interval between heel of switch and point of fast point .. = „ $b_1^3 \times 1\frac{1}{2}$ ft.
Length of splay of fast point = „ $b_1^3 \times 1\frac{3}{4}$ ft.

Main Line Curved.

For reverse curve, which curves in the same direction as the main line,

Interval between heels of switches is equal to

$$\sqrt{\text{radius of reverse curve in feet}} \times 1\tfrac{1}{3} - \text{lead of } b_1^3 \times 3 \text{ ft.};$$

Interval between heel of switch, and point of fast point, is equal to

$$\sqrt{\text{radius of reverse curve in feet}} \times 1\tfrac{3}{10} - \text{lead of } b_1^3 \times 3 \text{ ft.};$$

Length of splay of the fast point is equal to

$$\sqrt{\text{radius of reverse curve in feet}} \times \tfrac{1}{3}.$$

For reverse curve, which curves in the opposite direction to the main line,

Interval between heels of switches is equal to

$$\sqrt{\text{radius of reverse curve in feet}} \times \tfrac{3}{5} - \text{lead of } b_1^3 \times 3 \text{ ft.};$$

Interval between heel of switch, and point of fast point, is equal to

$$\sqrt{\text{radius of reverse curve in feet}} \times \tfrac{4}{7} - \text{lead of } b_1^3 \times 3 \text{ ft.};$$

Length of splay of the fast point is equal to

$$\sqrt{\text{radius of reverse curve in feet}} \times \tfrac{1}{6} \text{ ft.}$$

The radii r_2^b, r_1^b must be found by means of the formulæ for broad-gauge roads given in Division III., Chapter V. If the length of the splays of the fast points be given to the foreman platelayer, the following rules will show him how to put the fast points in their proper position:

Rule I.—The fast point with the longest splay belongs to the reverse curve, which turns in the opposite direction to the main line, when the main line is curved.

Rule II.—The clearance at the point of the fast point must be 2 inches.

Rule III.—Put all the crossings in their proper position, and the switches and stock rails of the two inner rails of each main road. Top and straighten the rails of the broad and narrow gauge cross-over from the points of both sets of switches past the last crossings of the opposite main road. The gauges will determine the positions of the remaining part of the cross-over, and of the fast points.

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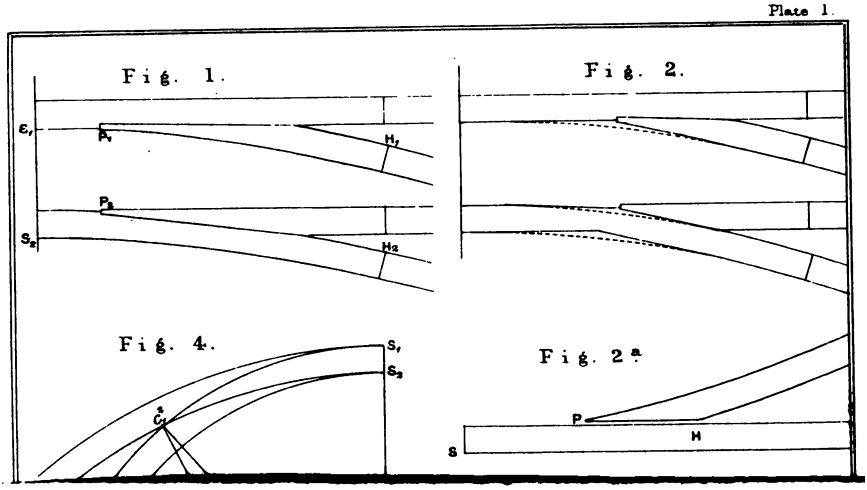
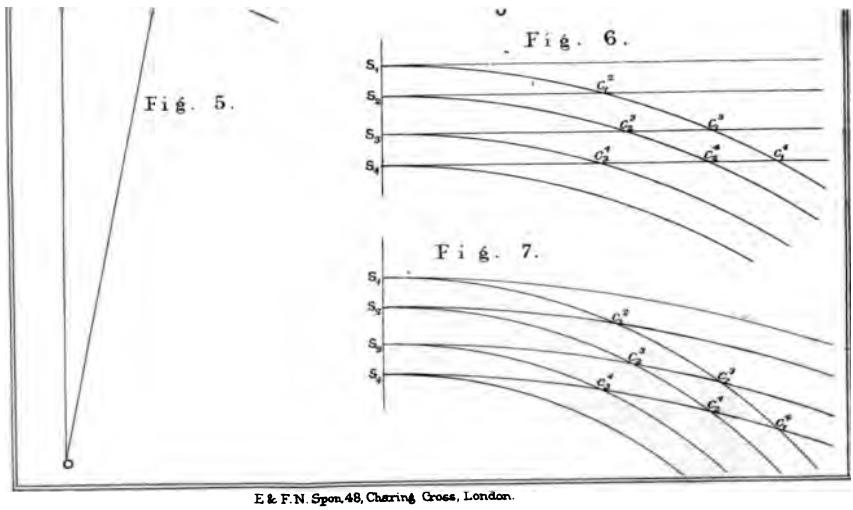


Shortly will be published, a Pocket-book of Tables, constructed from the Formulæ in Division II., Chapters V. and VI., of this book, for all the Principal Gauges used at home and in the Colonies, for the use of Platelayers.



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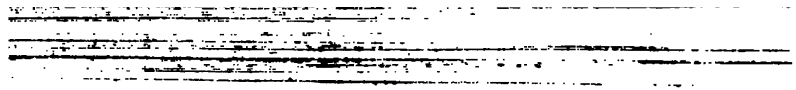

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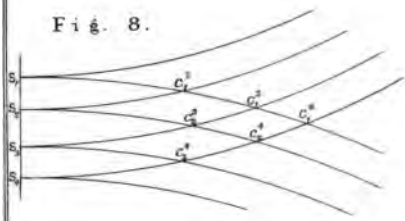

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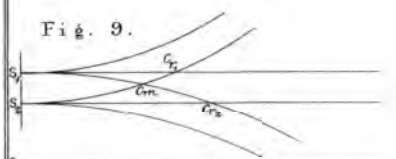

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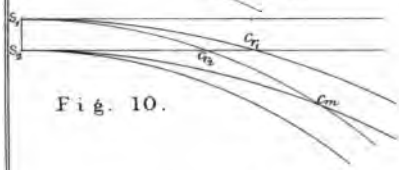

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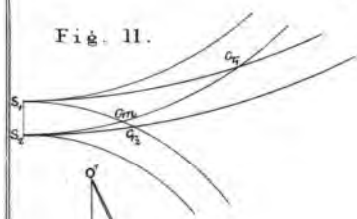

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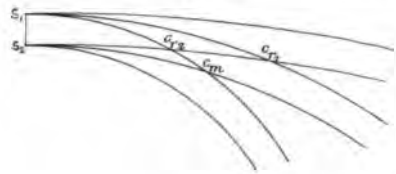

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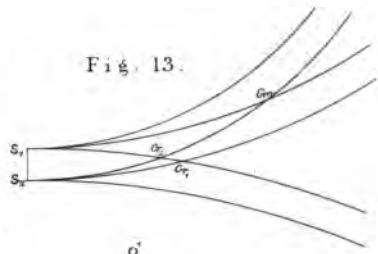

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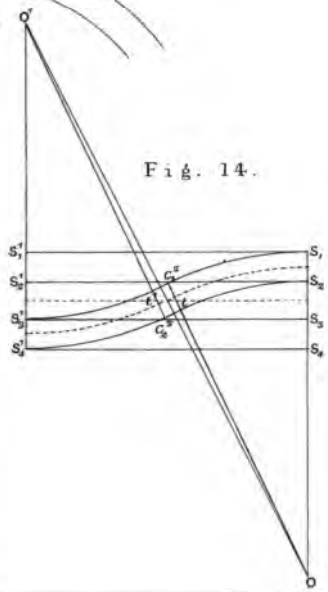

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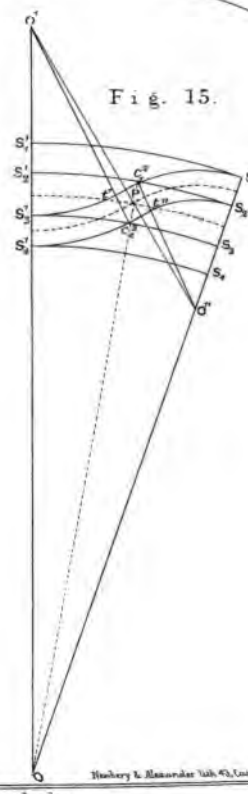

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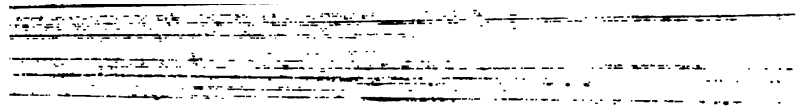

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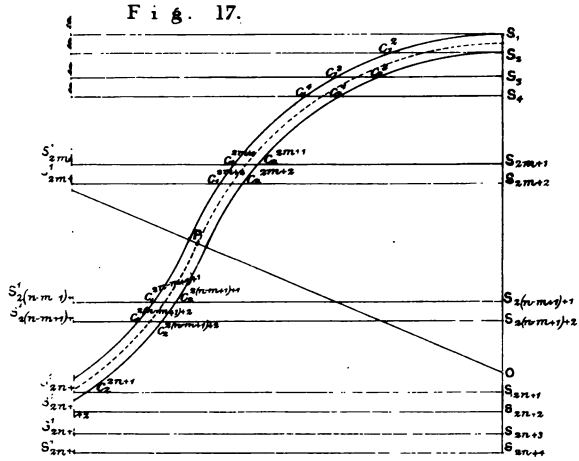

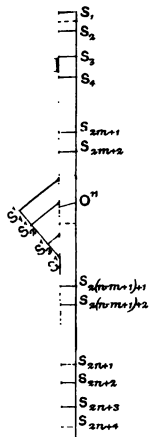

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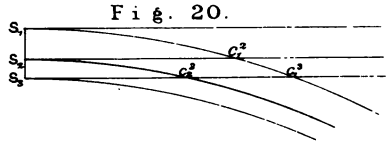

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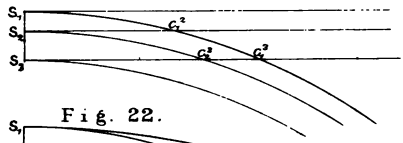

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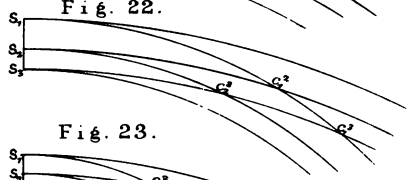

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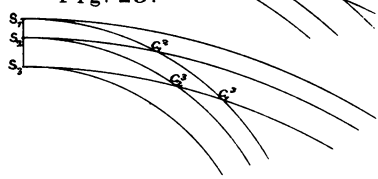

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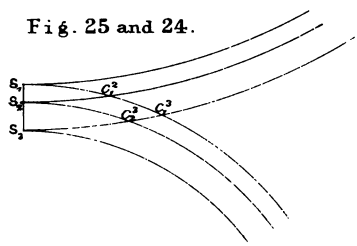

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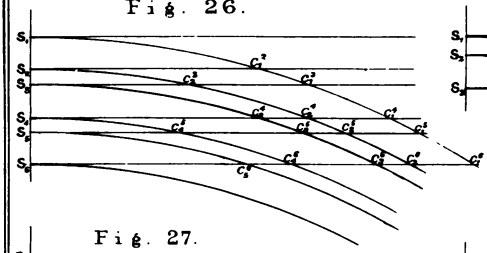

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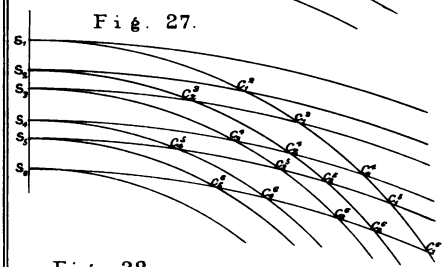

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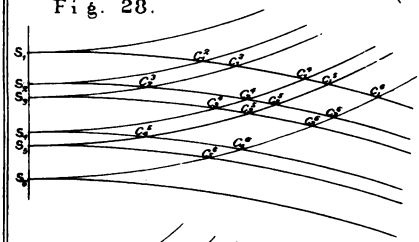

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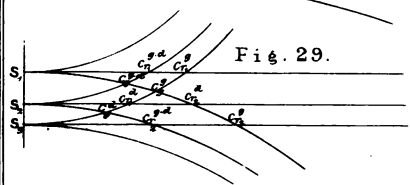

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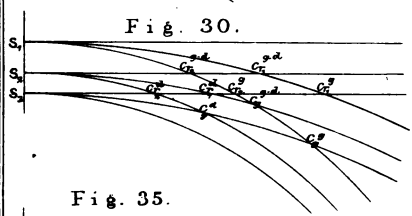

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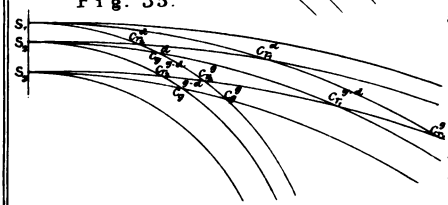

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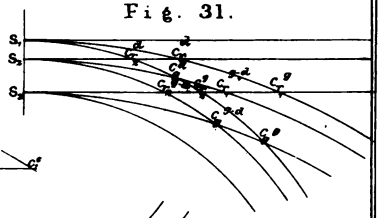

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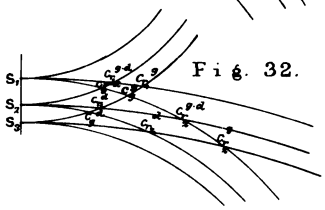

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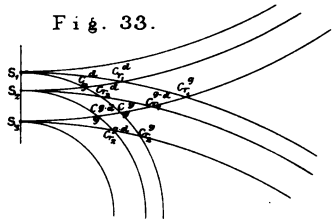

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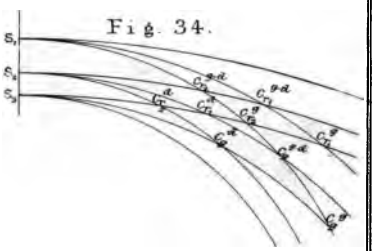

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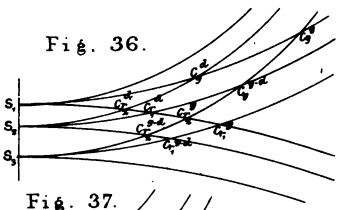

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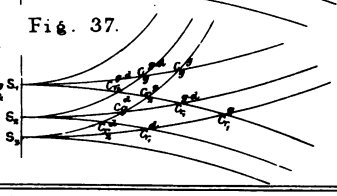

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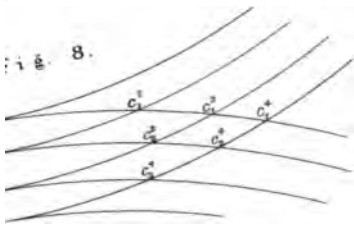

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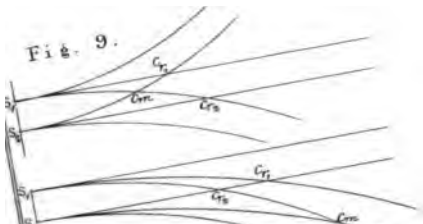

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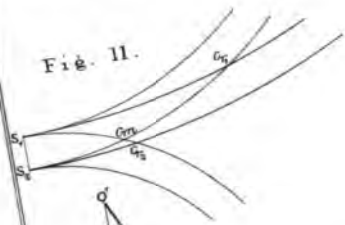

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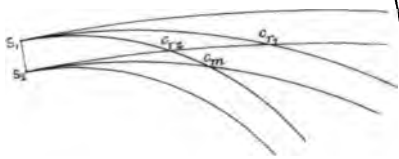

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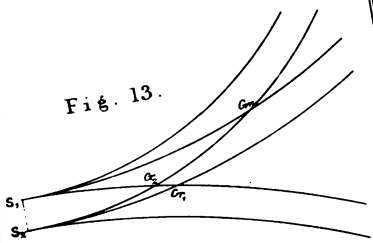

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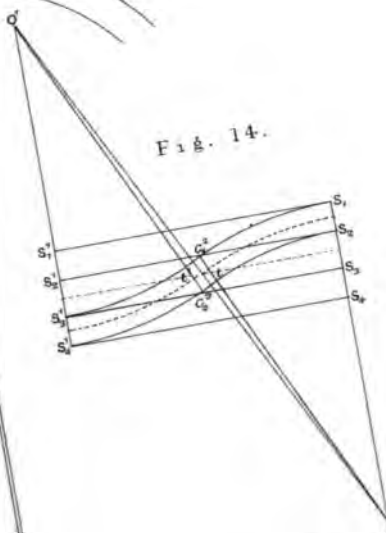

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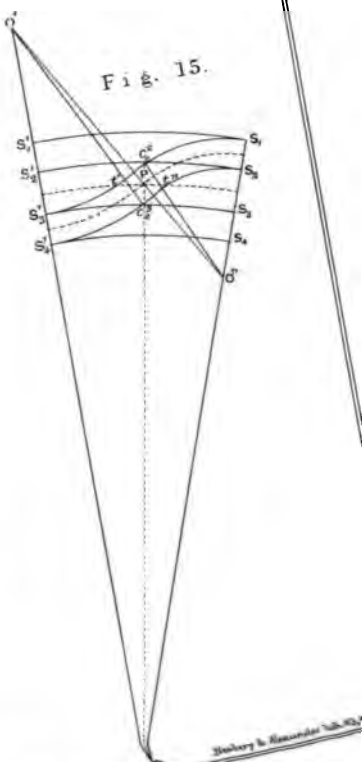

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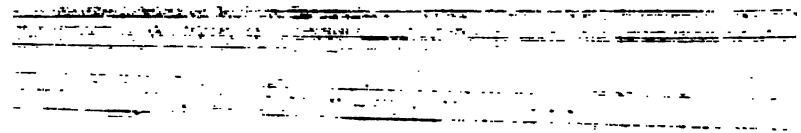

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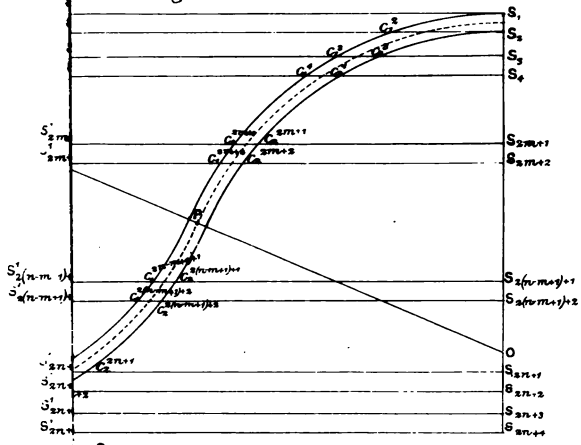

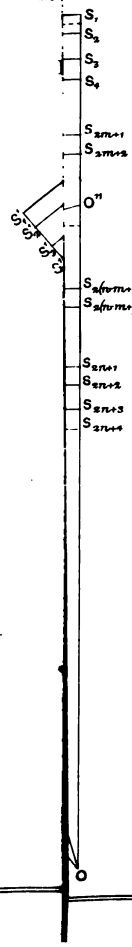

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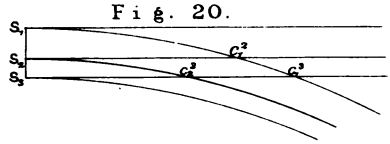

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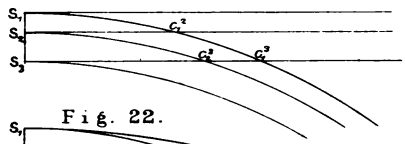

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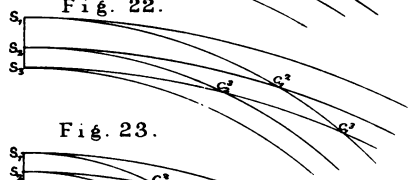

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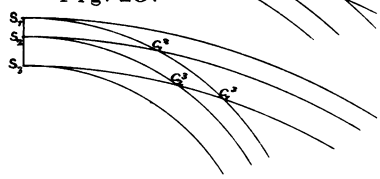
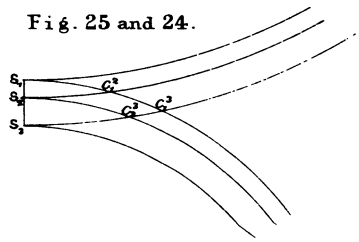


Fig. 25 and 24.



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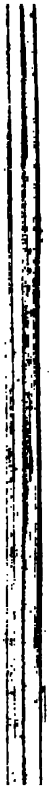

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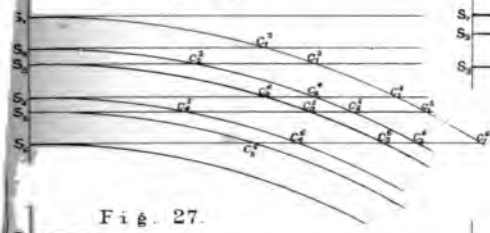

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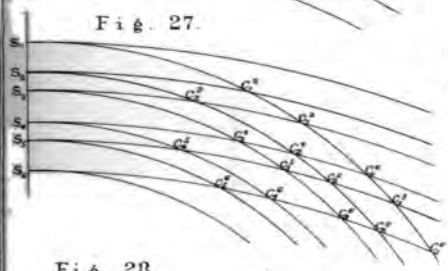

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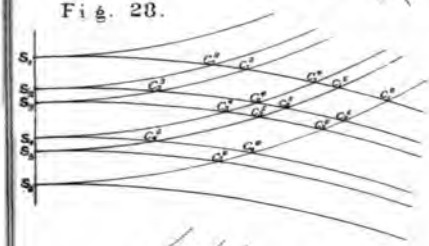

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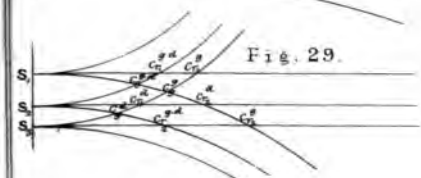

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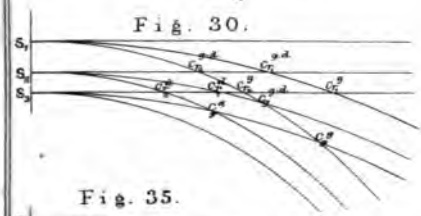

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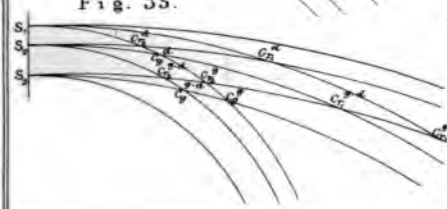

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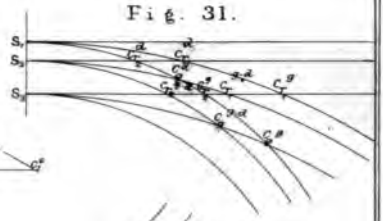

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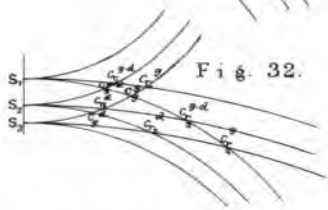

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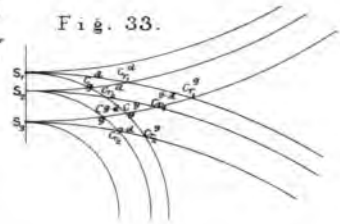

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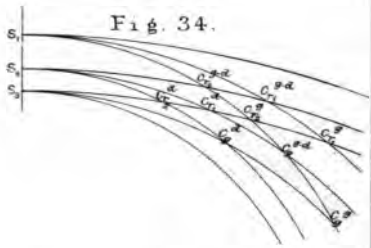

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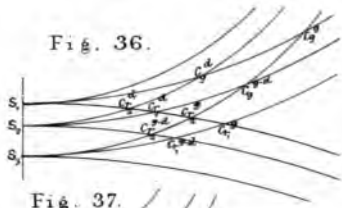

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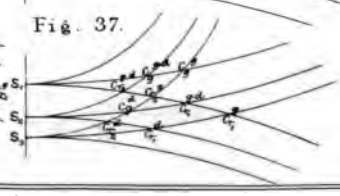

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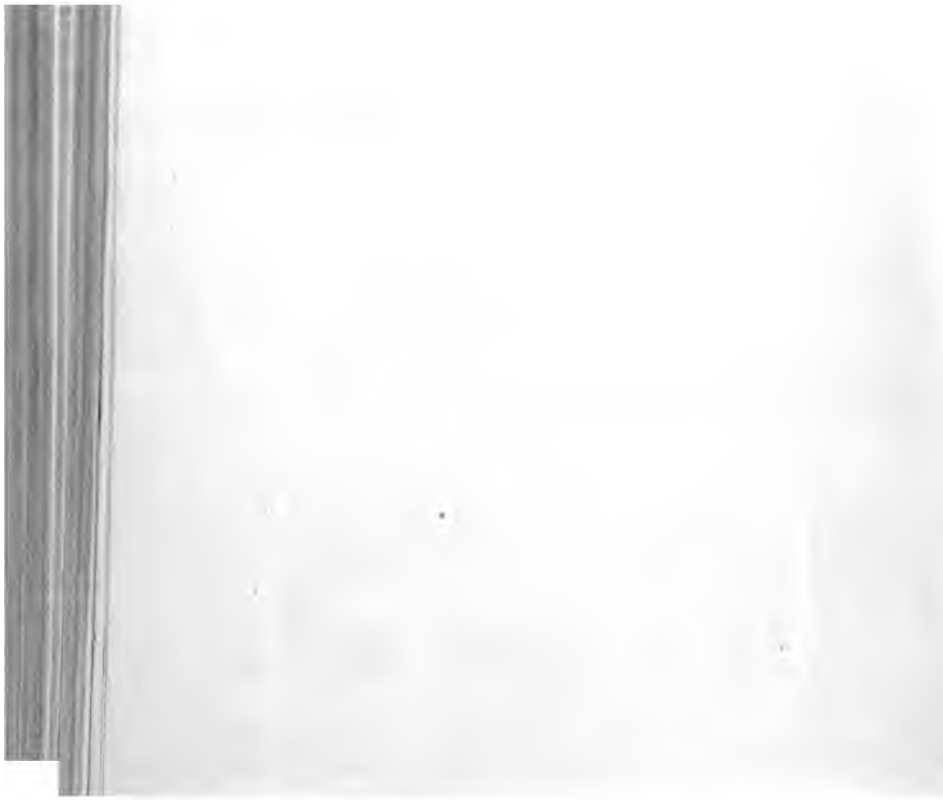


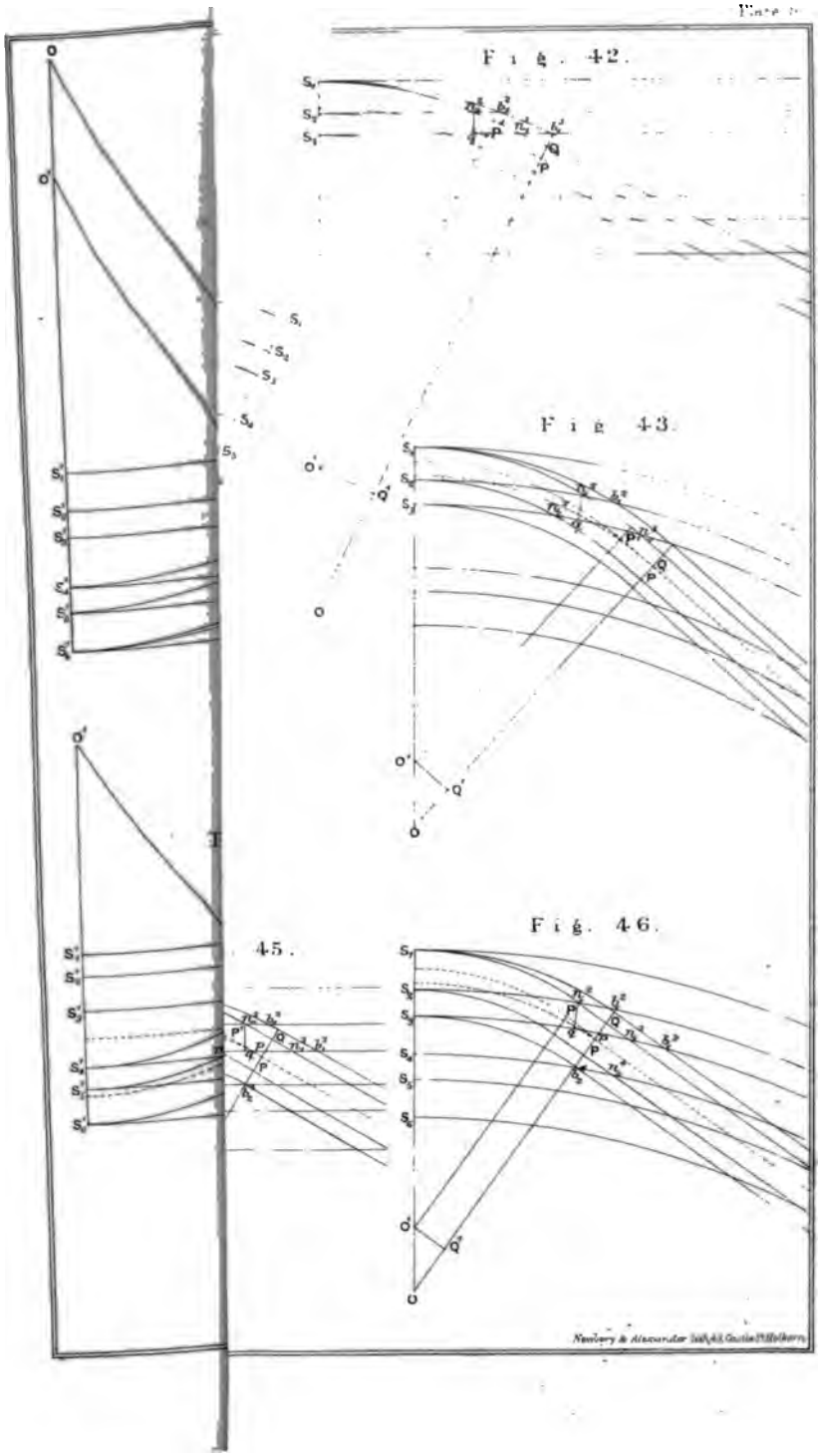
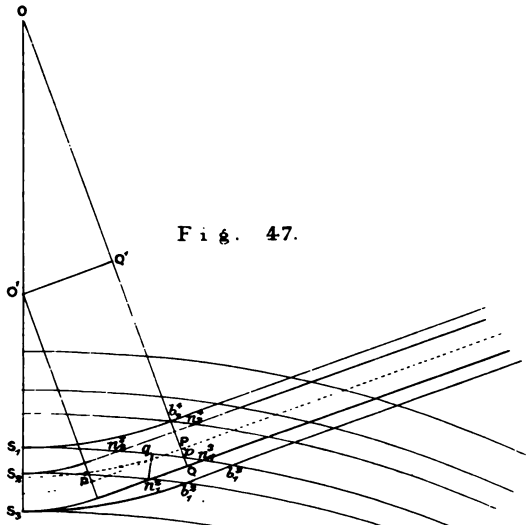


Fig. 42.
Fig. 43.
Fig. 45.
Fig. 46.



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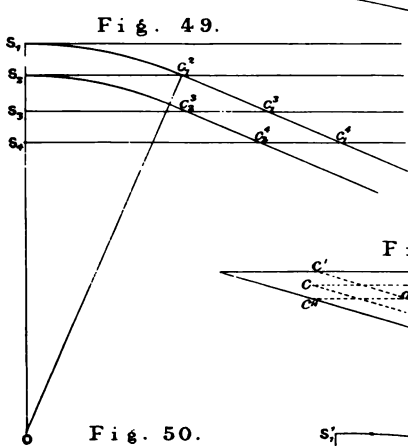

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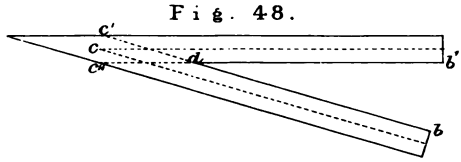

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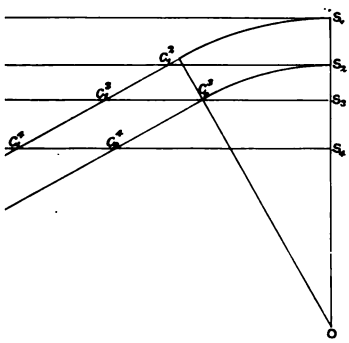

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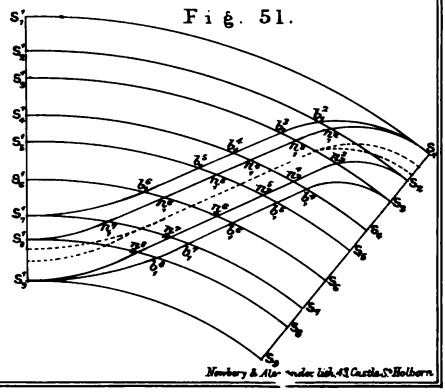

Fig. 51.

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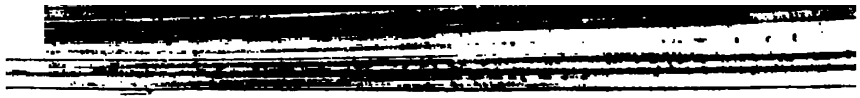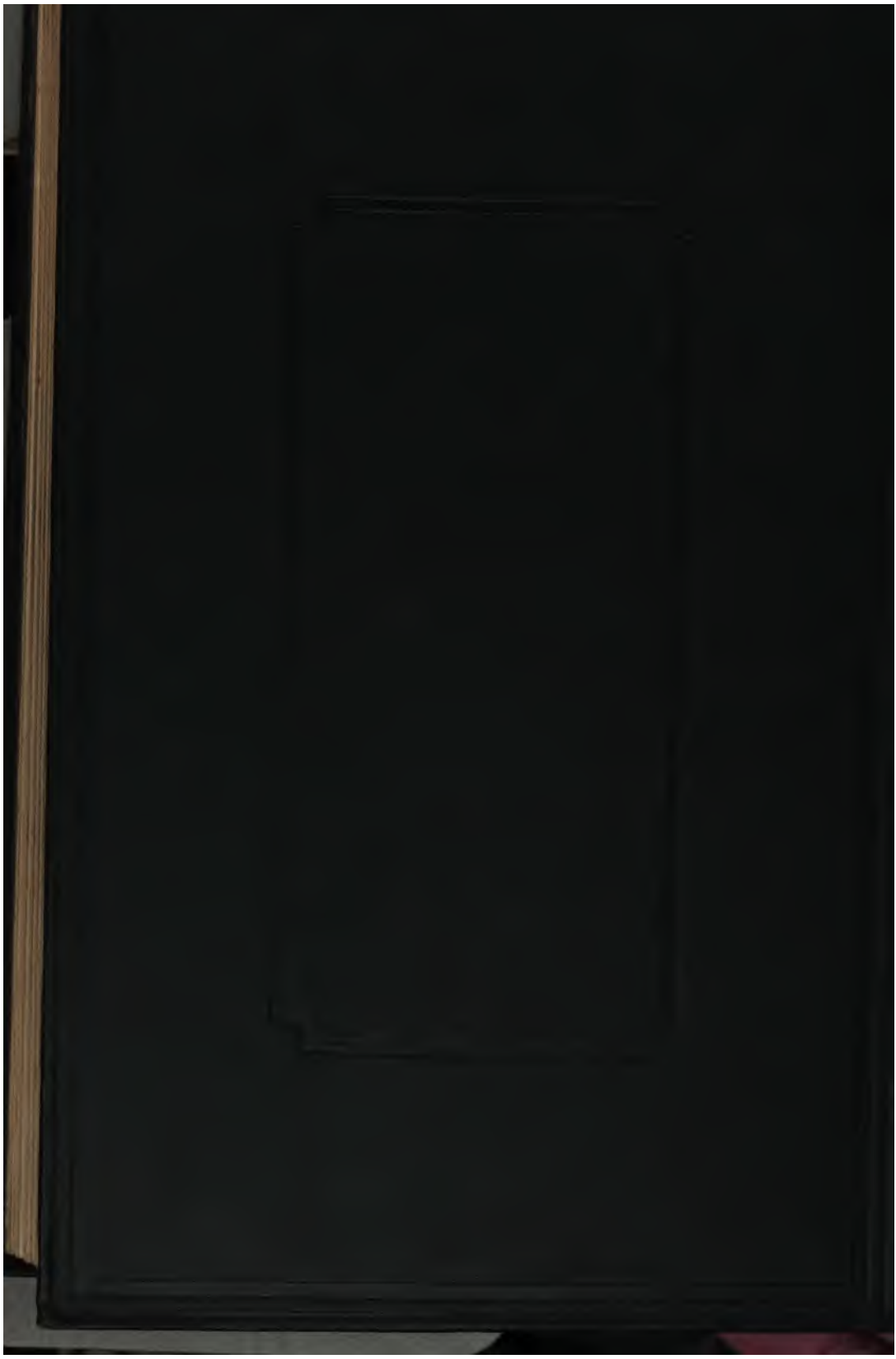
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